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Two-Dimensional Problems of Orthogonal Grids

by

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Summary. The subject of the considerations is a grid, the deformation of which is described by the translations $\mathbf{u}=(0, 0, u^3)$ and the rotations $\boldsymbol{\varphi}=(\varphi^1, \varphi^2, 0)$. By introducing the vector $\boldsymbol{\chi}$ the set of three difference equations (2.3) is reduced to the set of simple difference equations (2.5). A few examples of solution are given in which the Fourier finite sine transformation and exponential transformation are used.

1. Introduction. Let us consider an orthogonal grid of bars as represented in Fig. 1. Let us denote by (x^1, x^2, x^3) a node of this grid, the neighbouring nodes being denoted by (x^1+1, x^2, x^3), etc. We assume that the grid is regular and that the nodes are spaced by equal distances l_1, l_2, l_3 in the direction of the axes x^1, x^2, x^3. Let the rigidity for bending, torsion and tension be the same in all the three directions which are orthogonal.

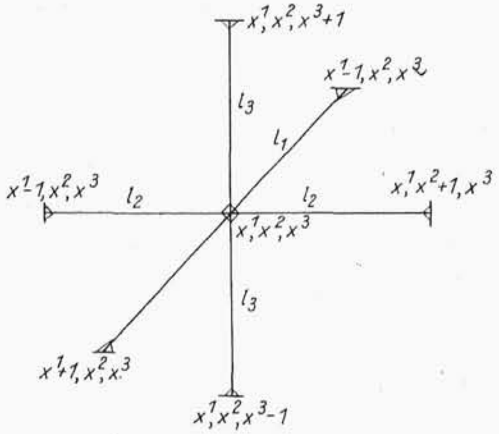

Fig. 1

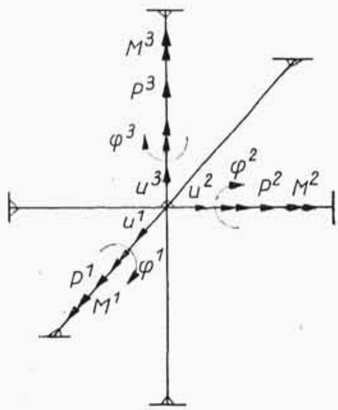

Fig. 2

It is assumed that the external loads, composed of forces $\mathbf{P}=(P^1, P^2, P^3)$ and moments $\mathbf{M}=(M^1, M^2, M^3)$, act on the nodes only thus producing a deformation of the system. The nodes of the grid undergo displacements $\mathbf{u}=(u^1, u^2, u^3)$ and rotations $\boldsymbol{\varphi}=(\varphi^1, \varphi^2, \varphi^3)$. The positive directions of $\mathbf{u}$, $\boldsymbol{\varphi}$, $\mathbf{P}$ and $\mathbf{M}$ are shown in Fig. 2.

On substituting the forces exerted by the bars on the nodes into the equilibrium equations of the latter, we obtain a set of six equations, the unknowns being the components of the displacement $\mathbf{u}$ and the rotation $\boldsymbol{\varphi}$ [1, 2]. These equations are

$$
(1.1)\qquad \begin{cases} [\lambda_1\, \Delta_1^2+\eta_{32}''\, \Delta_2^2+\eta_{23}''\, \Delta_3^2]\, u^1+\eta_{23}'\, C_3\, \varphi^2-\eta_{32}'\, C_2\, \varphi^3=-P^1, \\ [\eta_{31}''\, \Delta_1^2+\lambda_2^2\, \Delta_2^2+\eta_{13}''\, \Delta_3^2]\, u^2+\eta_{31}'\, C_1\, \varphi^3-\eta_{13}'\, C_3\, \varphi^1=-P^2, \\ [\eta_{21}''\, \Delta_1^2+\eta_{12}''\, \Delta_2^2+\lambda_3\, \Delta_3^2]\, u^3+\eta_{12}'\, C_2\, \varphi^1-\eta_{21}'\, C_1\, \varphi^2=-P^3 \end{cases}
$$

and

$$
(1.2)\qquad \begin{cases} [-\beta_1\, \Delta_1^2+\eta_{12}\,(\Delta_2^2+6)+\eta_{13}\,(\Delta_3^2+6)]\, \varphi^1+\eta_{12}'\, C_2\, u^3-\eta_{13}'\, C_3\, u^2=M^1, \\ [\eta_{21}\,(\Delta_1^2+6)-\beta_2\, \Delta_2^2+\eta_{23}\,(\Delta_3^2+6)]\, \varphi^2+\eta_{23}'\, C_3\, u^1-\eta_{21}'\, C_1\, u^3=M^2, \\ [\eta_{31}\,(\Delta_1^2+6)+\eta_{32}\,(\Delta_2^2+6)-\beta_3\, \Delta_3^2]\, \varphi^3+\eta_{31}\, C_1\, u^2-\eta_{32}'\, C_2\, u^1=M^3. \end{cases}
$$

and constitute a set of difference equations, in which the following difference operators have been introduced

$$
(1.3)\qquad \begin{aligned} &\Delta_1^2\, \varphi^1\,(x^1, x^2, x^3)=\varphi^1\,(x^1+1, x^2, x^3)-2\varphi^1\,(x^1, x^2, x^3)+\varphi^1\,(x^1-1, x^2, x^3), \\ &C_1\, \varphi^3\,(x^1, x^2, x^3)=\varphi^3\,(x^1+1, x^2, x^3)-\varphi^3\,(x^1-1, x^2, x^3), \\ &C_2\, \varphi^2\,(x^1, x^2, x^3)=\varphi^2\,(x^1, x^2+1, x^3)-\varphi^2\,(x^1, x^2-1, x^3), \quad \text{etc.} \end{aligned}
$$

The following symbols will denote the rigidities for torsion

$$
\beta_1=\frac{GC^1}{l_1}, \quad \beta_2=\frac{GC^2}{l_2}, \quad \beta_3=\frac{GC^3}{l_3},
$$

where $\beta_1, \beta_2, \beta_3$ concern the bars l_1, l_2, l_3, respectively. C^1, C^2, C^3 are quantities depending on the form of the cross-sections of the bars l_1, l_2, l_3 and G is the shear modulus.

The symbols

$$
\lambda_1=\frac{EA^1}{l_1}, \quad \lambda_2=\frac{EA^2}{l_2}, \quad \lambda_3=\frac{EA^3}{l_3},
$$

denote the rigidities for tension and A^1, A^2, A^3 — the cross-sectional areas of the bars, the lengths of which are l_1, l_2, l_3, respectively. E is Young's modulus.

The rigidities for bending are denoted by two indices. Thus, the flexural rigidities η_{21}, η_{31} are connected with the direction x^1 (the bar of length l_1) and η_{12}, η_{32} with the direction x^2 and η_{13}, η_{23} with the direction x^3. As an example $\eta_{21}=\dfrac{2EI^2}{l_1}$ denotes the flexural rigidity of the bar l_1, the moment of inertia being taken about the x^1-axis which passes through the centre of gravity of the cross-section of the bar.

Knowing the displacements $\mathbf{u}$ and the rotations $\boldsymbol{\varphi}$ of the nodes, we can determine the forces and the moments at the nodes from the following transformation equations of structural mechanics [3]

$$
(1.4)\qquad \begin{aligned} &M_{ik}=\eta\,(2\varphi_i+\varphi_k)+\eta'\,(v_i-v_k), \qquad M_{ki}=\eta\,(2\varphi_k+\varphi_i)+\eta'\,(v_i-v_k), \\ &T_{ik}=-\eta'\,(\varphi_i+\varphi_k)-\eta''\,(v_i-v_k), \qquad T_{ki}=-\eta'\,(\varphi_k+\varphi_i)-\eta''\,(v_i-v_k), \\ &H_{ik}=\lambda\,(u_k-u_i), \quad H_{ki}=\lambda\,(u_k-u_i), \quad \mathfrak{M}_{ik}=\beta\,(\psi_k-\psi_i), \quad \mathfrak{M}_{ki}=\beta\,(\psi_k-\psi_i). \end{aligned}
$$

Fig. 3 shows the nodal bending moments M_{ik}, M_{ki}, torques $\mathfrak{M}_{ik}$, $\mathfrak{M}_{ki}$, shear forces T_{ik}, T_{ki} and axial forces H_{ik}, H_{ki}. Finally, the figure shows the positive directions of the displacements u_i, u_k, v_i, v_k and rotations $\varphi_i, \varphi_k, \psi_i, \psi_k$.

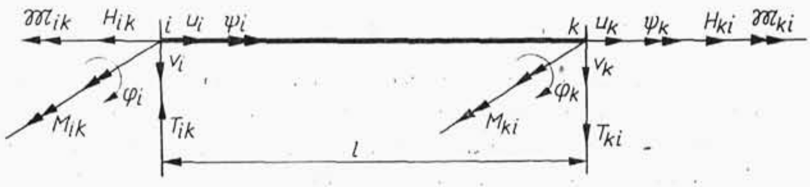

Fig. 3

2. The plane deformation. Similarly to the theory of elasticity the plane state of deformation will be considered by assuming that all the causes and results are independent of the variable x^3. As a result the set of equations (1.1) (1.2) is separated into two independent sets of equations.

In the first set the unknown quantities are the following components of the vectors $\mathbf{u}$ and $\boldsymbol{\varphi}$:

(2.1) $$\mathbf{u}=(u^1, u^2, 0), \quad \boldsymbol{\varphi}=(0, 0, \varphi^3).$$

In the second set the unknowns are

(2.2) $$\mathbf{u}=(0, 0, u^3), \quad \boldsymbol{\varphi}=(\varphi^1, \varphi^2, 0).$$

In what follows we shall be concerned with the latter plane state of deformation, for which the equilibrium equations are

(2.3) $$\begin{cases} [-\beta\Delta_1^2+\eta(\Delta_2^2+6)+6\gamma]\,\varphi^1+\eta' C_2 u^3=M^1, \\ [-\bar\beta\Delta_2^2+\bar\eta(\Delta_1^2+6)+6\bar\gamma]\,\varphi^2-\bar\eta' C_1 u^3=M^2, \\ \eta' C_2\varphi^1-\bar\eta' C_1\varphi^2+(\bar\eta''\Delta_1^2+\eta''\Delta_2^2)u^3=-P^3, \end{cases}$$

where the following symbols have been introduced

$$\eta=\frac{EI^1}{l_2}, \quad \bar\eta=\frac{EI^2}{l_1}, \quad \gamma=\frac{E\hat I^1}{l_3}, \quad \bar\gamma=\frac{E\hat I^2}{l_3}, \quad \beta=\frac{GC^1}{l_1}, \quad \bar\beta=\frac{GC^2}{l_2}.$$

The set of Eqs. (2.3) is characterized by a symmetric matrix of coefficients, which is a consequence of the reciprocity Betti theorem. Let us introduce the following expressions of the quantities $\varphi^1, \varphi^2, u^3$ in terms of a stress functions constituting a vector $\boldsymbol{\chi}=(\chi^1, \chi^2, \chi^3)$

(2.4) $$\begin{cases} \varphi^1=(D_2D_3-\bar\eta'^2C_1^2)\chi^1-\eta'\bar\eta' C_1C_2\chi^2-\eta' C_2D_2\chi^3, \\ \varphi^2=-\eta'\bar\eta' C_1C_2\chi^1+(D_1D_3-\eta'^2C_2^2)\chi^2+\bar\eta' C_1D_1\chi^3, \\ u^3=-\eta' C_2D_2\chi^1+\bar\eta' C_1D_1\chi^2+D_1D_2\chi^3, \end{cases}$$

where the following difference operators have been introduced

$$D_1=-\beta\Delta_1^2+\eta(\Delta_2^2+6)+6\gamma, \quad D_2=-\bar\beta\Delta_2^2+\bar\eta(\Delta_1^2+6)+6\bar\gamma, \quad D_3=\bar\eta''\Delta_1^2+\eta''\Delta_2^2.$$

On substituting (2.4) into Eqs. (2.3) we obtain a set of three separate equations

$$\Omega\chi^1 = M^1, \quad \Omega\chi^2 = M^2, \quad \Omega\chi^3 = -P^3, \tag{2.5}$$

where

$$\Omega = D_1 D_2 D_3 - \bar{\eta}'^2 C_1^2 D_1 - \eta'^2 C_2^2 D_2 .$$

Let us observe that the vector $\boldsymbol{\chi}$ plays the same role as the Galerkin vector in static problems of the theory of elasticity. The solution procedure is as follows. For prescribed right-hand members of Eqs. (2.3) we solve Eqs. (2.5). After determining the vector $\boldsymbol{\chi}$ we find the functions $\varphi^1, \varphi^2, u^3$ from the expressions (2.4).

Let us consider an infinite lattice of bars in plane deformation. To solve Eqs. (2.5) use will be made of a Fourier transformation developed by Babuška [4] in the form

$$\text{(2.6)} \quad \left\{ \begin{aligned} &\mathscr{F}[\varphi(x^1, x^2] = \tilde{\varphi}(\alpha^1, \alpha^2) = \sum_{x^1=-\infty}^{x^1=\infty} \sum_{x^2=-\infty}^{x^2=\infty} \varphi(x^1, x^2) \exp[i(\alpha^1 x^1 + \alpha^2 x^2)], \\ &\mathscr{F}^{-1}[\tilde{\varphi}(\alpha^1, \alpha^2)] = \varphi(x^1, x^2) = \\ &\qquad = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \tilde{\varphi}(\alpha^1, \alpha^2) \exp[-i(\alpha^1 x^1 + \alpha^2 x^2)]\, d\alpha^1 d\alpha^2, \end{aligned} \right.$$

where $\tilde{\varphi}(\alpha^1, \alpha^2)$ is the Fourier transform.

In further consideration the following relation will be essential

$$\mathscr{F}[\varphi(x^1 \pm p^1, x^2 \pm p^2) = \tilde{\varphi}(\alpha^1, \alpha^2) \exp[\pm i(\alpha^1 p^1 + \alpha^2 p^2)]. \tag{2.7}$$

Let us perform the Fourier transformation on Eq. $(2.5)_1$. We find

$$\sum_{x^1=-\infty}^{x^1=\infty} \sum_{x^2=-\infty}^{x^2=\infty} (\Omega\chi^1 - M^1) \exp[i(\alpha^1 x^1 + \alpha^2 x^2)] = 0 . \tag{2.8}$$

Bearing in mind the definition of the Fourier transformation (2.6) and the relations

$$\mathscr{F}(\Delta_1^2 \chi^1) = (e^{i\alpha^1} - 2 + e^{-i\alpha^1})\, \tilde{\chi}^1 = 2(\cos\alpha^1 - 1)\, \tilde{\chi}^1 ,$$
$$\mathscr{F}(C_1 \chi^1) = (e^{i\alpha^1} - e^{-i\alpha^1})\, \tilde{\chi}^1 = 2i \sin\alpha^1\, \tilde{\chi}^1, \text{ etc.}$$

occurring when (2.7) is used, Eq. (2.8) can be transformed to obtain

$$\tilde{\Omega}(\alpha^1, \alpha^2)\, \tilde{\chi}^1(\alpha^1, \alpha^2) = \tilde{M}^1(\alpha^1, \alpha^2), \tag{2.9}$$

where

$$\tilde{\Omega}(\alpha^1, \alpha^2) = \mathscr{D}_1 \mathscr{D}_2 \mathscr{D}_3 + \bar{\eta}'^2 \gamma_1^2 \mathscr{D}_1 + \eta'^2 \gamma_2^2 \mathscr{D}_2$$

and

$$\mathscr{D}_1 = \eta\sigma^1 - \beta\sigma^2 + 6(\eta + \gamma), \qquad \mathscr{D}_2 = \bar{\eta}\sigma^2 - \bar{\beta}\sigma^1 + 6(\bar{\eta} + \bar{\gamma}),$$
$$\mathscr{D}_3 = \bar{\eta}''\sigma^1 + \eta''\sigma^2,$$
$$\sigma^1 = 2(\cos\alpha^1 - 1), \qquad \sigma^2 = 2(\cos\alpha^2 - 1),$$
$$\gamma_1 = 2\sin\alpha^1 \qquad \gamma_2 = 2\sin\alpha^2 .$$

On performing the inverse transformation we have

$$(2.10)\quad \chi^1=\frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\frac{\tilde{M}^1(\alpha^1,\alpha^2)}{\tilde{\Omega}(\alpha^1,\alpha^2)}\exp\left[-i(\alpha^1x^1+\alpha^2x^2)\right]d\alpha^1,d\alpha^2,\ \text{etc.}$$

Let us consider the case in which a force $P^3=\delta_{\xi^1x^1}\delta_{\xi^2x^2}$ acts at the point (ξ^1,ξ^2) (unit forces acting on the nodes in the direction of the x^3 axis, along a line crossing the point (ξ^1,ξ^2). From Eqs. $(2.6)_1$ we have

$$\tilde{P}^3=\sum_{x^1=-\infty}^{x^1=\infty}\sum_{x^2=-\infty}^{x^2=\infty}\delta_{\xi^1x^1}\delta_{\xi^2x^2}\exp\left[i(\alpha^1x^1+\alpha^2x^2)\right]=\exp\left[i(\alpha^1\xi^1+\alpha^2\xi^2)\right].$$

In our case $M^1=0$, $M^2=0$ and we have $\chi^1=0$, $\chi^2=0$. From Eq. $(2.5)_3$ we have

$$(2.11)\quad \chi^3(x^1,x^2;\xi^1,\xi^2)=-\frac{1}{\pi^2}\int_0^{\pi}\int_0^{\pi}\frac{\cos\alpha^1(x^1-\xi^1)\cos\alpha^2(x^2-\xi^2)}{\tilde{\Omega}(\alpha^1,\alpha^2)}d\alpha^1\,d\alpha^2$$

From the expressions (3.4) we find

$$(2.12)\qquad \varphi^1=-\eta' C_2D_2\chi^3,\quad \varphi^2=\bar{\eta}' C_1D_1\chi^3,\quad u^3=D_1D_2\chi^3.$$

where the quantities φ^1,φ^2,u^3 may be considered as Green's functions for rotations and translations, due to the action of the concentrated force $P^3=1$.

Let us consider now a region of a frame grid bounded in the x^1x^2 plane and extending indefinitely in the direction of the positive and negative x^3-axis.

Let us assume that the translation u^3 vanishes along the straight lines $x^1=0,m$, $x^2=0$, n.

In agreement with this assumption let us assume for u^3 the following double trigonometric series

$$(2.13)\qquad u^3=\sum_{\mu=1}^{m-1}\sum_{\nu=1}^{n-1}u^3_{\mu\nu}\sin\alpha_\mu x^1\sin\beta_\mu x^2,$$

where

$$\alpha_\mu=\frac{\mu\pi}{m},\qquad \beta_\nu=\frac{\nu\pi}{n},\qquad \mu=1,2,\dots,m;\qquad \nu=1,2,\dots,n.$$

From Eqs. (2.3) we find the next functions

$$\varphi^1=\sum_{\mu,\nu}\varphi^1_{\mu\nu}\sin\alpha_\mu x^1\cos\beta_\nu x^2,\qquad \varphi^2=\sum_{\mu,\nu}\varphi^2_{\mu\nu}\cos\alpha_\mu x^1\sin\beta_\nu x^2,$$

$$M^1=\sum_{\mu,\nu}M^1_{\mu\nu}\sin\alpha_\mu x^1\cos\beta_\nu x^2,\qquad M^2=\sum_{\mu,\nu}M^2_{\mu\nu}\cos\alpha_\mu x^1\sin\beta_\nu x^2,$$

$$P^3=\sum_{\mu,\nu}P^3_{\mu\nu}\sin\alpha_\mu x^1\sin\beta_\nu x^2.$$

On substituting these into Eqs. (2.5) we solve them, thus finding

$$(2.14)\qquad \chi^1=\sum_{\mu,\nu}\frac{M^1_{\mu\nu}}{\Omega_{\mu\nu}}\sin\alpha_\mu x^1\cos\beta_\nu x^2,\ \text{etc.}$$

Here

$$\Omega_{\mu\nu}=D^1_{\mu\nu}D^2_{\mu\nu}D^3_{\mu\nu}+\bar{\eta}'^2\gamma^2_\mu D^1_{\mu\nu}+\eta'^2\gamma^2_\nu D^2_{\mu\nu},$$

where

$$D^1_{\mu\nu}=\eta\sigma_\mu-\beta\sigma_\nu+6(\eta+\gamma),\quad D^2_{\mu\nu}=\bar{\eta}\sigma_\nu-\bar{\beta}\sigma_\mu+6(\bar{\gamma}+\bar{\eta}),\quad D^3_{\mu\nu}=\bar{\eta}''\sigma_\mu+\eta''\sigma_\nu,$$

and

$$\sigma_\mu=2(\cos\alpha_\mu-1),\qquad \sigma_\nu=2(\cos\beta_\nu-1),$$
$$\gamma_\mu=2\sin\alpha_\mu,\qquad \gamma_\nu=2\sin\beta_\nu;$$

use having been made of the relations

$$\Delta^2_1(\sin\alpha_\mu x^1)=\sin\alpha_\mu(x^1+1)-2\sin\alpha_\mu x^1+\sin\alpha_\mu(x^1-1)=\sigma_\mu\sin\alpha_\mu x^1,$$
$$C^2_1(\sin\alpha_\mu x^1)=\sin\alpha_\mu(x^1+2)-2\sin\alpha_\mu x^1+\sin\alpha_\mu(x^1-2)=-\gamma^2_\mu\sin\alpha_\mu x^1,\ \text{etc.}$$

Let us consider the particular case of forces $P=1$ acting along the x^3-axis at the point (ξ^1, ξ^2). Then, for $P^3=\delta_{\xi^1x^1}\,\delta_{\xi^2x^2}$ we have

$$P^3_{\mu\nu}=\frac{4}{mn}\mathop{\mathsf{S}}_0^m\mathop{\mathsf{S}}_0^n\delta_{\xi^1x^1}\,\delta_{\xi^2x^2}\sin\alpha_\mu x^1\sin\beta_\nu x^2=\frac{4}{mn}\sin\alpha_\mu\xi^1\sin\beta_\nu\xi^2.$$

In this case $\boldsymbol{\chi}=(0,0,\chi^3)$, and

$$\chi^3=-\frac{4}{mn}\sum_{\mu,\nu}\frac{\sin\alpha_\mu\xi^1\sin\beta_\nu\xi^2}{\Omega_{\mu\nu}}\sin\alpha_\mu x^1\sin\beta_\nu x^2. \tag{2.15}$$

The rotations φ^1, φ^2 and the translation u^3 will be found from (2.12).

Let us consider the particular case of a plane grid. In this case the grid is no more continuous in the direction of the x^3-axis, therefire we must set $\gamma=0$, $\bar{\gamma}=0$ in (2.3) [5].

Let us consider such a plane grid in the form of an infinite strip of width $a_1=ml_1$. This problem can be solved by successive application of the Fourier sine and exponential transformation. Solution will be given for a concentrated force acting at the point (ξ^1, ξ^2) with $M^1=M^2=0$. The function χ^3 is expressed by the equations

$$\chi^3=-\frac{2}{m\pi}\sum_{\mu=1}^{m-1}\sin\alpha_\mu\xi^1\sin\alpha_\mu x^1\int_0^\pi\frac{\cos\alpha^2(x^2-\xi^2)\,d\alpha^2}{\Omega_{\mu\alpha}}, \tag{2.16}$$

where

$$\Omega_{\mu\alpha}=D^1_{\mu\alpha}D^2_{\mu\alpha}D^3_{\mu\alpha}+\bar{\eta}'^2\gamma^2_\mu D^2_{\mu\alpha}+\eta'^2\gamma^2_2 D^1_{\mu\alpha},$$

and

$$D^1_{\mu\alpha}=\eta\sigma_\mu-\beta\sigma^2+6\eta,\quad D^2_{\mu\alpha}=\bar{\eta}\sigma^2-\bar{\beta}^2\sigma_\mu+6\bar{\eta},\quad D^3_{\mu\alpha}=\bar{\eta}''\sigma_\mu+\eta''\sigma^2,$$
$$\sigma^2=2(\cos\alpha^2-1),\qquad \gamma^2_2=4\sin^2\alpha^2.$$

The quantities φ^1, φ^2, u^3 will be found from Eqs. (2.12). Let us consider also the particular case in which crossed bars are connected but do not penetrate each other. In this case the rigidities for torsion of crossed bars should be set equal to zero $(\beta=\bar{\beta}=0)$.

In this particular case the solution of the set of equations (2.3) is simplified considerably. With flexural rigidities in the directions x^1, x^2, we obtain for $M^1 = M^2 = 0$, the equation

$$Du^3 = \frac{1}{2\eta''} (\Delta_1^2+6)(\Delta_2^2+6) P^3, \tag{2.17}$$

where

$$D = \Delta_1^4(\Delta_2^2+6) + \Delta_2^4(\Delta_1^2+6).$$

Let us reconsider Eqs. (2.3) for loads independent of the variable x^2. We have the equations

$$K_1 \varphi^1 = M^1, \quad K_2 \varphi^2 - \bar{\eta}' C_1 u^3 = M^2, \quad -\bar{\eta}' C_1 \varphi^2 + K_3 u^3 = -P^3, \tag{2.18}$$

where

$$K_1 = -\beta\Delta_1^2 + 6(\eta+\gamma), \quad K_2 = \bar{\eta}(\Delta_1^2+6) + 6\bar{\gamma}, \quad K_3 = \bar{\eta}'' \Delta_1^2.$$

The angle φ^1 is independent of the loads M^2, P^3. The set of equations (2.18) can be solved in a very simple manner as a set of ordinary difference equations of the second order. On introducing the expressions

$$\varphi^2 = K_3 \chi^2 + \bar{\eta}' C_1 \chi^3, \quad u^3 = \bar{\eta}' C_1 \chi^2 + K_2 \chi^3 \tag{2.19}$$

we find the functions χ^2, χ^3 by solving the equations

$$\Omega\chi^2 = M^2, \quad \Omega\chi^3 = -P^3 \tag{2.20}$$

where

$$\Omega = K_2 K_3 - \bar{\eta}'^2 C_1^2.$$

Knowing χ^2, χ^3 from the solution of Eqs. (2.20), we find φ^2, u^3 from the relations (2.19).

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В. Новацки, **Двумерная задача для ортогональных ростверков**

Содержание. В работе рассматривается такой правильный пространственный ростверк, в котором деформация описывается через перемещения $\mathbf{u}=(0, 0, u^3)$ и вращение $\boldsymbol{\varphi}(\varphi', \varphi^2, 0)$. Путем введения вектора $\boldsymbol{\chi}$, в простую систему разностных уравнений (2.5) вводится система трех разностных уравнений (2.3). Приводятся нескольких примеры решений полученных с применением конечного синусового преобразования и экспоненциального преобразования Фурье.