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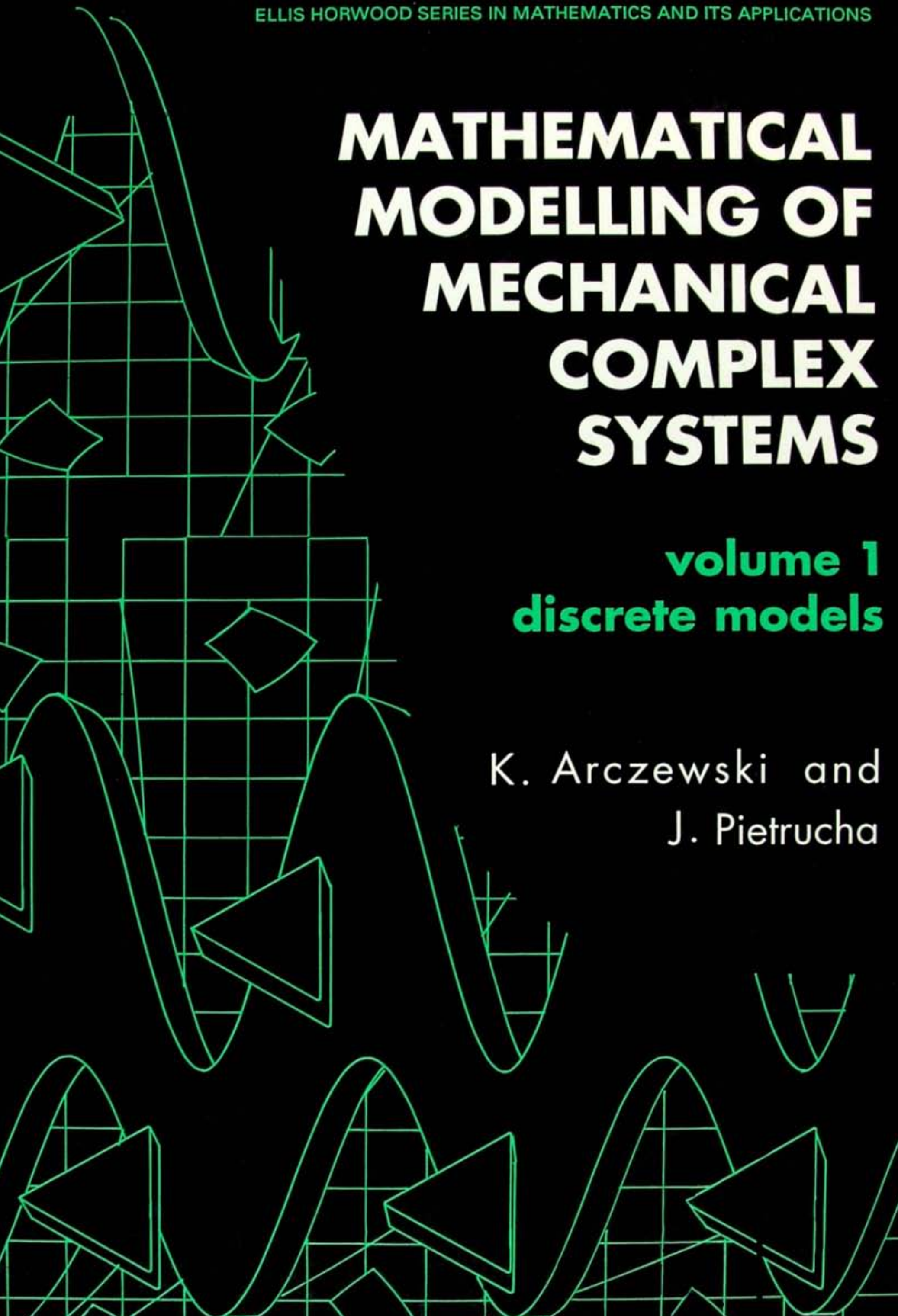

MATHEMATICAL MODELLING OF MECHANICAL COMPLEX SYSTEMS

volume 1
discrete models

K. Arczewski and
J. Pietrucha

MATHEMATICAL MODELLING OF COMPLEX MECHANICAL SYSTEMS
Volume 1: Discrete Models

MATHEMATICS AND ITS APPLICATIONS

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MATHEMATICAL MODELLING OF COMPLEX MECHANICAL SYSTEMS
Volume 1: Discrete Models

Dr K. ARCZEWSKI
Institute of Aeronautics and Applied Mechanics,
Warsaw University of Technology
and
Dr J. PIETRUCHA
Institute of Aeronautics and Applied Mechanics,
Warsaw University of Technology
Translation Editor:
Dr C. M. LEECH
Department of Mechanical Engineering,
Applied Mechanics Division, UMIST

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To our teachers, from whose wisdom we drew, and to our students, to whom we want to transmit our experience

Wisdom begins when you sincerely want to learn. To desire Wisdom is to love her

Wisdom of Solomon 6:17

Table of contents

Preface

This book has arisen from efforts to formulate a rational methodology for modelling complex mechanical systems. A stimulus for these efforts was provided by two observations made during more than twenty years of our didactic and scientific work in the Power and Aeronautical Faculty of the Warsaw University of Technology. The first of these two observations concerns the results of the traditional teaching process that is typical of many technical universities. In this process a student of a mechanical faculty gains knowledge within the confines of numerous separate courses, such as mechanical engineering, thermodynamics, theory of elasticity, fluid mechanics and theory of vibrations, and then of even more specific courses such as flight mechanics, heat transmission, magnetohydrodynamics or aeroelasticity. Knowledge gained in this manner, although quite broad, often lacked adequate integrity. This means that laws that have common roots, or are simply different forms of the same law (e.g. rate of change of linear momentum, or conservation of energy) were perceived by students as rather different, and having not too much in common. A student educated in this manner was usually helpless when confronted with the problem of creating a model of a complex phenomenon, in which one had to apply knowledge exceeding the domain of just one narrow discipline. Similarly, much difficulty was caused by questions of the type:

— Why does the continuity equation, so commonly used in the mechanics of fluids, seem to be completely ignored in the theory of elasticity?
— What do Kepler's second law, which states that the areal velocity of each planet is constant, and the postulate of symmetry for a stress tensor in continuous media have in common?
— In the first course of mechanics two basic laws of particle motion, i.e. the one concerning the time rate of change of angular momentum and that of work and kinetic energy, are usually derived by means of a simple transformation of the law of time rate of change of linear momentum (Newton's second law), and this strongly suggests mutual dependence between these three laws. At the same time these three laws are treated as apparently independent when the mathematical models of heat conducting fluids are formed. The question arises: are the laws of energy conservation, linear and angular momentum change interdependent or not?

Should one, though, be surprised that such questions cause difficulties for students, who are just acquiring knowledge, when many textbooks refer to the symmetry of the stress tensor either as an axiom, a law, or a property of the medium, and do not show that this symmetry is a consequence of the angular momentum balance law and of the assumption that a given medium cannot possess an internal angular momentum (spin)? If, on the other hand, a given medium does not satisfy this assumption, i.e. it *can* store spin, the stress tensor can be arbitrary and the corresponding medium is called a micropolar medium, or alternatively a Cosserat continuum. Moreover, in the literature—and this is the second observation—models are often presented in their final, neat and elegant form. In reality there are many steps, choices, doubts and even errors before the modeller reaches a satisfactory model. These features are usually hidden, and hence the beginner fails to see them. Due to this fact the usefulness of results presented in some scientific papers is either very limited, or not evident to the reader. These two observations formed the motive for creating a facultative course, usually taken at the beginning of the second half of studies, entitled *Physical System Modelling—Theory and Practice*, whose two main but equivalent objectives are: (1) Integration and ordering of knowledge gained to date through, for example, noting common roots of methods developed in particular domains of mechanics; (2) provision of a rational methodology for creating mathematical models for various levels of complex physical phenomena, e.g. of the thermo-aero-elastic vibrations of a turbine blade. The objectives of both volumes of the present book are exactly the same. These objectives have significantly influenced the shape and contents of the book, and will in turn greatly influence the potential readership.

The book is addressed to students in the senior years of technical universities, to engineers and applied scientists. It is also addressed to teachers all over the world who are involved in teaching mathematical modelling. It is assumed that the reader has studied mathematics through a first course in ordinary differential equations and has passed examinations in engineering mechanics and strength of materials. Although it is not absolutely necessary, a reader would find it helpful to have studied fluid mechanics, classical elasticity and thermodynamics. As this implies, although we use multiple notions, information and results coming from the domains of the disciplines mentioned above, we do not explain or derive them. Nevertheless, some of them are accompanied by comments explaining their utility in the process of modelling.

The book is structured into two volumes. The first volume is concerned with discrete models and the second with continuous ones. The present volume is composed of five chapters, the first two of which are of preparatory character.

In the first chapter we explain what modelling is. There we define the notions fundamental to modelling, present modelling against the background of other activities in engineering, describe subsequent stages of modelling and indicate to what extent the purpose of modelling influences the shape of a model. We have also tried to show that each step in the process of modelling requires a deep understanding of a variety of concepts and specific approaches blended with a combination of experience, intuition and foresight. This makes model building both a science and an art.

The art aspect of modelling is difficult to teach in any formal sense. Thus, we have striven to transmit our experience to the students through numerous comments accompanying the notions, concepts and laws presented, as well as through detailed presentation of a wide scope of case studies in which subsequent stages of model creation have been highlighted.

A special difficulty in the teaching of the science aspects is the breadth and depth of material needed for model building. The approach we have taken in Chapter 2 and, to some extent, in subsequent chapters is as follows. We indicate common roots, similarities and a common basis, in the form of balance laws, for a wide range of phenomena acting as subjects for modelling, then we identify five sets of topics whose elements appear in each causal model of a complex phenomenon.

The intention behind Chapter 2 was to create a basis of notions for the purpose of formulating both discrete and continuous models for a broad class of mechanical phenomena. In Chapter 2 we recall several notions of classical mechanics, adding to them comprehensive comments concerning their use in model development. A present-day engineer has to cope more and more often with situations in which notions and methods of classical mechanics are insufficient. Thus, having in mind the needs of modelling of phenomena exceeding the scope of narrowly understood mechanics, we have provided a section in which we define the notions of so-called 'integrated mechanics'.

It appears that in all the domains of classical physics there exist two main different approaches to the formulation of equations describing the dynamic behaviour of a given system. The first approach is based on the use of balance laws, while the second refers to so-called variational principles. And so it is in mechanics. The first approach was established by Sir Isaac Newton in his famous *Philosophiae Naturalis Principia Mathematica* (1687), and then developed by Leonard Euler. This approach is alternatively called geometrical or Newtonian.

The variational approach is intimately linked with the names of Johann Bernoulli, Jean le Rond d'Alembert, Joseph Louis Lagrange and Sir William Rowan Hamilton. It was Lagrange who summarized the subsequent epoch of important discoveries, forming an entirely different (to that founded by Newton) variational approach, in his monumental *Mécanique Analitique* (1788), and its subject matter is called today either Lagrangian or analytical mechanics.

The division sketched out here into two methods of generating equations of motions is completely reflected in the book. Thus, Chapter 3 is devoted to modelling by means of balance laws, and Chapter 4 presents the variational approach to the modelling problem.

In cases when a system becomes complex, due to a large number of elements and a differentiated structure of connections between them (in other words the system has an intricate topology) a graph reflecting the structure becomes an extremely valuable tool of modelling. Methods which make use of graphs (irrespective of the fact that they can also refer to balance or variational methods for the development of mathematical model of system components) are called graph-theoretical methods and two of them are presented in Chapter 5.

Numerous examples are provided in the book, mostly from the field of aircraft engineering, but many examples may be seen to be directly relevant to civil and mechanical engineering. The examples provided, however, are of distinctly different levels of difficulty. Simple examples are used only for the purpose of illustrating the corresponding theoretical considerations. Problems are also considered, though, taken from a wide variety of contemporary engineering situations. Some of them were the subjects of scientific publications, and the others might be. The range of topics appearing as examples in the book is drawn from many university-level disciplines and it is also an objective of this book to demonstrate that the principles of mechanics and the methods of modelling are remarkably universal for such a range of topics, in spite of the impression often made when the topics are studied in separate courses.

In spite of its strong connections with mechanics the present book cannot be regarded to be a manual of either engineering mechanics or of any discipline of mechanics. The book, though, undoubtedly recapitulates and integrates knowledge from several disciplines of mechanics, organizing it around the fundamental subject of modelling. Attentive tracking of the subjects treated in this book would certainly contribute to a deeper understanding of many specific disciplines of mechanics and to a broadening of the field of application of knowledge already acquired.

It is a pleasure to acknowledge the help we have received from different individuals and institutions. We are particularly indebted to Professor R. Gutowski who first introduced us to the methods of analytical mechanics, and to J. T. Szuster who first drew our attention to the important problem of modelling. This work has benefited greatly from discussions with our colleagues Dr F. Dul, Dr Z. Goraj and W. Michalski, to each of whom we express our thanks.

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Krzysztof P. Arczewski
Józef A. Pietrucha