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WITOLD NOWACKI

THERMAL STRESSES IN SHELLS

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SYMPOSIUM ON
NON-CLASSICAL
SHELL PROBLEMS

THERMAL STRESSES IN SHELLS

General Report

Witold Nowacki [1]

Thermal stresses have attracted an increasing attention during the post war years. In this period of time numerous new papers on this subject have been published. The theory of thermal stresses, which previously was regarded as a narrow branch of strength of materials and the theory of elasticity, now became an independent subject connecting the theory of elasticity and thermodynamics. There appear many new synthetic works and monographs concerning the theory of thermal stresses.

The development of the theory of thermal stresses has been marked by a great progress in many fields of technology, during the two last decades. Thus, the numerous problems when the thermal stresses play an important, sometimes the decisive role, occur in the machine building, particularly in steam and gas turbines, in aviation structures, chemical engineering and especially in nuclear engineering.

Before the second world war the main interest in the investigation was concentrated on stationary thermal stresses, now we are mainly faced with the problems of non-stationary flow of heat. The unsteady thermal stresses can be divided into two groups, concerning quasi-static and dynamic thermal stresses respectively . In the case when the variation of temperature is slow in time then deformations as well as stresses may regarded to be quasi-static, thus, the inertia terms appearing in the fundamental equations of the theory of thermal stresses can be neglected. The time variable occuring in the solutions is then regarded as a parameter. When the temperature change is rapid / for example, a body is

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[1] Professor Polish Academy of Sciences, Warsaw

suddenly heated or cooled / the inertia terms cannot be omitted. The body produces the elastic vibrations and the problem becomes a dynamic one.

It is a known fact that a sudden heating of an elastic semi-space produces an elastic wave inward the semi-space. This wave possesses a finite jump of stresses on its front. Similarly, a sudden collapse of a nuclear reactor produces a sudden decay of heat sources in the reactor blanket, and consequently the vibrations of the blanket.

On the other hand, one can observe the trend toward a generalization of the theory of thermal stresses on the bodies of macroscopic and anisotropic structure, on the media of a non-homogeneous material, and on the viscoelastic bodies. When more elevated temperatures are used one must take into account that the properties of the material become functions of temperature.

Finally, in the recent years, the general theory connecting the deformation field with the temperature field has been developed on the basis of irreversible thermodynamic processes. This theory is termed thermoelasticity.

The linear theory of thermal stresses takes the assumptions of the classical, linear theory of elasticity and that the mechanical as well as thermal material properties are constant. These assumptions restrict the obtained solutions to definite ranges of temperature. Thus, the fundamental equations of the theory of thermal stresses in elastic, homogeneous and isotropic bodies take the following form

$$\sigma_{ij} = 2\mu\,\varepsilon_{ij} + (\lambda\varepsilon_{kk} - \gamma T)\delta_{ij} \qquad \gamma = (3\lambda + 2\mu)\alpha_t \qquad /1/$$

$$\varepsilon_{ij} = \frac{1}{2}\,(u_{i,j} + u_{j,i}) \qquad /2/$$

$$\sigma_{ij,j} + X_i = \rho\,\ddot{u}_i \qquad /3/$$

$$T_{,jj} - \frac{1}{\varkappa}\,T = -\frac{Q}{\varkappa} \qquad /4/$$

The first equation is the Duhamel-Neumann relation, the second one gives the relations between strains and displacements. The third equation is the equation of motion while the fourth, the equation of thermal conductivity. Eliminating between these equations the stresses and deformations we obtain the complete set of the equations of the theory of thermal stresses [1]

$$\mu u_{i,kk} + (\lambda+\mu) u_{k,ki} + X_i = \rho \ddot{u}_i + \gamma T_{,i}, \tag{5}$$

$$T_{,kk} - \frac{1}{\varkappa}\dot{T} = -\frac{Q}{\varkappa} \tag{6}$$

The Lame constants μ , λ appearing in the equations are adiabatic. The procedure of solving the equations is the following. We determine from Eq. /6/ the temperature, then knowing the temperature, we find displacements from Eq. /5/. It is apparent that in order to determine the displacements one can use the "body forces" analogy, regarding the terms $\gamma T_{,i}$ as fictitious body forces.

The general structure of the displacement equations of the elastic shells is analogous to Eqs. /5/ . The influence of temperature is responsible for the additional term which implies the non-homogeneity of the equations. Therefore , solving problems of thermal stresses in shells we must, first of all, determine the particular solutions of the non-homogeneous equations of the shell theory .

In the case of the generalized theory, i.e. in thermo-elasticity, the conductivity equation are modified. The Lame constants μ, λ are determined for isothermal state /1/. Then we obtain the following system of equations [2] ;

$$\mu u_{i,kk} + (\lambda+\mu) u_{k,ki} + X_i = \rho \ddot{u}_i + \gamma T_{,i}, \tag{7}$$

$$T_{,kk} - \frac{1}{\varkappa}\dot{T} - \eta \ddot{u}_{k,k} = -\frac{Q}{\varkappa} \tag{8}$$

The above equations are coupled. It is apparent that the body and surface forces produce a field of temperature which effects on the damping and dispersion of elastic waves. Equations /7/ and /8/ can be regarded as general equations of the classical elastokinetics coupling two interacting fields : namely those of deformation and temperature.

Let us return to the equations of the theory of thermal stresses and focus our attention on the un-coupled equation of thermal conductivity /6/. The solution of this equation for shells in the three-dimensional case is difficult since the variables in this equation can be separated only for very few systems of coordinates. Accordingly, there exists a number of approximate methods of solution of this equation. The most frequent method is due to Bolotin [3] and it consists in the assumption that the temperature distribution is linear within the shell thickness:

$$T=\theta_N(\alpha_1,\alpha_2)+\alpha_3\,\theta_M(\alpha_1,\alpha_2) \qquad /9/$$

where

$$\theta_N=\frac{1}{h}\int_{-\frac{h}{2}}^{+\frac{h}{2}} T\,d\alpha_3\,,\quad \theta_M=\frac{1}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}} T\alpha_3\,d\alpha_3$$

Departing from the variational principle Bolotin splits the three-dimensional equation /6/ into a system of two mutually independent, parabolic, two-dimensional equations for the functions θ_N and θ_M . In the particular case of a thin plate the Bolotin equations pass into the known equations of Marguerre [4] .

Another way of an approximate solution is proposed by V.I. Danilovskaja [5] . Assuming a parabolic distribution of temperature within the shell thickness she satisfies the equation of thermal conductivity in approximate way, while the boundary conditions are exactly satisfied on the internal and external surfaces of the shell and approximately satisfied on the remaining boundaries.

Finally, M.A. Brull and J.R. Vinson [6] introduce the approximations analogous to those assumed for the displacements in thin shells. Thus, an approximate form of the conductivity equation is obtained which in the case of shells of revolution can be split into the set of three ordinary equations. In the result we obtain the solution consisting of two parts. The first part corresponds to the solution for a thin plate while the second one gives an additional correcting solution.

So far a number of problems referring to quasi-static thermal stresses in shells has been solved. Thus, H. Parkus [7] considers thermal stresses in shells of revolution subjected to the axially symmetric field of temperature. Departing from the equations of Love and Meissner and generalizing them on the case of thermal effects he has shown the method of determination of the parti ular solutions for a spherical shell and for rotationally symmetric cylindrical and conical shells.

Another method of solution of the same problem has been given by V. M. Maysel [8] . The point of departure of his considerations is the generalization of the Betti reciprocal theorem to the case of thermal stresses. Assuming in this theorem that the action of the temperature and the stresses and the displacement produced by it are the first state of loading and that the action of the unit concentrated force 1_i applied at point $(\vec{\xi})$ and directed along the x_i axis is the second state of loading, we obtain the following formula

$$u_i(\vec{\xi},t) = \alpha_t \int_V T(\vec{x},t)\, \sigma_{jj}^{(i)}(\vec{x},\vec{\xi})\, dV(\vec{x}) \qquad /10/$$

for the required displacement $u_i\,(\vec{\xi},t)$
Here $\sigma_{jj}^{(i)}(\vec{x},\vec{\xi})$ denotes the sum of the normal stresses at point $(\vec{x})$ produced in an elastic body / in isothermal state / by the concentrated force 1_i applied at the point $(\vec{\xi})$. The expression $\sigma_{jj}^{(i)}$ is the Green function. By the application of the procedure described roughly above, V. M. Maysel obtained a number of particular solutions for the shells of revolution and axially symmetric state of stresses. The advantages of this method permit to apply it also to the other types of shells, particularly when the Green function is readily obtainable.

Also the paper of R.P. Nordgren [9] is worth being mentioned. This author departing from the Bolotin equations derived the Green functions for temperature and afterwards for displacements and stresses.

Finally, a very interesting procedure has been devised by O.A. Conrad and W. Flugge [10]. Proceeding from L.H. Donnell's equations and completing them by thermal terms they have arrived at the solutions of these equations for the nucleous of temperature in the exact form. The influence surfaces / Green's functions / thus obtained can be used for the determination of the state of stresses produced by an arbitrary distribution of the temperature.

Numerous papers have been published on the problem of the quasi-static thermal stresses in elastic shells, on the contrary, very few concern the dynamical problems of the thermal effects in shells. So far I have come across three papers referring to dynamic problems. The first of these deals with the vibrations of a cylindrical viscoelastic shell exerted by its sudden heating [11], the second work concerns the forced harmonic vibrations of a thickwalled conical shell [12], while the third paper refers to the propagation of the elastic wave in a spherical shallow shell. The fourth paper [14], presented on this Symposium, deals with the propagation of the elastic wave in an infinite thin-walled conical shallow shell.

In my opinion, the development of this trend of investigation is necessary because of its practical significance. A sudden heating or cooling of a shell exerts the vibrations resulting in a rapid increase of stresses considerably exceeding the values of the stresses in quasi-static problems.

Also the investigation of the process of propagation of the elastic wave produced in shells by a thermal shock is important from the point of view of theory as well as for practice. The structure of this wave is entirely different from that produced by mechanical disturbances. It always consists of two terms, the first one of diffusive character while the second one of the character of an elastic wave.

Also interesting and quantitatively new results can be expected in examining the dynamic thermoelastic problems in shells when the coupling of the fields of deformation and temperature is taken into account. According to my knowledge, these problems have not been so far discussed. Hitherto, the investigations have been restricted to the propagation of waves in the bodies of simple geometric forms / infinite space, semi-space, layer, infinite cylinder /. Recently, the vibration problems of plates and discs have been examined [15], [16].

The general theories accounting the effects of the external and body forces have already been established also for anisotropic elastic shells. The generalization of these theories to the case of thermal stresses presents no difficulty. The practice and the growing application of the shells with properties of macroscopic anisotropy in many types of structures exert influence on the development of the theory of thermal stresses in this kind of shells.

The further, anticipated trend of the investigations will presumably be the study of thermal stresses in non-homogeneous elastic shells. The non-homogeneity can be discontinuous / the layers of various density / or can vary continuously. In this latter case the density as well as the material mechanical and thermal properties are the functions of position. Hence, the equations of the theory of shells and the equation of the thermal conductivity remain linear but of variable coefficients.

Let us observe that the homogeneous structure of a shell may become non-homogeneous at elevated temperatures. In this instance the thermal and mechanical coefficients become the functions of temperature. For a stationary flow of heat the equation of thermal conductivity becomes nonlinear whereas the displacement equations remain linear but with the position dependent coefficients. In this branch there exist a few works referred to one-dimensional cases [18], [19] applied to a thick-walled layer, a spherical shell and a cylindrical shell.

This trend of investigations can be extended, first of all, to the shells of revolution which are kept in an axially-symmetric temperature field.

At the elevated temperature many structural materials exhibit visco-elastic properties / creep /. Experiments show that the materials subjected to a constant load and temperature and kept in this state exhibit an increase of deformations with time. The redistribution of internal forces and deformations developes in time. The model of a perfectly elastic body cease to be sufficient for the description of the stress and strain state. Accordingly, designing the shells subjected to the action of the elevated temperatures one must take into account the rheological effects. Examining the creep of a material one also must have in view the fact that the coefficients characterizing the viscosity of the material are very sensitive to the increase of temperature. The dependence of the viscosity coefficient on the temperature is much more pronounced than that of the elastic modulus or the coefficient of linear expansion. Consequently, the state of stresses in shells can be described by the non-linear theory of viscoelasticity. Unfortunately, no works have been published in this field; the known results are based on the linear theory of elasticity under the assumption that the material coefficients are independent of the temperature.

The investigations accounting for the effect of temperature on the stresses in the physically and geometrically nonlinear shells will certainly be interesting and of practical value. Here, under the notion "physically nonlinear" shells we understand the shells made of a material for which the relations between the stresses, strains and the temperature are no longer linear. The assumption of "physical nonlinearity" leads to the nonlinear displacement equations.

Under the notion of "geometrically nonlinear" theory of shells we understand the theory for which the linear assumptions of the Duhamel--Neumann relations remain valid but finite displacements are taken into account. It is apparent that one can imagine the shells which are simultaneously physically and geometrically nonlinear.

Finally, let us observe that for an isotropic body subjected to the action of temperature the deformation consists of two parts

$$\varepsilon_{ij} = \varepsilon^{o}_{ij} + \varepsilon'_{ij} \qquad /11/$$

where $\varepsilon^{o}_{ij} = \alpha_t T \delta_{ij}$ are the initial deformations while ε'_{ij} are the deformations connected with the stresses σ'_{ij} for which the compatibility equations hold.

The thermal deformations $\varepsilon^{o}_{ij} = \alpha_t T \delta_{ij}$, however, are restricted only to the initial normal deformations. These deformations are only a particular case of more general distorsions producing in the body the state of initial stresses. The distorsion may develop in many ways as shrinkage deformations, initial deformations which arise during the process of forming the shells, and finally initial stresses resulting from the inaccuracies of the assemblage process.

Thus, there exists a possibility to extend the theory of shells to the more general state of initial deformations, as compared with thermal deformations.

Though the papers presented to this Symposium concern only some of here roughly described directions, they may serve as an illustration of the contemporary trends in the theory of thermal stresses in shells. The pronounced majority of the papers refers to the linear theory of elastic shells subjected to the stationary or unsteady temperature fields. Static and quasi-static problems are prevailing, only one paper is devoted to the propagation of stresses in a shallow shell. Two works concern the nonlinear theory of shells.

I. STATIC AND QUASI-STATIC PROBLEMS

1. K. Apeland, Bending of orthogonally anisotropic and asymetrically ribreinforced shallow shells subjected to temperature changes and surface loads.

In this paper the author generalises the equations theory of the large deformations of isotropic shells / Marguerre's equations / to the case of shells composed of orthotropic layers. The influence of temperature, the distribution of which is linear within the thickness of the layers, is taken into account.

After the linearization of the equations a uniform shell of orthogonal anisotropy is considered.

Making use of the Huber-type approximation for the shear modulus and assuming that the ratio of the coefficients of the thermal expansion in both directions of orthotropy are equal to the ratio of the appropriate elastic moduli the author obtains the model of a shell the deformation of which is described by the equations analogous to those for isotropic shells. Singular solutions for translational shells are discussed with reference to local normal loads and hot spots. This latter problem is examined in details for a cylindrical shell, for an almost cylindrical shell, and for a shell of the form of elliptical paraboloid. The approximate solutions for these shells are given in a closed form.

2. E.M. Barrowman, R.M. Kenedi, The use of the influence technique in the analysis of thermal effects in shell

Similarly as in case of plates where by means of the influence surfaces deflection and internal forces can be determined, the analogous method can be applied to shells. The authors extends the method of the influence line valid for the external loadings of shells, devised by R.M. Kenedi, to the case of thermal effects. The use is made of the Green functions, obtained by N. Flügge and D.A. Conrad for a nucleous of temperature acting in a shallow shell. The application of the influence line technique is explained on an example of a spherical shell. The results are compatible with those obtained from the experiment.

3. C.E. Caliari, Effets thermiques et phenomenes d'adaptation dans les voiles cylindriques minces

The paper is an extension and application of the previous author's works to the problems of thermal stresses and distorsional stresses produced by the excess of the elastic limit in cylindrical shells. The author gives a variant of an approximation method assuming that the shell may be represented by an arch in the transversal direction and by a beam in the longitudinal direction. The accuracy of the method can be estimated by comparison of the obtained results with the exact solutions / for example, on the basis of the Donnell theory / .

4. F. Essenburg, Thermal elastic cylindrical shells with surface constraints

The author considers a thin axially symmetric cylindrical shell subjected to the action of temperature. A part of the surface is constrained from deflection by a contact with a rigid concentric cylindrical surface. In order to determine the contact forces the more accurate theory than the elementary one, namely, the theory which accounts for the effect of transverse shear deformation, is taken into consideration.

The solution to the problem presents serious difficulties since the width of the contact is unknown and has to be determined from a transcendental equation.

5. J.E. Goldberg, J.L. Bogdanoff, and D.W. Alspaugh, Stresses in spherical domes under arbitrary loading including thermal and shrinkage effects

The purpose of the paper is to present a convenient method of analysis of spherical domes, appropriate for application to electronic computers. The authors use the Love and Reissner forms of the equations of shells. In contrast to the common reduction of the general equations to the system of three displacement equations, here, the equations are transformed into a system of eight first order ordinary differential equations in the eight intrinsic dependent variables. These variables are the three components of displacement of the middle surface, the rotation of the tangent to the generatrix, the normal force and the bending moments in the direction of the generatrix, and shear stresses in both directions. The advantages of this method are: a simple form of the boundary conditions and the possibility of the shell thickness variation in the meridional direction.

The intrinsic dependent variables are represented in the form of single trigonometric series, and consequently eight ordinary differential equations for the eight harmonics are obtained. These equations can be integrated by means of the Runge-Kutta fourth order process.

6. M. Gradowczyk, On thermal stresses in thin shallow shells

The point of departure of the considerations are the non-linear equations of Marguerre for shallow shells. The terms corresponding to thermal effects are included. These equations are linearized and adopted to translational shells, simply supported on the boundary. The author gives the solution of these equations in the form of a double trigonometric series for parabolic shells with rectangular boundary, under the assumption that the temperature in the direction of the thickness varies linearly. The author shows that in the case when the temperature is linear only along the thickness then the problem can be reduced to homogeneous equations with non-homogeneous boundary conditions.

In the second part of the paper the author examines a spherical shallow shell in axially symmetric state of stresses. The general solution of the problem is represented by the Thomson functions while the particular solution in the form of an infinite power series. Finally, the approximate asymptotic solution to the problem is given.

Interesting and extensive numerical work allows to estimate the solutions based on approximate models, on the membrane theory, and on the inextensional theory of flexure of shells.

7. O. Matsuoka, Approximation theory of thin shells

In the first part of the paper the author derives, in the tensorial form, the fundamental geometric and static equations and the relations between the stresses and deformations as well as the compatibility equations. The Love-Kirchhoff assumptions are employed.

In the second part of the paper, the general approximate equations of the theory of shells are specialized to the case of spherical shells.

8. Z. Mazurkiewicz, Shallow shells with variable curvatures subjected to non-homogeneous temperature field

The author employes the equations of the technical theory of shells due to V.Z. Vlasov. The V.Z. Vlasov assumption that the curvature is constant, is relaxed and now the curvature is assumed to be variable. The bending of the shell is produced by the temperature $T = x_3 \, T(x_1, x_2)$ linearly varying in the direction of the shell thickness.

The finite double Fourier transform is performed upon the equations of the technical theory of shells. The author assumes that the moments and normal deflections are prescribed on the rectangular boundary of the shell.

After performing the transform the system of two differential equations for the function of deflection w and the function of stresses f is reduced to an infinite system of algebraic non-homogeneous equations. Here, besides the quantities w_{nm} , φ_{nm} / the transforms of functions w and φ / occur the coefficients $B^1_{nm}, B^2_{nm}, H_{nm}$ which are dependent on the boundary conditions. In the case when the shell is simply supported the coefficients $B^1_{nm}, B^2_{nm}, H_{nm}$ vanish. In the other cases / e.g. the clamped boundary / the coefficients must be eliminated from the boundary conditions.

The method presented is very general and was successfully applied to problems of the plates theory.

It would be desirable to verify the convergence of the solution, at least in the simplest case of the shell simply supported, in spite of the serious difficulties which may be encountered.

II. DYNAMIC PROBLEMS OF THE THEORY OF THERMAL STRESSES

1. C.S. Hsu and P.M. Naghdi, Propagation of thermoelastic waves in a shallow conical shell

The authors depart from the linearized Marguerre equations in the special case of a conical shell. The dynamic problem of the theory of stresses is considered. The field of temperature is assumed to be axially symmetric. The considered shell is shallow and infinite. If a plane π , perpendicular to the axis of the cone, passes through the apex then the distance from a point z on the generatrix of the cone to plane π is equal $z = -\varepsilon r$, where ε is small as compared with unity. This assumption permits to apply the perturbation method to the / Bolotin's / equations of thermal conductivity and to the displacement equations. The dependent functions may be represented by the series

$$u = u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \dots \qquad /1/$$

For $\varepsilon \to 0$ the equations of the conical shell degenerate to equations valid for an infinite plate. The authors restrict their considerations to two terms of the series /1/ .

Two particular cases are discussed. The first case deals with an axially symmetric nucleous of temperature on the surface of the conical shell, the second with a source of heat concentratet at the apex. It is assumed that the temperature varies harmonically in time.

The solution obtained in the paper consists of two parts. The first part, given in a closed form, refers to an infinite plate, the second part gives the complementary term in the form of an improper integral, which, for large values of the argument, can be replaced by the asymptotic expression. The paper is very interesting from the standpoint of theory and gives a significant contribution to the development of the dynamic problems of the theory of theory of thermal stresses.

III. NONLINEAR THEORY OF THERMAL STRESSES IN SHELLS

1. Cz. Woźniak, Momentless thermal states in the nonlinear theory of thin shells

The author envisages the thin shells which, as a result of heating, suffer deflections whose magnitude is comparable with the dimensions of the shell. Among the numerous states of deformation of the shell / simply supported on the boundaries / the author selects these states, which in the effect of heating, develop the momentless state of stress. It turns out that the momentless states may be simultaneously the stressless states, it may so occur when the temperature of the medium is constant. Two conditions of the momentless state are given. The first one determines the shape of the undeformed shell and gives the relation between the difference of temperature and the value of the mean curvature. The second condition gives the relation between the deformed shape of the shell and its initial form. The discussion of these conditions and their dependence on the sign of the Gaussian curvature concludes the paper.

IV. BUCKLING OF SHELLS

1. M. Kozarov, Thermal stability of structurally orthotropic cylindrical shells

The author considers the differential equations of cylindrical shells in the case of the static buckling. The effect of heating is included in terms T_1^o , T_2^o and S_2^o which express the influence of longitudinal and shear forces. In the case when the temperature T varies in the directions x, y and the normal. to the surface the author gets a system of linear homogeneous differential equations with variable coefficients. These equations are solved for a simply supported shell by means of the double Fourier series. The problem has been reduced to the infinite system of linear homogeneous equations with the unknown parameter, namely, the critical force of the system. Equating the determinant of the system to zero the author obtains the condition of buckling of the structure. Two special cases and the possibility of simplification of the results are examined.

V. MISCELLANEOUS PROBLEMS

1. S. Kaliski, Magneto-elastic vibrations of perfectly conductive cylindrical shells in a constant magnetic field

This paper obviously does not fall within the framework of the considered thermal stresses. It is discussed only because of the formal analogy existing between this problem and thermoelasticity.

If a strong initial and constant magnetic field $\vec{H}$ arises in an elastic body then the loading, varying in time, produces an electromagnetic field in the body as well as in the surrounding vacuum. The additional terms, the Lorentz body forces, must be added in the displacement equations of the elasticity theory . Also, additional terms must be taken into account in the Maxwell equations of the electrodynamics. These terms take the form of the time derivatives of the displacements.

In the case of a perfect electrical conductor the displacement equations have the following form

$$\mu \nabla^2 \vec{u} + (\lambda + \mu)\, \mathrm{grad}\, \mathrm{div}\, \vec{u} + \vec{X} = \rho \ddot{\vec{u}} + \frac{\mu_0}{4\pi c}\left[\mathrm{rot}\,\mathrm{rot}(\dot{\vec{u}} \times \vec{H})\right] \times \vec{H}$$

It is apparent from these equations that the introduction of the constant initial magnetic field $\vec{H}$ developes the anisotropy of the material which vanishes when the magnetic field is removed.

Departing from the linear theory of shells, S. Kaliski adds to the right side of the equations the terms coupling the deformation and electromagnetic fields. The author makes use of these equation solving an example of the forced radial vibrations of an infinite cylindrical shell. The vibrations are exerted by external tractions varying harmonically in time. It is apparent that the deformation is accompanied by the radiation of electromagnetic waves into the vacuum.

The author claims that the problems of the magnetoelastic cylindrical shells may have the applications in the aeromagneto-flutter of the tubes conducting a ionized liquid, and in plasmatrons and plasmaguides.

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