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THERMAL EXCITATIONS IN COUPLED FIELDS

S.Kaliski, W.Nowacki /Warsaw/

Introduction

The problems of thermal excitations in the theory of coupled fields belong to the subject called briefly the thermoelectromagnetoelasticity and constitute an extension of the classical thermoelasticity to the phenomena of coupling with the electromagnetic field. Evidently we may also speak more generally about thermoelectromagnetoplasticity, at present however we lack serious papers in this field and consequently we confine ourselves to the thermoelectromagnetoelasticity.

The richness of the equations and hence the variety of solutions and physical phenomena create a wide field of practical applications for various particular forms of the theory which is now in the initial stage of its development and has so far comperatively few effective solutions of the fundamental problems ; the same concerns to an even greater degree the practical problems.

Approximately in the beginning of the sixties we observed a rather rapid and intensive development of the theory of coupled fields containing the problems of magnetoelasticity of media without and with spin. We witnessed also a birth of electroelasticity /piezoelectricity/ in connection with the development of the nonlinear mechanics of continuous media on one hand and on the other hand in connection with the discovery of new practical possibilities in the theory of ultrasonics and hypersonics in the application of semiconductors.

At the same time there began a rather slower development of the thermoelectromagnetoelasticity, both in the field of thermomagnetoelasticity and electroelasticity.

The contribution of the Polish school to the development of the theory of coupled fields is considerable and sometimes pioneering, in particular to the thermoelectromagnetoelasticity. The purpose of the present paper is a brief outline of the existing achievements and an indication of the possible trends and new problems.

The thermoelectric, magnetoelectric effects and the fundamental physical relations of these effects to the elastic field have been known in physics for a long time /see e.g. [1], we shall not therefore deal with them here.

On the other hand, from the point of view of the thermal, elastic electromagnetic fields the problem was elaborated, as mentioned before in the beginning of the sixties. A systematic account of the equations of thermomagnetoelasticity was presented in papers [2] and [3].

The thermodynamic foundations of the thermoelectromagnetic processes on the basis of the thermodynamics of irreversible processes were systematized in the monograph [4] while the complete system of equations of thermoelectromagnetoelasticity based on a thermodynamic analysis was given in [5].

In [5] we also deduced new equations of the so-called "wave" theory of thermoelectromagnetoelasticity, in which on the basis of a modification of the Fourier law we constructed approximate phnomenological equations of thermoelectromagnetoelasticity characterized by a finite velocity of propagation of thermal electromagnetic and elastic excitations.

The equations of thermopiezoelectricity including a thermodynamic analysis were presented systematically in the paper [6]. Further, in [7] Eringen deduced the equations on the nonlinear theory of the thermoelectromagnetoelastic field describing finite deformations.

Parallelly to the fundamental papers concerning the construction of the equations of the coupled thermoelectromagnetoelastic fields, there began to appear particular papers concerning either certain definite solutions of general theorems.

Thus, the fundamental one-dimensional problem for the elastic semispace subject to a thermal shock on its surface, in the casecf a perfect and real conductor in the magnetic field, was solved in the papers [7] and [8]. In [9], [10] and [11] particular problems were solved for the one-dimensional periodic plane waves in thermomagnetoelasticity, for perfect and real conductors, whereas in [18] the same problems were examined for thermopiezoelectricity. In the papers [19] and [20] variational theorems were deduced for the perfect and real elastic conductors, while in [21] and [22] the reciprocity theorems for the wave equations of the thermomagnetoelasticity and thermopiezoelectricity were investigated. In the paper [23] the problem of acceleration waves in the nonlinear thermomagnetoelasticity was examined.

Besides papers of the field-mathematical nature a number of papers appeared of practical character, e.g. [24], where the problem of conversion of the energy of laser radiation into the heat and elastic energies was considered, and other papers of this type. Further some papers were published of physical nature concerning the character of the physical relations between the fields, for instance [25] and [26] where the construction of the equations of thermomagnetoelasticity was presented, for media without and with spin, and further papers [27], [28] in which it was proved that under the influence of the temperature gradient at sufficiently low temperatures there may exist new types of thermomagnetic waves with specific stability properties. These problems are included into the scope of plasma dynamics /also in solids/.

For obvious reasons it is difficult in this lecture to embrace this wide field of problems. Thus, we confine ourselves first of all to the papers on thermoelectromagnetoelasticity developed on the basis of the mechanics of continuous media, in its specific language and methods. In the last section we mention qualitatively some wider aspects of the problem and further trends.

The structure of the present lecture is the following.

After a general introduction in § 2. we present the general linear system of equations of thermoelectromagnetoelasticity including the thermodynamic foundations, i,e, the system of equations of thermomagnetoelasticity, thermopiezoelectricity, thermomagnetomicroelasticity and the wave equation of the thermoelectromagnetoelasticity.

In § 3. we deal with the generalization to the nonlinear case of the equations of thermomagnetoelasticity.

In § 4. we review the fundamental solutions of the thermoelectromagnetoelasticity and finally in § 5. we briefly consider further problems in thermoelectromagnetoelasticity.

2. General systems of equations of thermoelectromagnetoelasticity

Thermodynamic foundations

In this section we briefly present the equations and the relevant thermodynamic discussion of the linear theories of thermomagnetoelasticity, thermopiezoelectricity, extensions of these equations to wave phenomena and the equations of thermomagnetomicroelasticity. We do not examine the assumptions, referring the reader to the papers [5] and [26].

2.1. The equations of thermomagnetoelasticity of conductors

According to [2], [3] and [5] the equations of thermomagnetoelasticity of real anisotropic conductors in a magnetic field have the form in the RMKS system

$$\mathrm{rot}\,\vec{h} = \vec{j} + \dot{\vec{D}}, \quad \mathrm{rot}\,\vec{E} = -\dot{\vec{b}}, \quad \mathrm{div}\,\vec{b} = 0, \quad \mathrm{div}\,\vec{D} = \rho_e$$

$$\rho\ddot{u}_i = \sigma_{ik,k} + (j \times B_0)_i + \rho_e E_i + P_i \qquad /2.1/$$

$$\beta\dot{T} + \lambda_{ij}\dot{e}_{ij} - (k_{ij}T_{,j})_{,i} + (\pi_{ik}j_k)_{,i} = F$$

Here

$$b_i = u_{ik}h_k \,; \quad D_i = \varepsilon_{ik}\left[E_k + (\dot{\vec{u}} \times \vec{B}_0)_k - \frac{1}{C^2}(\dot{\vec{u}} \times \vec{H}_0)\right]$$

$$\sigma_{ik} = E_{ikmn}e_{mn} - \alpha_{ik}T - \int_0^t R_{ikmn}(t-\tau)\left[e_{mn}(\tau) - \alpha_{0ik}T(\tau)\right]d\tau \qquad /2.2/$$

$$j_i = \eta_{ik}E_k - \varkappa_{ik}T_{,k} + \eta_{ik}(\dot{\vec{u}} \times \vec{B}_0)_k + \rho_e\dot{u}_i$$

where

$$\alpha_{ik} = E_{ikmn}\alpha_{0mn}\,, \qquad e_{ik} = \frac{1}{2}(u_{i,k} + u_{k,i})$$

The system of equations /2.1/ consitutes the system of the Maxwell equations, elasticity equations and the heat conduction, respectively, with the appropriate couplings, while Eqs /2.2/ are the equations of state and the relations between the generalized forces and fluxes, respectively. The expressions $\rho_e E_i$, $\rho_e\dot{u}_i$ are to be neglected in accordance with the linearization. The notations for the temperature, displacements and the field components are the usual ones.

The tensors have the following meaning:

σ_{ik} - stress tensor

E_{ikmn}, R_{ikmn} - tensors of elastic and relaxation moduli

$\mu_{ik}, \varepsilon_{ik}$ - tensors of magnetic and electric permeabilities

η_{ik} - tensor of electric conductivity

$\alpha_{ik}, \alpha_{0ik}$ - tensors of thermal expansion

k_{ik} - tensors of thermal conduction

λ_{ik} - tensor describing the influence of the strain on the temperature field

π_{ik} - tensor describing the influence of the current intensity on the heat flux

$\varkappa_{ik}$ - tensor connecting the temperature gradient with the electric current

P_i - vector of body forces

f - density of thermal sources

$\vec{H}_0, \vec{B}_0$ - vectors of the initial magnetic field and the magnetic induction.

In the isotropic case the systems of equations /2.1/ and /2.2/ after certain transformations take the form

$$\mathrm{rot}\,\vec{h} = \vec{j} + \varepsilon\dot{\vec{E}} + \frac{\varepsilon\mu c^2 - 1}{c^2}\left(\ddot{\vec{u}} \times \vec{H}_0\right)$$

$$\mathrm{rot}\,\vec{E} = -\mu\dot{\vec{h}} \qquad /2.3/$$

$$\rho\ddot{\vec{u}} = G\nabla^2\vec{u} + (\lambda_0 + G)\,\mathrm{grad\,div}\,\vec{u} + \rho_e\vec{E} + \frac{1}{c}\left(\vec{j} \times \vec{B}_0\right) + \vec{P} - 3\alpha_0 K\,\mathrm{grad}\,T$$

$$\beta\dot{T}+\lambda\, div\,\vec{\dot{u}}+\pi\, div\vec{j}-k\nabla^2 T=f$$

$$\vec{j}=\eta\left[\vec{E}+(\vec{\dot{u}}\times\vec{B}_0)\right]-\varkappa\, grad\, T+\rho_e\vec{\dot{u}}$$

where $K=\lambda_0+\frac{2}{3}G$

In the coordinate system connected with the medium the constitutive equations /2.2/ take the form

$$b_i^0=\mu_{ik}h_k^0\ ;\qquad D_i^0=\varepsilon_{ik}E_k^0\ ;\qquad j_i^0=\eta_{ik}E_k^0-\varkappa_{ik}T_{,k}^0$$

To derive the symmetry and energy relations we briefly discuss the first and second laws of thermodynamics.

The energy equation for the thermomagnetoelastic field is the following:

$$-\int_A g_i\,dA_i-\int_A N_i\,dA_i-\int_A \rho w\dot{u}_i\,dA_i+\int_A \dot{u}_i\sigma_{ij}\,dA_j-$$
$$-\frac{\partial}{\partial t}\int_V \rho w\,dV-\frac{\partial}{\partial t}\frac{1}{2}\int_V (H_iB_i+E_iD_i)\,dV=0 \qquad /2.4/$$

where

$$H_i=H_{0i}+h_i\,,\qquad B_i=B_{0i}+b_i\,,\qquad w=\frac{1}{2}\dot{u}_i^2+w_0$$

w - internal energy mechanical per unit mass ,

N_i - Umov-Pointing vector , q_i - heat flux.

Making use of the Gauss formula, the expression for the Lorentz force and the Fourier law we have

$$q_i=-h_{ij}T_{,i}+\pi_{ik}j_k \qquad /2.5/$$

Bearing in mind that the energy equation of the electromagnetic field independently of the contribution of the mechanical and thermal fields has the form

$$\int_A N_i\, dA_i + \frac{1}{2}\frac{\partial}{\partial t}\int_V (H_i B_i + E_i D_i)\, dV + \int_V E_{ij} j_i\, dV = 0 \qquad /2.6/$$

and taking into account the complete linearization we can represent the first law of thermodynamics for the thermomagnetoelasticity in the form

$$\rho W_0 = E_i j_i + \sigma_{ij}\epsilon_{ij} - (F_i + P_i)\dot{u}_i + (h_{ij} T_{,j})_{,i} - (\pi_{ik} j_k)_{,i} \qquad / 2.7/$$

where

$$F_i = \rho_e E_i + (\vec{j} \times \vec{B})_i$$

We now proceed to the second law:

$$\frac{Q}{T} \leqslant \frac{\partial}{\partial t}\int_V \rho s\, dV + \int_A \rho s \dot{u}_i\, dA_i \qquad /2.8/$$

Here s - is the entropy per unit mass, Q - quantity of heat entering V . Introducing the density of entropy production σ we can write

$$\int_V \dot{\sigma}\, dV - \int_A \frac{q_i}{\theta}\, dA_i = \frac{\partial}{\partial t}\int_V \rho s\, dV + \int \rho s \dot{u}_i\, dA_i \qquad /2.9/$$

where

$$\theta = T_0 + T, \qquad \frac{T}{T_0} \ll 1$$

Hence we obtain

$$\dot{\sigma} - \left(\frac{q_i}{\theta}\right)_{,i} = \rho \dot{s} \qquad /2.10/$$

and independently

$$\rho \dot{w}_0 = \rho \theta \dot{s} + \sigma_{ij} \dot{e}_{ij} \qquad /2.11/$$

After appropriate transformations /2.7/ can be written in the the form

$$\rho \dot{w}_0 = E_{oi} j_{oi} + \sigma_{ij} \dot{e}_{ij} + (h_{ij} T_{,j})_{,i} - (\pi_{ik} j_k)_{,i} \qquad /2.12/$$

/for $P_i = f = 0$ / where

$$\vec{E}_0 = \vec{E} + (\vec{u} \times \vec{B}_0)\ ; \qquad \vec{j}_0 = \vec{j} + \rho_e \vec{u} \approx \vec{j}_0 \qquad /2.13/$$

Thus, from /2.11/ and /2.12/ we obtain

$$\rho \dot{s} = \frac{E_{oi} j_{oi}}{\theta} + \frac{(h_{ij} T_{,j})_{,i}}{\theta} - \frac{(\pi_{ik} j_{ok})_{,i}}{\theta} = \frac{E_{oi} j_{oi}}{\theta} - \left(\frac{q_i}{\theta}\right)_{,i} - \frac{q_i T_i}{\theta^2} \qquad /2.14/$$

and hence, making use of /2.10/

$$\dot{\sigma} = \frac{E_{oi} j_{oi}}{\theta} - \frac{q_i T_{,i}}{\theta^2} \qquad /2.15/$$

where $E_{oi} j_{oi}$ is the term describing the Joule heat.

If in /2.15/ we choose in an appropriate manner the forces and the fluxes, making use of the Onsager principle we arrive at the symmetry relations.

We set

$$j_{oi} = \eta_{ik} \theta \frac{E_{ok}}{\theta} - \varkappa_{ik} \theta^2 \frac{T_{,k}}{\theta^2} = \bar{\eta}_{ik} \frac{E_{ok}}{\theta} - \bar{\varkappa}_{ik} \frac{T_{,k}}{\theta^2} \qquad /2.16/$$

$$q_i = \pi_{il}\,\eta_{ik}\,\theta\,\frac{E_{0k}}{\theta} - \theta^2(h_{ik} + \pi_{il}\,\varkappa_{lk})\,\frac{T_{,k}}{\theta^2} = \bar{\gamma}_{ik}\,\frac{E_{0k}}{\theta} - \bar{\nu}_{ik}\,\frac{T_{,k}}{\theta^2}$$

Then the symmetry relations take the form

$$\bar{\eta}_{ik}(\vec{B}) = \bar{\eta}_{ki}(-\vec{B}); \quad \bar{\nu}_{ik}(\vec{B}) = \bar{\nu}_{ki}(-\vec{B}); \quad \bar{\gamma}_{ik}(\vec{B}) = \bar{\gamma}_{ki}(-\vec{B}) \qquad /2.17/$$

For the rate of entropy we have the expression

$$\dot{\sigma} = \bar{\eta}_{ik}^{-1}\,\dot{j}_{0i}\,\dot{j}_{0k} + \frac{1}{\theta^4}\left(\bar{\nu}_{ik} - \bar{\varkappa}_{li}\,\bar{\varkappa}_{sk}\,\bar{\eta}_{sl}^{-1}\right) T_{,i}\,T_{,k} \qquad /2.18/$$

which implies the conditions of its positiveness.

In the isotropic case we have

$$\dot{\sigma} = \frac{\dot{j}_{0i}^{2}}{\bar{\eta}} + \frac{1}{\theta^4}\left(\bar{\nu} - \frac{\bar{\varkappa}^2}{\bar{\eta}}\right) T_{,i}^{2} \qquad /2.19/$$

where $\nu \cdot \theta \frac{\varkappa^2}{\eta} = h$, which requires the positiveness of and η .

2.2. Thermopiezoelectricity

In the case of dielectrics the coupling of which with the elastic and electric fields occurs by means of the piezo - effect the equations of the coupled fields of thermopiezoelectricity in the linearized form take the form [5], [6]

$$\mathrm{rot}\,\vec{H} = \dot{\vec{D}}; \qquad \mathrm{rot}\,\vec{E} = -\dot{B}; \qquad \mathrm{div}\,\vec{B} = 0; \qquad \mathrm{div}\,\vec{D} = \rho_e$$

$$\rho\ddot{u}_i = \sigma_{ik,k}\,, \qquad (k_{ij}T_{,j})_{,i} = T_0\dot{s} \qquad /2.20/$$

where

$$\sigma_{ik}=\overset{ETH}{E_{ikmn}}e_{mn}-\overset{EH}{\alpha_{ik}}T-\overset{TH}{\gamma_{ikl}}E_l$$

$$D_i=\overset{TH}{\gamma_{ikl}}e_{kl}+\overset{TeH}{\varepsilon_{ij}}E_j+\overset{eH}{p_i}T \qquad /2.21/$$

$$s=\overset{EH}{\alpha_{ik}}e_{ik}+\overset{EH}{p_i}E_i+\overset{eEH}{\beta_0}T$$

$$B_i=\overset{TeH}{u_{ik}}H_k$$

The upper indices denote the thermodynamic constancy of the quantities for which the tensor is defined. In the above equations the notations are analogous to those in the proceding section, and moreover, the following have been introduced:

$\overset{TH}{\gamma_{ikl}}$ - tensor of piezoelectric constants at a constant temperature and magnetic field,

p_i - vector connecting the electric induction with the temperature.

Substituting from /2.21/ we write in full the system /2.20/

$$E_{iklm}u_{m,ni}-\gamma_{ikl}E_{l,i}-\alpha_{ik}T_{,i}=\rho\ddot{u}_k$$

$$\varepsilon_{ijk}H_{k,j}=\frac{1}{2}\gamma_{ikl}(\dot{u}_{k,l}+\dot{u}_{l,k})+\varepsilon_{ij}\dot{E}_j+p_i\dot{T} \qquad /2.22/$$

$$\varepsilon_{ijk}E_{j,k}=\mu_{ik}\dot{H}_k$$

$$\alpha_{ij}\dot{u}_{i,j}-p_i\dot{E}_i+\beta_0\dot{T}-T_0^{-1}(k_{ij}T_{,j})_{,i}=0$$

where

$$\theta=T_0+T \; ; \quad \frac{T}{T_0}\ll 1$$

The principle of conservation of energy yields

$$-\int_A N_i\, dA_i - \int_A q_i\, dA_i + \int_A \sigma_{ij}\, \dot{u}_j\, dA_i = \int_V (\dot{K}+\dot{U})\, dV \qquad /2.23/$$

where

$N_i = \varepsilon_{ijk} E_j H_k$ - Umov-Pointing vector,

$K = \frac{1}{2}\rho \dot{u}_i^2$ - density of the kinetic energy,

U - density of the internal energy.

Applying the Gauss formula to /2.23/, after transformations we obtain

$$\dot{U} = \sigma_{ij}\dot{e}_{ij} + (E_i\dot{D}_i + H_i\dot{B}_i) - q_{i,i} \qquad /2.24/$$

According to the definition of entropy

$$q_{i,i} = -\theta\dot{s} \approx -T_0\dot{s}$$

and the Fourier law

$$q_i = -k_{ij}\theta_{,j} = -k_{ij}T_{,j} \qquad /2.25/$$

we obtain introducing the free energy

$$U_0 = U - T_0 s \qquad /2.26/$$

$$\dot{U}_0 = \sigma_{ij}\dot{e}_{ij} + (E_i\dot{D}_i + H_i\dot{B}_i) + \dot{T}s = \dot{U}_0(e_{ij}, D_i, B_i, s) \qquad /2.27/$$

Hence

$$\sigma_{ij} = \frac{\partial U_0}{\partial e_{ij}}\ ; \quad H_i = \frac{\partial U_0}{\partial B_i}\ ; \quad E_i = \frac{\partial U_0}{\partial D_i}\ ; \quad T = \frac{\partial U_0}{\partial s}\ ; \qquad /2.28/$$

If we choose

$$U_0=\frac{1}{2}E^{DsB}_{ikmn}e_{ij}e_{mn}+\frac{1}{2}\beta^{seB}_{ij}D_iD_j+\frac{1}{2}b^{eDB}s^2- \qquad /2.29/$$

$$-h^{sB}_{kij}D_ke_{ij}-g^{eB}_i sD_i-\gamma^{DB}_{ij}se_{ij}+\frac{1}{2}(\mu_{ik})^{-1}B_iB_k$$

where

$$E^{DsB}_{ikmn}=E^{DsB}_{kimn}=E^{DsB}_{iknm}=E^{DsB}_{mnik}$$

$$/2.30/$$

$$\beta^{seB}_{ij}=\beta^{seB}_{ji}\,;\quad h^{sB}_{kij}=h^{sB}_{kji}\,;\quad \gamma^{DB}_{ij}=\gamma^{DB}_{ji}\,,\quad \mu_{ik}(\mu_{ik})^{-1}=1$$

we arrive at the expressions for σ_{ij}, P, E_i, H_j

Transforming these expressions by means of the coefficients used in /2.21/, [5], we arrive exactly at the relations /2.21/. In the equations of thermopiezoelectricity in general the influence of the magnetic induction may be neglected which results in a simplification of the equations and makes it possible to introduce the potential of electric field.

A discussion of the second law of thermodynamics implies

$$\dot{s}+\left(\frac{q_i}{\theta}\right)_{,i}\geqslant 0 \qquad /2.31/$$

which, making use of the Fourier law leads to the condition

$$-\frac{\theta_{,i}q_i}{\theta^2}=\frac{k_{ij}\theta_{,i}\theta_{,j}}{\theta^2}\approx\frac{k_{ij}T_{,i}T_{,j}}{\theta^2}\geqslant 0 \qquad /2.32/$$

this, in turn, requires the symmetry $k_{ij} = k_{ji}$ and $|k_{ij}| > 0$;

$$\begin{vmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{vmatrix} > 0 ; \qquad k_{11} > 0$$

2.3. The wave equations of thermoelectromagnetoelasticity

It was proved in [5] that generalizing in an appropriate manner the Fourier law we can extend the systems of equations of thermomagnetoelasticity and thermopiezoelectricity to wave equations, i.e. equations in which the disturbances of all fields, including the thermal field, are propagated with a finite velocity.

This effect cannot be obtained /see [5] by taking into account relativistic effects in the systems of equations. In view of limited scope we do not quote here the considerations and the modifications of the thermodynamics of irreversible processes referring the reader to [5] but we write down the final wave equations. These equations contain therefore strongly non-stationary processes in contrast to the above considered parabolic-hyperbolic equations which are true for stationary or weakly non-stationary processes.

The wave equations of thermomagnetoelasticity, according to [5] have the form

$$\mathrm{rot}\,\vec{h} = \vec{j} + \dot{\vec{D}} ; \qquad \mathrm{rot}\,\vec{E} = -\dot{\vec{b}} ; \qquad \mathrm{div}\,\vec{b} = 0 ; \qquad \mathrm{div}\,\vec{D} = \rho_e$$

$$\rho \ddot{u}_i = \sigma_{ik,k} + (\vec{j} \times \vec{B}_0)_i + \rho_e E_i + P_i$$

$$\tau \dot{q}_{i,i} + q_{i,i} + (k_{ij} T_{,j})_{,i} - (\pi_{ik} j_k)_{,i} = -F$$

$$b_i = \mu_{ik} h_k ; \quad D_i = \varepsilon_{ik} [E_k + (\dot{\vec{u}} \times \vec{B}_0)_k] - \frac{1}{c^2} (\dot{\vec{u}} \times \vec{H}_0)_i$$

$\sigma_{ik} = E_{ikmn} \epsilon_{mn} - \alpha_{ik} T$ /the relaxation has been disregarded/

$$j_i = \eta_{ik} E_k + \varkappa_{ik} [k_{kj}^{-1} q_j - k_{kj}^{-1} \pi_{js} \dot{j}_s] + \eta_{ik} (\dot{\vec{u}} \times \vec{B}_0)_k + \rho_e \dot{u}_i$$

where the constant τ has the character of the relaxation constant and follows from the generalized Fourier law

$$\tau \dot{q}_{i,i} + q_{i,i} = -(k_{ij} T_{,j})_{,i} + (\pi_{jk} j_k)_{,i} \qquad /2.34/$$

as $\tau \to 0$ Eqs /2.33/ are transformed into the equations of /2.1/.

In the case of an isotropic medium Eqs /2.33/ take the form

$$\operatorname{rot} \vec{h} = \vec{j} + \varepsilon [\dot{\vec{E}} + (\ddot{\vec{u}} \times \vec{B}_0)] ; \quad \operatorname{rot} \vec{E} = -\dot{\vec{b}} ; \quad \operatorname{div} \vec{b} = 0 ; \quad \operatorname{div} \vec{D} = 0 \quad (\rho_e = 0)$$

$$\rho \ddot{\vec{u}} = G \nabla^2 \vec{u} + (\lambda + G) \operatorname{grad} \operatorname{div} \vec{u} - \alpha \operatorname{grad} T + (\vec{j} \times \vec{B}_0) \qquad /2.35/$$

$$\tau \beta \varphi \ddot{T} + \beta \dot{T} + \tau \lambda \varphi \operatorname{div} \ddot{\vec{u}} + \lambda \operatorname{div} \dot{\vec{u}} - k \varphi \nabla^2 T = 0$$

$$\vec{j} + \tau \varphi \dot{\vec{j}} + \eta \{ \vec{E} + \tau \dot{\vec{E}} + (\dot{\vec{u}} + \tau \ddot{\vec{u}}) \times \vec{B}_0 \} - \varkappa \operatorname{grad} T$$

where

$$\lambda = T_0\alpha\ ; \qquad \beta = \beta_0 T_0\ ; \qquad \varphi = 1 + \frac{\varkappa\pi}{\eta}$$

The corresponding equations of thermopiezoelectricity have the form

$$E_{ikmn} u_{m,ni} - \gamma_{ikl} E_{l,i} - \alpha_{ik} T_{,i} = \rho \ddot{u}_k$$

$$\varepsilon_{ijk} H_{kj} = \frac{1}{2} \gamma_{ikl} (\dot{u}_{k,l} + \dot{u}_{l,k}) + \varepsilon_{ij} \dot{E}_j + p_i \dot{T} \qquad /2.36/$$

$$\varepsilon_{ijk} E_{j,k} = \mu_{ik} \dot{H}_k$$

$$T_0 \alpha_{ij} (\tau \ddot{u}_{i,j} + \dot{u}_{i,j}) + T_0 p_i (\tau \ddot{E}_i + \dot{E}_i) + T_0 \beta (\tau \ddot{T} + \dot{T}) - (k_{ij} T_{,j})_{,i} = 0$$

As $\tau = 0$ they are transformed into /2.22/.

It can readily be verified that both /2.33/ and /2.36/ consitute hyperbolic systems of equations /see [5] /. The idea of an experimental verification of the wave pnenomenon on the basis of the Cherenkov effect in the coupled fields was presented in [30] .

In our considerations we have not dealt with the boundary conditions. In general they follow directly from the physical nature of the relations on the boundary and can be expressed in terms of the field components and the constitutive relations. It has been necessary to confine our review mainly to the equations and the following physical effects of field couplings.

2.4. Thermomagnetomicroelasticity

In the case of a medium without spin we can easily generalize the equations of thermomicroelasticity /with 6 local degrees of freedom/ to the equations of thermomagnetoelasticity [26] . In view of the lack of time we omit here also the thermodynamic considerations, quoting only the final system of equations.

In the particular case of the centrosymmetric body [26]

$$rot\,\vec{h}=\vec{j}+\varepsilon\dot{\vec{E}}-\frac{\varepsilon\mu c^2-1}{c^2}(\ddot{\vec{u}}\times\vec{H}_0); \qquad rot\,\vec{E}=-\mu\dot{\vec{h}}$$

$$\rho\ddot{\vec{u}}=(\mu+\alpha)\nabla^2\vec{u}+(\lambda+\mu-\alpha)\,grad\,div\,\vec{u}+2\alpha\,rot\,\vec{\varphi}+\rho_e\vec{E}+$$

$$+(\vec{j}\times\vec{B}_0)-3\alpha_0 K\,grad\,T+\vec{P} \qquad /2.37/$$

$$m\ddot{\vec{\varphi}}=(\gamma+\varepsilon)\nabla^2\vec{\varphi}+(\beta+\gamma-\varepsilon)\,grad\,div\,\vec{\varphi}+2\alpha\,rot\,\vec{u}-4\alpha\vec{\varphi}+\vec{M}$$

$$\beta\dot{T}+\lambda\,div\,\dot{\vec{u}}+\pi\,div\,\vec{j}-k\nabla^2T=F$$

$$\vec{j}=\eta\left[\vec{E}+(\dot{\vec{u}}\times\vec{B}_0)\right]-\varkappa\,grad\,T+\rho_e\dot{\vec{u}}$$

/the terms $\rho_e\vec{E}$, $\rho_e\dot{\vec{u}}$ drop out in view of the linearization; we have written them down to indicate the structure of the equations/.

The equations of thermomagnetomicroelasticity for a medium with a spin are far more complicated. In fact, in this case the equation of micromoments and the Landau spin equation require a deeper physical analysis in order to determine the nature of the couplings [27] . Similarly, the equations of micropiezoelectricity are more complicated; here the basic problem is the connection between the polarization and the micromoments.

We have omitted here the boundary conditions, referring the reader for the details to [26] . However, these conditions follow either from the variational equations or from the fundamental physical considerations if we make use of the appropriate state relations for the stresses, moments, inductions, etc. and the components of the displacement, microrotations field vectors, etc.

3. Nonlinear equations of thermomagnetoelasticity

For the lack of space we do not intend to discuss the nonlinear /finite deformations/ equations of thermopiezoelectricity, or more generally, the thermoelectroelasticity of dielectrics. These equations can easily be derived on the basis of the nonlinear equations, [34] , for the general dynamics of dielectrics, by completing them by the equations of heat transport and thermal couplings.

Consider now briefly the nonlinear equations of thermomagnetoelasticity [24] the investigation of which has begun only recently. Here, similarly to the linear theory we can consider jointly the problems of thermoelectroelasticity and thermomagnetoelasticity, but in view of the physical nature of the problem and different ranges of applications of dielectrics and conductors, a seperate consideration is simpler and more expedient. Similarly to the linear case we do not deal here with the media with spin. These problems will be examined in the last section.

Denote the natural, initial and current coordinates by

$$x_\alpha\,,\ X_k\,,\ y_i$$

respectively. The deformations at an arbitrary instant of time are described by the relations

$$\underset{\sim}{y}=\underset{\sim}{y}\,(\underset{\sim}{x}t) \qquad /3.1/$$

We follow here the notations of Truesdell and his collaborators.

Neglecting the mechanical body forces, the system of nonlinear equations of thermomagnetoelasticity takes the following form

1. The Maxwell equations

$$\varepsilon_{ijk}\frac{\partial e_k}{\partial y_j}+B_i^*=0\,; \qquad \frac{\partial B_i}{\partial y_i}=0$$

$$/3.2/$$

$$\varepsilon_{ijk}\frac{\partial u_k}{\partial y_i}-D_i^*=j_i\,; \qquad \frac{\partial D_i}{\partial y_i}=q_i$$

where

$$e_i = E_i + \varepsilon_{i\gamma s} v_\gamma B_s \, ; \; \mu_i = H_i - \varepsilon_{i\gamma s} v_\gamma D_s \, ; \; D_i = \varepsilon E_i + \alpha \varepsilon_{i\gamma s} v_\gamma H_s \qquad /3.3/$$

$$B_i = \mu H_i - \alpha \varepsilon_{i\gamma s} v_\gamma E_s \, ; \quad j_i = E_i \eta \, ; \quad \alpha = \mu\varepsilon - \mu_0 \varepsilon_0$$

the vectors have the same meaning as before, and $\varepsilon, \mu, \varepsilon_0, \mu_0$ denote the electric and magnetic permeabilities in the medium and in vacuo ; ε_{ijk} is the unit pseudotensor. We have also denoted

$$f_i^* = \frac{\partial f_i}{\partial t} + v_j \frac{\partial f_i}{\partial y_j} - f_j \frac{\partial v_i}{\partial y_j} + f_i \frac{\partial v_j}{\partial y_i} \qquad /3.4/$$

2. The equations of motion

$$\frac{\partial t_{ij}}{\partial y_j} + q_i e_i + \varepsilon_{i\gamma s} j_\gamma B_s = \rho \dot{v}_i \qquad /3.5/$$

3. The equations of energy balance

$$\rho \dot{\Sigma} = t_{ij} \frac{\partial v_i}{\partial y_j} + e_i j_i - \frac{\partial q_i}{\partial y_i} \qquad /3.6/$$

where t_{ij} is the Cauchy tensor and Σ is the internal energy.

The above equations have to be completed by the equations of state and transport

$$t_{ij} = t_{ij}\,(p_{i\alpha}, \theta); \qquad q_i = q_i\,(\theta_{,j}, p_{j\alpha}, \theta) \tag{3.7}$$

where θ is the temperature

$$\theta_{,i} = \frac{\partial \theta}{\partial y_i}\;, \qquad p_{i\alpha} = \frac{\partial y_i}{\partial X_\alpha}$$

The functions t_{ij} have to be chosen in such a way that the symmetry conditions are satisfied, and, moreover, the Clausius- Duhen inequality

$$\rho \dot{s} - \frac{e_i j_i}{\theta} - \frac{\partial}{\partial y_i}\left(\frac{q_i}{\theta}\right) \geqslant 0 \tag{3.8}$$

holds true.

A shortcoming of the above equations consists in the fact that some derivatives are with respect to x while some are with respect to y .

The considered equations can be simplified by reffering them to the natural configuration by means of the transformation

$$\begin{aligned} b_\alpha(\underset{\sim}{x}t) &= JX_{\alpha,i}\, B_i(\underset{\sim}{x}t); & h_\alpha(\underset{\sim}{x}t) &= p_{i\alpha}\, H_i(\underset{\sim}{x}t) \\ d_\alpha(\underset{\sim}{x}t) &= Jx_{\alpha,i}\, D_i(\underset{\sim}{x},t); & \bar{e}_\alpha(\underset{\sim}{x}t) &= p_{i\alpha}\, E_i(\underset{\sim}{x}t) \\ g_\alpha &= Jx_{\alpha,i}\, j_i(\underset{\sim}{x}t)\;; & Q_e(\underset{\sim}{x}t) &= Jq_e(\underset{\sim}{x}t) \end{aligned} \tag{3.9}$$

$$\bar{\varepsilon}_{\alpha\beta\gamma} = J^{-1} p_{i\alpha} p_{j\beta} p_{k\gamma} \varepsilon_{ijk}, \qquad v_\alpha(\underset{\sim}{x}t) = x_{\alpha,i} v_i(\underset{\sim}{x}t)$$

$$T_{\alpha i}(\underset{\sim}{x}t) = J x_{\alpha,j} t_{ij}(\underset{\sim}{x}t) \qquad \rho_0 = J\rho\,; \qquad \theta_\alpha(\underset{\sim}{x}t) = J x_{\alpha,i} q_i(\underset{\sim}{x}t)$$

where

$$x_{\alpha,i} = \frac{\partial x_\alpha}{\partial y_i} \qquad J = /p_{i\alpha}/$$

Then the equations take the form

$$\bar{\varepsilon}_{\alpha\beta\gamma} \frac{\partial e_j}{\partial x_\beta} + \dot{b}_\alpha = 0\,; \qquad \frac{\partial b_\alpha}{\partial x_\alpha} = 0$$

$$\bar{\varepsilon}_{\alpha\beta\gamma} \frac{\partial \mu_\gamma}{\partial x_\beta} - \dot{d}_\alpha = q_\alpha\,; \qquad \frac{d\, d_\alpha}{d x_\alpha} = Q_e \qquad /3.10/$$

Here we have

$$e_\alpha = \bar{e}_\alpha + \bar{\varepsilon}_{\alpha\beta\gamma} v_\beta b_\gamma\,; \qquad \mu_\alpha = h_\alpha - \varepsilon_{\alpha\beta\gamma} v_\beta d_\gamma$$

$$d_\alpha = J c^{-1}_{\alpha\beta} (\varepsilon e_\beta + \alpha J \bar{\varepsilon}_{\beta\mu\gamma} v_\mu c^{-1}_{\gamma\omega} h_\omega) \qquad /3.11/$$

$$b_\alpha = J c^{-1}_{\alpha\beta} (\mu h_\beta - \alpha J \bar{\varepsilon}_{\beta\mu\gamma} v_\mu c^{-1}_{\gamma\omega} e_\omega)$$

$$g_\alpha = \eta J c^{-1}_{\alpha\beta} e_\beta$$

where

$$c^{-1}_{\alpha\beta} = \frac{\partial x_\alpha}{\partial y_s} \frac{\partial x_\beta}{\partial y_s}$$

and

$$\frac{\partial T_{\alpha i}}{\partial x_\alpha} = \mu_{\alpha,i} \left\{ Q_i e_\alpha + \bar{\varepsilon}_{\alpha\beta\gamma} g_\beta b_\gamma \right\} = \rho_0 \dot{v}_i \qquad /3.12/$$

$$\rho_0 \dot{\Sigma} = T_{\alpha i}\frac{\partial v_i}{\partial x_\alpha} + e_\alpha g_\alpha - \frac{\partial Q_\alpha}{\partial x_\alpha}$$

$$\rho_0 \dot{s} = \frac{e_\alpha q_\alpha}{\theta} - \frac{\partial}{\partial x_\alpha}\left(\frac{Q_\alpha}{Q}\right) \geqslant 0$$

The equations are considerably simplified if we assume $\varepsilon = \varepsilon_0, \ \mu = \mu_0$ and neglect the effects connected with the velocity of light. A further simplification is obtained for perfect conductors. For details the reader is reffered to [24] . The latter paper contains also a consideration of the problem of propagation of the acceleration waves for the equations derived in this section.

4. Review of the fundamental solutions of magnetothermoelasticity

4.1. One-dimensional problem of magnetothermoelasticity

In this field there appeared papers concerning the propagation of plane magnetothermoelastic waves. The deal mainly with real conductors. The problem of propagation of a plane wave in an infinite medium was first considered in a paper by G.Paria [10] and then, under wider assumtions, by A.J.Willson [11] . Paria assumed the orthogonality of the initial vector of the magnetic field to the direction of propagation of the plane wave ; A.J.Willson, on the other hand, assumed that the initial magnetic field has also a component in the direction of propagation of the longitudinal wave. The initial field is described by the vector $H=/H_1,H_2,0/$ and all quantities changing with the deformation, temperature and electromagnetic field depend on the variables x_1 and t.

If we assume that $H_1=0$, $H_2 \neq 0$, we find that the transverse wave is not coupled with the temperature and electromagnetic fields ; the coupling appears only in the longitudinal wave. If $H_1 \neq 0, H_2=0$ there exists a coupling of the displacement and temperature fields in the longitudinal wave and a coupling of the deformation and electromagnetic fields in the transverse wave. Evidently, in the case $H_1 \neq 0$, $H_2 \neq 0$ we have the coupling of the deformation, temperature and electromagnetic fields both in the longitudinal and transverse waves.

Two particular cases of propagation of a plane wave were examined by W.Nowacki [12] . They are produced by the action of either a plane heat source of the type $Q(x,t)= = Q_0\,\delta(x_1)\,e^{-i\omega t}$ or body forces $P(x,t)=P_0\,\delta(x_1)\,e^{-i\omega t}$

The equation for the longitudinal wave in the case of a perfect conductor has the form

$$[(\partial_1^2-\frac{1}{\varkappa}\,\partial_t)(\partial_1^2-\frac{1}{c_0^2}\,\partial_t^2)-\eta m_0 \partial_t \partial_1^2]\,u_1=0 \qquad /4.1/$$

This equation differs from the equation of propagation of the thermoelastic plane wave by the terms c_0 and

$m_0 = \frac{m}{c_0^2}$ where $c_0 = c_1^2(1+\alpha)$, $\alpha = \frac{a_0^2}{c_1^2}$. Here $a_0^2 = \frac{H_2^2 \mu_0}{4\pi\rho}$ and a_0 is the Alfven velocity. The electromagnetic excitation is described by the quantity α.

It follows from the structure of Eq. /4.1/ that the plane wave undergoes a dispersion and damping.

Another one-dimensional problem soluble in a closed form is the propagation of a plane wave in the elastic semi-space $x_1 \geqq 0$, due to a sudden application of temperature of the plane $x_1 = 0$ bounding the semi-space. At the instant t = 0 the temperature has been applied and held at this value. Under the action of the thermal shock $\theta(0,t) = \theta_0 H(t)$ a magnetothermoelastic wave is propagated in the medium, depending on two variables x_1 and t . The considered problem was discussed in two papers by S.Kaliski and W.Nowacki [8], [9], in the first for a perfect conductor while in the second for a real conductor. To derive results simple enough to be examined analytically the coupling between the deformation and temperature fields was neglected. Finally it was assumed that the boundary $x_1 = 0$ is free of tractions. The closed expression for the displacement $u_1(x_1,t)$ the stress $\sigma_{11}(x_1,t)$ and the component $h_3(x_1,t)$ were deduced.

It turns out that the stress $\sigma_{11}(x_1,t)$ consists of two parts, first of the nature of an elastic wave mowing with a phase velocity $c_0 = c_1/1+\alpha/^{1/2}$ and the second, of diffusional nature. When the elastic wave passes through the plane

$x_1 = const$ at the instant $t = x_1/c_0$ there appears a jump in the stress of a constant value $\gamma\theta_0/\vartheta(1+\beta)$ where $\gamma = (3\lambda + 2\mu)\alpha_t$; here λ, μ are the Lamé constants in the adiabatic case and α_t is the coefficient of linear thermal expansion, $\vartheta = \frac{c_0^2}{c_1}$ and $\beta = \frac{n}{\alpha}\beta_0$ where

$$\beta_0 = \frac{\varkappa H_3}{4\pi\rho c_0^2}, \qquad n = \frac{\mu_0 c_0^3 H_3}{\varkappa c^2}$$

The modified electromagnetic wave is propagated with the same velocity as the modified elastic wave and in the plane $x_1 = const$ at the instant $t = x_1/c_0$ has a discontinuity. There is a wave h_0 moving with the light velocity c radiated out into the vacuum. If we assume that the initial magnetic field $\vec{H} = (0,0,H_3)$ vanishes, then the results become those obtained by J.V.Danilovskaya in the theory of thermal stresses.

4.2. Two-dimensional problem of magnetothermoelasticity

In this field only particular cases were considered, concerning the propagation of cylindrical wave in a perfect and real conductor /W.Nowacki/, [13], [14], [15]. If the plane in which the excitation is propagated in the plane x_1, x_2 , then the production of cylindrical waves is possible only when the initial magnetic field has the direction of the x_3 -axis. Assuming that $H = /0,0,H_3/$, we arrive at the following system of equations of magnetothermoelasticity for a perfect conductor:

$$\mu \nabla_1^2 u_\alpha + (\lambda+\mu+a_0^2\rho) u_{\beta,\beta\alpha} + X_\alpha = \rho \ddot{u}_\alpha + \gamma\theta_{,\alpha} \quad \alpha = 1,2 \qquad /4.2/$$

$$(\nabla^2 - \frac{1}{\varkappa}\partial_t)\theta - \eta\partial_t \operatorname{div} \dot{\vec{u}} = -\frac{Q}{\varkappa} \qquad /4.3/$$

where

$$a_0^2 = \frac{H_3^2\mu_0}{4\pi\rho} \qquad \nabla_1^2 = \partial_1^2 + \partial_2^2$$

Eqs /4.2/ are the displacement equations in which the influence of the electromagnetic field is expresses by the term $a_0^2\rho$. Eq. /4.3/ is the heat conduction equation. When $a_0 = 0$ Eqs /4.2/ and /4.3/ become the equations of thermoelasticity.

Decomposing the displacement field ane the body forces into the potantial and solenoidal parts

$$u_1 = \partial_1\Phi - \partial_2\Psi \qquad u_2 = \partial_2\Phi + \partial_1\Psi \qquad /4.4/$$

$$X_1 = \rho(\partial_1\vartheta - \partial_2\chi), \qquad X_2 = \rho(\partial_2\vartheta + \partial_1\chi)$$

and eliminating the temperature θ we obtain the following wave equations:

$$(\Box_1^2\Box_3^2 - \eta m_0 \partial_t \nabla_1^2)\Phi = \frac{m_0 Q}{\varkappa} - \frac{1}{c_0^2}\Box_3^2\vartheta \qquad /4.5/$$

$$\Box_2^2\Psi = -\frac{1}{c_2^2}\chi \qquad /4.6/$$

We have introduced here the notations

$$\Box_1^2 = \nabla_1^2 - \frac{1}{c_0^2}\partial_t^2, \qquad \Box_2^2 - \nabla_1^2 - \frac{1}{c_2^2}\partial_t^2, \qquad \Box_3^2 = \nabla_1^2 - \frac{1}{\varkappa}\partial_t$$

$$c_0^2 = c_1(1+\alpha) \qquad \alpha = \frac{a_0^2}{c_1^2}$$

Eq. /4.5/ represents the longitudinal wave undergoing a dispersion and damping. In the infinite space the factors producing the longitudinal wave are the heat sources and body forces of the form $X_\alpha = \rho\dot{\vartheta}_{,\alpha}$, $\alpha = 1,2$

The transverse waves are produced in the infinite space by the body forces $X_1 = -\rho\partial_2\chi$, $X_2 = \rho\partial_1\chi$. They are not dispersed or damped and are propagated with the acoustic velocity c_2 . These waves in the infinite space are not accompanied by the temperature field. However, there exists an electromaghetic field, for

$$\vec{E} = \frac{\mu_0 H_3}{c}(-\partial_1\dot{\psi}, \partial_2\dot{\psi}, 0) \quad \vec{h} = 0, \quad \vec{j} = 0$$

The system of equations /4.2/ /4.3/ can also be uncoupled by using three functions: the vector $\vec{\varphi} = (\varphi_1, \varphi_2, 0)$ and the scalar ξ . This method constitutes a generalization of the method applied by M.Iacovache to the problems of dynamic elasticity and the B.G.Galerkin method in the static elasticity.

The paper [7] contains two particular problems: the action of a linear heat source $Q(\gamma,t) = Q_0 \frac{\delta(\gamma)}{2\pi r} e^{i\omega t}$ and a linear center of pressure $\vartheta_0 = \vartheta_0 \frac{\delta(\gamma)}{2\pi r} e^{-i\omega t}$

In [8] a two-dimensional problem was examined for a medium with a finite conductivity. The system of displacement equations

$$\mu\nabla_1^2 u_1 + (\lambda+\mu)\partial_1 e + X_1 - \gamma\partial_1\theta - \frac{\mu_0 H_3}{4\pi}\partial_1 h_3 = \rho\ddot{u}_1 \qquad /4.7/$$

$$\mu\nabla_1^2 u_2 + (\lambda+\mu)\partial_2 e + X_2 - \gamma\partial_2\theta - \frac{\mu_0 H_3}{4\pi}\partial_2 h_3 = \rho\ddot{u}_2$$

and the heat conduction equation

$$\left(\nabla_1^2 - \frac{1}{\varkappa}\partial_t\right)\theta - \eta\partial_t e = -\frac{Q}{\varkappa} \qquad e = \partial_1 u_1 + \partial_2 u_2 \qquad /4.8/$$

are completed by the field equation

$$\nabla_1^2 h_3 - \beta\partial_t h_3 = -\beta H_3\partial_t e, \qquad \beta = \frac{4\pi\lambda_0\mu_0}{c^2} \qquad /4.9/$$

The above mentioned representations reduce the system /4.7/÷ /4.9/ to the following wave equations

$$\left(\nabla_1^2 - \frac{1}{c_1^2}\partial_t^2\right)\Phi - m\vartheta - \frac{\mu_0 H_3 h_3}{4\pi\rho c_1^2} = -\frac{1}{c_1^2}\vartheta \qquad /4.10/$$

$$\left(\nabla_1^2 - \frac{1}{c_2^2}\partial_t^2\right)\Psi = -\frac{1}{c_2^2}\chi \qquad /4.11/$$

$$\left(\nabla_1^2 - \frac{1}{\varkappa}\partial_t\right)\theta - \eta\partial_t\nabla^2\Phi = -\frac{Q}{\varkappa} \qquad /4.12/$$

$$\left(\nabla_1^2 - \beta\partial_t\right)h_3 - \beta H_3\partial_t\nabla_1^2\Phi = 0 \qquad /4.13/$$

Here, Eq. /4.10/ represents the longitudinal wave, Eq. /4.11/ the transverse wave. Eq. /4.12/ is the heat conduction equation and /4.13/ the field equation. It is evident that in an elastic space the equation of the longitudinal wave is independent of the other equations. Eliminating h_3 nad θ from the above system we arrive at a complex equation for the longitudinal wave

$$D\Box_1^2\Box_3^2-\frac{1}{\varkappa}\partial_t\nabla_1^2(D\varepsilon_T+\varepsilon\Box_3^2)\ \Phi=-\frac{m}{\varkappa}DQ-\frac{1}{c_1^2}D\Box_3^2\vartheta \qquad /4.14/$$

where

$$D=\nabla_1^2-\beta\partial_t,\quad \varepsilon_T=\eta\varkappa m\quad \varepsilon_H=\alpha\beta\varkappa\quad \alpha=\frac{a_0^2}{c_1^2}$$

In the equation the coefficient ε_T describes the coupling between the deformation and temperature fields, while the coefficient ε_H couples the deformation and electromagnetic fields. The longitudinal waves undergo a dispersion and damping.

Well known simplifications are obtained by assuming an adiabatic process / for Q=0/, which leads to the wave equation

$$(D\Box_1^2-\alpha\beta\partial_t\nabla_1^2)\Phi=-\frac{1}{c_1^2}D\vartheta \qquad /4.15/$$

the structure of which resembles that of the wave equation in the coupled thermoelasticity.

Eq./4.14/ is considerably simplified if $\alpha=\frac{a_0^2}{c_1^2}\ll 1$ i.e. when the initial magnetic field $H=/0,0,H_3/$ is small. In this case of a considerable use may be the perturbation method.

4.3. General theorems of magnetothermoelasticity

One of the most interesting theorems of the theory of elasticity is the Betti reciprocity theorem. This is a very general theorem and contains a possibility of introducing methods of solving the equations of the elasticity theory by means of the Green function.

The reciprocity theorem was extended by V.Casimir-Jonescu to the problems of coupled thermoelasticity. The generalization of the reciprocity theorem to the problems of magnetothermoelasticity ,was given by S.Kaliski and W.Nowacki in the three papers [16], [17], [18]. In the first the theorem was examined for a perfect isotropic conductor, in the second for a real isotropic conductor and in the third for a real anisotropic conductor.

Assuming that the motion of the body begins at the instant t=0 and that the initial conditions are homogeneous we obtain from the constitutive relations the identity

$$\bar{\sigma}_{ij}\bar{\varepsilon}'_{ij}-\bar{\sigma}'_{ij}\bar{\varepsilon}_{ij}+\gamma(\bar{\theta}\bar{e}'-\bar{\theta}'\bar{e})=0 \qquad /4.16/$$

Here $\bar{\sigma}_{ij}$, $\bar{\varepsilon}_{ij}$, etc. denote the Laplace transforms of the functions σ_{ij}, ε_{ij} , etc.. Integrating /4.16/ over the volume of the body and applying the Gauss transformations we have

$$\int_V(\bar{X}_i\bar{u}'_i-\bar{X}'_i\bar{u}_i)\,dV+\int_A(\bar{p}_i\bar{u}'_i-\bar{p}'_i\bar{u}_i)\,dA+\gamma\int_V(\bar{\theta}\bar{e}'-\bar{\theta}'\bar{e})\,dV= \qquad /4.17/$$

$$=\int_V(\bar{T}_{ij}\bar{\varepsilon}'_{ij}-\bar{T}'_{ij}\bar{\varepsilon}_{ij})\,dV$$

Here $T_{ij}=\frac{\mu_0}{4\pi}(h_iH_j+H_ih_j-\delta_{ij}(h_kH_k))$

are the components of the Maxwell tensor and $p_i=\sigma_{ij}n_j$, in the traction on the surface A . From the heat conduction equation written down forthe two states we obtain

$$\int_V(\bar{\varepsilon}\bar{\theta}'-\bar{\varepsilon}'\bar{\theta})\,dV=\frac{1}{\varkappa\eta p}\int_V(\bar{Q}\bar{\theta}'-\bar{Q}'\bar{\theta})dV+\frac{1}{\eta p}\int_A(\bar{\theta}'\bar{\theta}_{,n}-\bar{\theta}\bar{\theta}'_{,n})\,dV \qquad /4.18/$$

where p is the Laplace transform parameter.

Eliminating the common terms from /4.17/ and /4.18/ we obtain for an isotropic body the following form of the reciprocity theorem:

$$\varkappa\eta p\Big[\int_V(\bar{X}_i\bar{u}'_i-\bar{X}'_i\bar{u}_i)dV+\int_A(\bar{p}_i\bar{u}_i-\bar{p}'_i\bar{u}_i)\,dA\Big]+$$
$$+\gamma\int_V(\bar{Q}\bar{\theta}'-\bar{Q}'\bar{\theta})dV+\gamma\varkappa\int_A(\bar{\theta}'\bar{\theta}_{,n}-\bar{\theta}\bar{\theta}'_{,n})dA=\eta\varkappa p\int_V(\bar{T}_{ij}\bar{\varepsilon}'_{ij}-\bar{T}'_{ij}\bar{\varepsilon}_{ij})\,dV \qquad /4.19/$$

For an unbounded body V Eq./4.19/ is considerably simplified. The integrals drop out in a bounded body as well, in the case of homogeneous boundary conditions. It can easily be proved that Eq. /4.19/ is decomposed into two parts

$$\eta \varkappa p \int_V (\bar{X}_i \bar{u}_i' - \bar{X}_i' \bar{u}_i)\, dV + \gamma \int_V (\bar{Q}\bar{\theta}' - \bar{\theta}\bar{Q}')\, dV = 0 \qquad /4.20/$$

$$p \int_V (\bar{T}_{ij} \bar{\varepsilon}_{ij}' - \bar{T}_{ij}' \bar{\varepsilon}_{ij})\, dV = 0$$

The validity of this decomposition follows from the symmetry of the system of the displacement equations and hence, the symmetry of the Green function.

Inverting the Laplace transform in Eqs /4.20/ we arrive at the final from of the reciprocity theorem for a perfect isotropic conductor

$$\eta \varkappa \Big\{ \int_0^t d\tau \int_V \Big[X(x,t-\tau) \frac{\partial u_i'(x,\tau)}{\partial \tau} - X'(x,\tau) \frac{\partial u_i(x,t-\tau)}{\partial \tau} \Big] dV(x) + \qquad /4.21/$$

$$+ \gamma \int_0^t d\tau \int_V \big[Q(x,\tau)\theta'(x,t-\tau) - \theta'(x\tau) Q'(x,t-\tau) \big] dV(x) \Big\} = 0$$

$$\int_0^t d\tau \int_V \Big[T_{ij}(x\tau) \frac{\partial \varepsilon_{ji}'(x,t-\tau)}{\partial \tau} - T_{ij}'(x,t-\tau) \frac{\partial \varepsilon_{ij}(x\tau)}{\partial \tau} \Big] dV = 0 \qquad /4.22/$$

On the basis of the above theorem a number of practical formulae can be deduced, possessing many applications. In particular we can obtain an extension of the Somigliana theorem of the problems of magnetothermoelasticity; this theorem can be used to construct integral equations for certain boundary value problems.

We shall not dwell here on the reciprocity theorems for a medium with a finite electric conductivity, isotropic and anisotropic. Of course, the formulae here will be much more complicated.

Finally variational theorems werde deduced for a perfect and real conductors, in which the displacement, entropy and field underwent a variation [20], [21].

These theorems make it possible to deduce the fundamental energy theorems which can be used to prove the uniqueness of the solution of the differential equations of thermomagnetoelasticity.

4.4.

In conclusion briefly about the general theorems of thermoelectroelasticity in piezoelectrics.

The general constitutive equation and the fundamental energy theorems were given by R.D.Mindlin [6]. The variational principle and the reciprocity theorem was presented by W.Nowacki in [19]. The reciprocity theorem for the wave equations of piezoelectricity is contained in the paper by S.Kaliski [22]. Similar theorems for the wave equations of thermomagnetoelasticity were given by S.Kaliski in [23].

5. Remarks on the problems of thermoelectromagnetoelasticity of more complicated media.

The problems considered in the preceding four sections concerned only seperate problems of thermomagnetoelasticity and thermoelectroelasticity for the simplest cases, i.e. in the first case for simple media without spin in which the coupling with the electromagnetic field appears only through the Lorentz force in the equations of motion and the coupling due to the internal mechanisms are present only for the temperature field. This excludes all ferromedia, i.e. media with a spin. This type of equations without discussing the transport phenomena of heat was derived in [32] . The heat problems are here very complicated not only due to the conductivity and the couplings but also in view of the production of heat in the field processes due to the effects of nonlinear irreversibility /the hysteresis loop/ ; this fact, similarly to the thermomagnetoplasticity introduces a very serious difficulty . These problems not only possess presently no solutions, but we lack sufficiently precise formulations /in the media with a spin the boundary problem of conditions is a problem in itself/. A similar, though somewhat simpler situation occurs in the case of the second group of problems /ferrodielectrics, etc./.

Another group of problems omitted in this lecture are the problems of semiconductors and semimetals in which, in view of their present importance and engineering applicability, a phenomenological construction of the theory of coupled thermoelectromagnetoelastic fields if of fundamental engineering importance, while the complexity of the equations and couplings contains a variety of physical phenomena. It is sufficient to mention as an example the possibility of amplification of thermal, acoustic and electric impulses in piezosemiconductors and semimetals and couplings of these fields with the spin waves in ferrobodies and further a number of phenomena connected with the thermoelectromagnetic waves in low temperatures [28], [29] or a number of magnetocaloric phenomena, in order to realize the importance of these investigations for the measurement technique in electronics, ultrasonics and in the methods of investigation of physics of solids. Recent applications of laser radiation to the investigation of the propagation of the shock and heat waves, propagation of microcracks, problems of conversion of the heat, mechanicsl and electromagnetic energies in these processes /see e.g. [25] / constitute further examples and perspectives of the theory of coupled thermoelectromagnetoelastic fields. A separate problem consitute the microquantum solutions for the lattice dynamics, taking into account the effects of coupled fields, problems of dynamics of dislocations in piezoferromedia, etc.

The scope of this paper allows to mention only the most important problems.

Bearing in mind further scientific possibilities it seems that the following several trends should be mentioned.

1. Mathematical solutions in the field theory of linear equations of thermomagneto and thermoelectroelasticity, and the practical applications.
2. Investigations of nonlinear equations /including plastic effects/.
3. Constructions of the theory of thermoelectromagnetoelasticity for the media with a spin /in general ferrobodies/.
4. Investigations of thermoelectromagnetoelastic fields in semiconductors and semimetals.
5. Microscopic investigations /lattice dynamics, dislocations, etc. of the above phenomena in coupled fields/.
6. All engineering problems, making use of the possibilities created by the application of the laser radiation to the investigation of physical media.

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