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APPLIED MECHANICS
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Two-dimensional Problem of Micropolar Magnetoelasticity

by

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Summary. In this paper the author is concerned with a micropolar, elastic, isotropic, homogeneous and centrosymmetric medium. The medium is assumed to be situated within a constant primary magnetic field **H**. Under the effect of body forces and moments acting on the medium it becomes deformed and, moreover, an electro-magnetic field is excited. In the first part of the paper the basic equations of magneto-elasticity are derived under the assumption that the medium evidences a perfect electric conductivity. In the second part the bidimensional problem is discussed independent of the variable x_3 assuming $\mathbf{H} \equiv (0, 0, H_3)$. Decomposing the displacement vector into the potential and solenoidal parts we succeeded in deriving the wave equations. In the third part bidimensional deformation field is considered; it depends on the variables x_1 and x_3 assuming, as above, $\mathbf{H} \equiv (0, 0, H_3)$.

The equations of motion could be decomposed into two systems of equations independent of each other. One of them is reduced to simple wave equations by decomposing the rotation vectors into the potential and solenoidal parts. The second one is reduced to wave equations, too, by means of the stress function.

1. General equations

In this note we shall consider a homogeneous, isotropic, centrosymmetric, elastic micropolar medium placed within a constant, strong primary magnetic field **H**. Under the effect of external forces acting on the micropolar body a deformation field will be formed in this body and, moreover, an electromagnetic field, too.

We assume further that the medium evidences a perfect electric conductivity.

The starting point of our considerations consists of two groups of equations, namely the equations of thermodynamics of slow-moving media [1] and equations of motion of micropolar elasticity with terms derived from Lorentz forces.

$$
(1.1)\qquad \begin{cases} \operatorname{rot}\mathbf{h}=\dfrac{4\pi}{c}\mathbf{j}, \qquad \operatorname{rot}\mathbf{E}=-\dfrac{\mu_0}{c}\dot{\mathbf{h}}, \\[2ex] \mathbf{E}=-\dfrac{\mu_0}{c}(\dot{\mathbf{u}}\times\mathbf{H}), \qquad \operatorname{div}\dot{\mathbf{h}}=0. \end{cases}
$$

where the symbols **h** and **E** stand for the vectors of magnetic and electric fields, respectively, **j** denotes the vector of current density, **H** is to denote the vector of constant primary magnetic field, **u** stands for the displacement vector, μ_0 denotes the magnetic permeability factor and, finally, c is the velocity of light.

The second group consists of equations of motion of a micropolar elastic medium subject to the action of constant primary magnetic field $\mathbf{H}$ [2÷4]:

$$\left\{\begin{aligned} &\Box_2\,\mathbf{u}+(\lambda+\mu-a)\operatorname{grad}\operatorname{div}\mathbf{u}+2a\operatorname{rot}\boldsymbol{\varphi}+\frac{\mu_0}{c}(\mathbf{j}\times\mathbf{H})+\mathbf{X}=0,\\ &\Box_4\,\boldsymbol{\varphi}+(\beta+\gamma-\varepsilon)\operatorname{grad}\operatorname{div}\boldsymbol{\varphi}+2a\operatorname{rot}\mathbf{u}+Y=0, \end{aligned}\right. \tag{1.2}$$

where

$$\Box_2=(\mu+a)\,\nabla^2-\rho\partial_t^2,\qquad \Box_4=(\gamma+\varepsilon)\,\nabla^2-4a-J\partial_t^2,\qquad \nabla^2=\partial_i\partial_i.$$

Here, in Eqs. (1.2) the symbol $\mathbf{X}$ denotes the vector of body forces, while $\mathbf{Y}$ that of body couples; $\mathbf{u}$ is the displacement vector as in Eq. (1.1); the rotation vector is denoted by the symbol $\boldsymbol{\varphi}$. The quantities a, β, γ, ε, μ, λ, are material constants. Finally, ρ denotes the density and J — the rotational inertia.

The state of stress is characterized by two asymmetric tensors: The tensor of force stresses σ_{ji} and that of couple stresses μ_{ji}.

The constitutive equations have the form:

$$\begin{aligned} &\sigma_{ji}=(\mu+a)\,\gamma_{ji}+(\mu-a)\,\gamma_{ij}+\lambda\gamma_{kk}\delta_{ji},\\ &\mu_{ji}=(\gamma+\varepsilon)\,\kappa_{ji}+(\gamma-\varepsilon)\,\kappa_{ij}+\beta\delta_{ji}\,\kappa_{kk},\qquad i,j=1,2,3. \end{aligned} \tag{1.3}$$

The strain tensor γ_{ji} and the curvature-twist tensor κ_{ji} are connected with the vectors $\mathbf{u}$ and $\boldsymbol{\varphi}$, respectively, by the following relations

$$\gamma_{ji}=u_{i,j}-\varepsilon_{kji}\,\varphi_k,\qquad \kappa_{ji}=\varphi_{i,j},\qquad ij=1,2,3. \tag{1.4}$$

In our subsequent considerations we shall assume that the primary magnetic field is reduced to the sole component $\mathbf{H}\equiv(0,0,H_3)$ along the direction of the x_3-axis.

In accordance with this assumption we derive from Eqs. (1.1) the following relations:

$$\begin{aligned} &\mathbf{E}=\frac{\mu_0\,H_3}{c}(-\partial_t\,u_2,\,\partial_t\,u_1,\,0),\\ &\partial_t\,\mathbf{h}=\frac{c}{\mu_0}(\partial_3\,E_2,-\partial_3\,E_1,\,\partial_2\,E_1,-\partial_1\,E_2),\\ &\mathbf{j}=\frac{c}{4\pi}(\partial_2\,h_3-\partial_3\,h_2,\,\partial_3\,h_1-\partial_1\,h_3,\,\partial_1\,h_2-\partial_2\,h_1). \end{aligned} \tag{1.5}$$

Expressing in the above equations the components of the vector $\mathbf{j}$ by those of the vector $\mathbf{u}$ and introducing them into Eqs. (1.2), we obtain the following equations of motion in terms of the displacement $\mathbf{u}$ and rotation $\boldsymbol{\varphi}$ vectors.

$$\begin{aligned} &\Box_2\,u_1+(\lambda+\mu-a+a_0^2\,\rho)\,\partial_1\,e+2a\,(\partial_2\,\varphi_3-\partial_3\,\varphi_2)+\\ &\qquad\qquad+a_0^2\,\rho\partial_3\,(\partial_3\,u_1-\partial_1\,u_3)+X_1=0,\\ &\Box_2\,u_2+(\lambda+\mu-a+a_0^2\,\rho)\,\partial_2\,e+2a\,(\partial_3\,\varphi_1-\partial_1\,\varphi_3)+\\ &\qquad\qquad+a_0^2\,\rho\partial_3\,(\partial_3\,u_2-\partial_2\,u_3)+X_2=0,\\ &\Box_2\,u_3+(\lambda+\mu-a)\,\partial_3\,e+2a\,(\partial_1\,\varphi_2-\partial_2\,\varphi_1)+X_3=0, \end{aligned} \tag{1.6}$$

and

$$\begin{aligned} &\Box_4 \varphi_1+(\beta+\gamma-\varepsilon)\,\partial_1 \kappa+2a(\partial_2 u_3-\partial_3 u_2)+Y_1=0,\\ &\Box_4 \varphi_2+(\beta+\gamma-\varepsilon)\,\partial_2 \kappa+2a(\partial_3 u_1-\partial_1 u_3)+Y_2=0,\\ &\Box_4 \varphi_3+(\beta+\gamma-\varepsilon)\,\partial_3 \kappa+2a(\partial_1 u_2-\partial_2 u_1)+Y_3=0,\end{aligned} \tag{1.7}$$

where:

$$e=\partial_k u_k, \quad \kappa=\partial_k \varphi_k, \quad a_0^2=\frac{\mu_0 H_3^2}{4\pi\rho}.$$

Here $e=\operatorname{div}\mathbf{u}$ is the dilatation, $\kappa=\operatorname{div}\boldsymbol{\varphi}$; the symbol a_0 denotes the Alfvén velocity. Eqs. (1.6) and (1.7) form a hyperbolic system of mutually coupled differential equations. The coupling of electromagnetic and deformation fields is described by the term $a_0^2\rho$. For $H_3\to 0$ Eqs. (1.6) and (1.7) assume the form of equations of micropolar elasticity.

2. The first two-dimensional problem

Now, considering the two-dimensional problem, let us observe that three individual cases can be discussed assuming that all causes and effects depend on the variables x_1x_2 or x_2x_3 or, finally, on x_1x_3. In this chapter we assume all the quantities, appearing in Eqs. (1.5)÷(1.7), to be independent of x_3.From the formula (1.5) we derive

$$\left\{\begin{aligned} &\mathbf{E}=\frac{\mu_0 H_3}{c}(-\dot{u}_2,\dot{u}_1,0), \qquad \dot{\mathbf{h}}=\frac{c}{\mu_0}(0,0,\partial_2 E_1-\partial_1 E_2)=H_3(0,0,-\dot{e}),\\ &\mathbf{j}=\frac{c}{4\pi}(\partial_2 h_3-\partial_1 h_3,0)=\frac{cH_3}{4\pi}(-\partial_2 e,\partial_1 e,0),\end{aligned}\right. \tag{2.1}$$

$$e=\partial_1 u_1+\partial_2 u_2.$$

The system of Eqs. (1.6) and (1.7) can also be split into two systems independent of each other. In the first of them the displacement functions u_1, u_2 and the rotation φ_3 appear as unknown quantities. We write:

$$\left\{\begin{aligned} &\Box_2 u_1+(\lambda+\mu-a+a_0^2\rho)\,\partial_1 e+2a\,\partial_2 \varphi_3+X_1=0,\\ &\Box_2 u_2+(\lambda+\mu-a+a_0^2\rho)\,\partial_2 e-2a\,\partial_1 \varphi_3+X_2=0,\\ &\Box_4 \varphi_3+2a(\partial_1 u_2-\partial_2 u_1)+Y_3=0.\end{aligned}\right. \tag{2.2}$$

Here

$$\Box_2=(\mu+a)\,\nabla_1^2-\rho\partial_t^2, \quad \Box_4=(\gamma+\varepsilon)\,\nabla_1^2-4a-J\partial_t^2, \quad \nabla_1^2=\partial_1^2+\partial_2^2.$$

In the second system of equations the role of unknown quantities is played by the displacement function u_3 and the rotations $\varphi_1\varphi_2$.

$$\left\{\begin{aligned} &\Box_4 \varphi_1+(\beta+\gamma-\varepsilon)\,\partial_1 \kappa+2a\partial_2 u_3+Y_1=0,\\ &\Box_4 \varphi_2+(\beta+\gamma-\varepsilon)\,\partial_2 \kappa-2a\partial_1 u_3+Y_2=0,\\ &\Box_2 u_3+2a(\partial_1 \varphi_2-\partial_2 \varphi_1)+X_3=0, \qquad \kappa=\partial_1 \varphi_1+\partial_2 \varphi_2.\end{aligned}\right. \tag{2.3}$$

Eqs. (2.2) are coupled with those describing the electromagnetic field (2.1). This coupling is characterized by the term, $a_0^2\rho$ absent in Eqs. (2.3). The causes Y_1, Y_2, Y_3 do not elicit any disturbances in the electromagnetic fields. The quantities φ_1, φ_2, u_3 do not appear in the field equations (2.1). In our subsequent considerations we shall be concerned solely with the system of Eqs. (2.2), The system of Eqs. (2.2) can be partially disjoined by introducing the decomposition of the vector of displacement $\mathbf{u}\equiv(u_1, u_2, 0)$ and of that of body forces $\mathbf{X}\equiv(X_1, X_2, 0)$ into two parts: potential and solenoidal

$$\text{(2.4)}\qquad \begin{cases} u_1=\partial_1\,\Phi+\partial_2\,\Psi, & u_2=\partial_2\,\Phi-\partial_1\,\Psi,\\ X_1=\rho(\partial_1\,\vartheta+\partial_2\,\chi), & X_2=\rho\,(\partial_2\,\vartheta-\partial_1\,\chi). \end{cases}$$

Introducing (2.4) into (2.2), we obtain the following system of Eqs. [5]:

$$\text{(2.5)}\qquad \mathring{\Box}_1\,\Phi+\rho\vartheta=0,\qquad \Box_2\,\Psi+2a\,\varphi_3+\rho\chi=0,\qquad \Box_4\,\varphi_3-2a\,\nabla_1^2\,\Psi+Y_3=0.$$

Now we eliminate functions φ_3 and Ψ from Eqs. $(2.5)_{1,2}$; in this way we obtain a system of two equations independent of each other

$$\text{(2.6)}\qquad \begin{aligned} &\mathring{\Box}_1\,\Phi+\rho\vartheta=0,\\ &(\Box_2\,\Box_4+4a^2\,\dot{\nabla}_1^2)\,\Psi=2a\,Y_3-\rho\Box_4\,\chi,\\ &(\Box_2\,\Box_4+4a^2\,\nabla_1^2)\,\varphi_3=-2a\rho\,\nabla_1^2\,\chi-\Box_2\,Y_3, \end{aligned}$$

where: $\mathring{\Box}_1=(\lambda+2\mu+a_0^2\,\rho)\,\nabla_1^2-\rho\partial_t^2$.

We obtain three wave equations. The first of them describes the longitudinal wave, while the second —the transverse displacement wave. The third equation describes the propagation of rotation φ_3. Only the first equation is coupled with that of electromagnetic field; the coupling factor $a_0^2\rho$ appears in the operator $\mathring{\Box}_1$. The longitudinal wave is elicited by the body forces $\mathbf{X}'=\rho$ grad ϑ.

Eqs. $(2.6)_{2,3}$ are of the same form as those describing the micropolar elasticity. The quantities Ψ and φ_3, due to the action of body forces $\mathbf{X}''=\rho\,\mathrm{rot}\,\boldsymbol{\chi}$, are independent of the quantity H_3 characterizing the primary magnetic field. The functions Φ, Ψ and φ_3 being determined from the wave equations (2.6), we may then obtain — from the formulae (2.1) (1.3) and (1.4) —all the components of the electromagnetic and deformation fields.

Let us now consider Eqs. (2.2) introducing therein the displacements u_1, u_2 and the rotation φ_3 expressed as the functions F_1, F_2, M_3 in the following relations

$$\text{(2.7)}\qquad \begin{aligned} &u_1=\mathring{\Box}_1\,\Box_4\,F_1-\partial_1\,[\partial_1(\mathring{\Gamma}\,F_1)+\partial_2(\mathring{\Gamma}\,F_2)]-2a\,\Box_3\,\partial_2\,M_3,\\ &u_2=\mathring{\Box}_1\,\Box_4\,F_2-\partial_2\,[\partial_1(\mathring{\Gamma}\,F_1)+\partial_2\,(\mathring{\Gamma}\,F_2)]+2a\Box_3\,\partial_1\,M_3,\\ &\varphi_3=2a\,\mathring{\Box}_1\,(\partial_2\,F_1-\partial_1\,F_2)+\Box_2\,\Box_3\,M_3. \end{aligned}$$

The following notations have been introduced here:

$$\Box_3=(\beta+2\gamma)\,\nabla_1^2-4a-J\partial_t^2,\qquad \mathring{\Gamma}=(\lambda+\mu-a+a_0^2\,\rho)\,\Box_4-4a^2.$$

Introduction of the relations (2.7) into Eqs. (2.2) leads to three differential equations, wherein only one function from among F_1, F_2 or M_3 appears

$$\begin{aligned}
&\mathring{\Box}_1 (\Box_2 \Box_4+4a^2 \nabla_1^2) F_1+X_1=0 ,\\
(2.8)\qquad &\mathring{\Box}_1(\Box_2 \Box_4+4a^2 \nabla_1^2) F_2+X_2=0 ,\\
&\Box_3 (\Box_2 \Box_4+4a^2 \nabla_1^2) M_3+Y_3=0 .
\end{aligned}$$

Eqs. (2.8) are a generalization of Sandru's equations [6] for micropolar elasticity. These equations may serve for the construction of singular solutions of Eqs. (2.2).

Let us observe that only the function M_2 is independent of Eqs. (2.1). For the case $F_1=F_2=0$, and thus $X_1=X_2=0$ we have from the relations (2.7) that the displacements u_1, u_2 and the rotation φ_3 do not depend on the quantity H_3. The effect of coupling of electromagnetic and deformation fields manifests itself in Eqs. (2.8) for F_1 and F_2 in the operator $\mathring{\Box}_1$.

Let us return to Eqs. (2.2). We assume all the causes and effects to be independent of the variable x_2. Under this assumption we get the following system of equations:

$$(2.9)\qquad \begin{cases}
[(\lambda+2\mu+a_0^2 \rho)\, \partial_1^2-\rho\partial_t^2]\, u_1+X_1=0,\\
[(\mu+a)\, \partial_1^2-\rho\partial_t^2]\, u_2-2a\, \partial_1\, \varphi_3+X_2=0 ,\\
[(\gamma+\varepsilon)\, \partial_1^2-4a-J\partial_t^2]\, \varphi_3+2a\, \partial_1\, u_2+Y_2=0 .
\end{cases}$$

In this one-dimensional problem the two last equations are coupled. The effect of disturbance of the electromagnetic field appears only in Eq. $(2.9)_1$. From the formulae (2.1) we have

$$\begin{aligned}
&\mathbf{E}=\frac{\mu_0\, H_3}{c}(-\dot{u}_2, \dot{u}_1, 0), \qquad \mathbf{h}=H_3\,(0, 0, -\partial_1\, u_1),\\
(2.10)\qquad &\mathbf{j}=\frac{cH_3}{4\pi}\,(0, \partial_1^2, u_1, 0).
\end{aligned}$$

The system of Eqs. (2.9) is connected with the state of stress in the following way:

$$\begin{aligned}
&\sigma_{11}=(2\mu+\lambda)\, \partial_1\, u_1 , \quad \sigma_{22}=\sigma_{33}=\lambda\partial_1\, u_1 ,\\
(2.11)\qquad &\sigma_{12}=(\mu+a)\, \partial_1\, u_2-2a\, \varphi_3 , \quad \sigma_{21}=(\mu-a)\, \partial_1\, u_2+2a\, \varphi_3 ,\\
&\mu_{13}=(\gamma+\varepsilon)\, \partial_1\, \varphi_3 , \quad \mu_{31}=(\gamma-\varepsilon)\, \partial_1\, \varphi_3 .
\end{aligned}$$

3. The second two-dimensional problem

We assume in our subsequent consideration that —under the assumed magnetic field $\mathbf{H}\equiv(0, 0, H_3)$ —all the causes and effects are the functions of the variables x_1 and x_3.

In this particular case the systems of Eqs. (1.6) and (1.7) will be disjoined into two systems of equations independent of each other

$$
(3.1)\quad \begin{cases} \Box_2 u_1+(\lambda+\mu-a+a_0^2 \rho)\,\partial_1 e-2a\partial_3 \varphi_2+a_0^2 \rho\,\partial_3(\partial_3 u_1-\partial_1 u_3)+X_1=0, \\ \Box_2 u_3+(\lambda+\mu-a)\,\partial_3 e+2a\,\partial_1 \varphi_2+X_3=0, \\ \Box_4 \varphi_2+2a\,(\partial_3 u_1-\partial_1 u_3)+Y_2=0, \end{cases}
$$

and

$$
(3.2)\quad \begin{cases} \Box_4 \varphi_1+(\beta+\gamma-\varepsilon)\,\partial_1 \kappa-2a\,\partial_3 u_2+Y_1=0, \\ \Box_4 \varphi_3+(\beta+\gamma-\varepsilon)\,\partial_3 \kappa+2a\,\partial_1 u_2+Y_3=0, \\ \Box_2 u_2+2a\,(\partial_3 \varphi_1-\partial_1 \varphi_3)+a_0^2 \rho\,\partial_3^2 u_2+X_2=0. \end{cases}
$$

Now the following notations have been introduced:

$$
\begin{cases} \Box_2=(\mu+a)\,\nabla_1^2-\rho\partial_t^2, \quad \Box_4=(\gamma+\varepsilon)\,\nabla_1^2-4a-J\partial_t^2, \\ \nabla_1^2=\partial_1^2+\partial_3^2, \quad e=\partial_1 u_1+\partial_3 u_3, \quad \kappa=\partial_1 \varphi_1+\partial_3 \varphi_3. \end{cases}
$$

It is seen that in both systems of equations there is a coupling of the components of the displacement $\mathbf{u}(x_1, x_3, t)$ and rotation $\boldsymbol{\varphi}(x_1, x_3, t)$ vectors with the electromagnetic field.

Let us consider first the system of Eqs. (3.2) which by the introduction of the relations

$$
(3.3)\quad \begin{aligned} &\varphi_1=\partial_1 \Omega+\partial_3 \Xi, \qquad \varphi_3=\partial_3 \Omega-\partial_1 \Xi, \\ &Y_1=J\,(\partial_1 \sigma+\partial_3 \eta), \quad Y_3=J\,(\partial_3 \sigma-\partial_1 \eta), \end{aligned}
$$

may be easily reduced to a system of wave equations:

$$
(3.4)\quad \begin{cases} \Box_3 \Omega+J\sigma=0, \\ \Box_4 \Xi-2a\,u_2+J\eta=0, \\ (\Box_2+a_0^2 \rho\partial_3^2)\,u_2+2a\,\nabla_1^2 \Xi+X_2=0, \end{cases}
$$

where: $\Box_3=(2\gamma+\beta)\,\nabla_1^2-4a-J\partial_t^2$.

Eliminating the functions u_2 and Ξ from the two last equations, we arrive at

$$
(3.5)\quad \begin{aligned} &(\hat{\Box}_2\,\Box_4+4a^2\,\nabla_1^2)\,\Xi=2a\,J\nabla_1^2\,\eta-\Box_4\,X_2, \\ &(\hat{\Box}_2\,\Box_4+4a^2\,\nabla_1^2)\,u_2=2aX_2-J\hat{\Box}_2\,\eta. \end{aligned}
$$

Here: $\hat{\Box}_2=\Box_2+a_0^2\,\rho^2\,\partial_3^2$.

The first of (3.4) equations represents the equation of the microrotational longitudinal wave; it is identical with that describing the wave of this type in undisturbed micropolar medium. Eqs. (3.5) represent the propagation of the transverse wave. The first of them corresponds to the microrotational transverse wave, while the second — to the displacement transverse wave. Both are disturbed. For $H_3 \to 0$ both of them reduce to the equations of micropolar elasticity. The system of

Eqs. (3.1) cannot be divided into potential and solenoidal parts by means of the decomposition of the vectors $\mathbf{u}\equiv(u_1, 0, u_3)$ and $\boldsymbol{\varphi}\equiv(0, \varphi_2, 0)$.

Let us express the displacements u_1, u_3 and the rotation φ_3 by the functions F_1, M_2, F_3 in the following way:

$$
(3.6)\quad \begin{cases}
u_1=(\square_1\,\square_4-\Gamma\partial_1^2)\,F_1+2a\,\square_1\,\partial_3\,M_2-\partial_1\,\partial_3\,\Gamma F_3,\\
\varphi_2=-2a\,\square_1\,\partial_3\,F_1+[\square_1\,\mathring{\square}_1-(\lambda+\mu-a)\,(\partial_1^2\,\mathring{\square}_1+\\
\qquad\qquad +\partial_3^2\,\square_1)]\,M_2+2a\,\mathring{\square}_1\,\partial_1\,F_3,\\
u_3=-\partial_1\,\partial_3\,\Gamma F_1-2a\,\mathring{\square}_1\,\partial_1\,M_2+(\mathring{\square}_1\,\square_4-\Gamma\partial_3^2)\,F_3,\\
\mathring{\square}_1=(\lambda+2\mu+a_0^2\,\rho)\,\nabla_1^2-\rho\partial_t^2,\quad \Gamma=(\lambda+\mu-a)\,\square_4-4a^2.
\end{cases}
$$

Substituting the above relations into Eqs. (3.1), we obtain the following equations —independent of each other —describing the functions F_1, M_2, F_3:

$$
(3.7)\quad \begin{cases}
[\square_1\,\square_4\,\mathring{\square}_1-\Gamma\,(\partial_1^2\,\mathring{\square}_1+\partial_3^2\,\square_1)]\,F_1+X_1=0,\\
[\square_1\,\square_4\,\mathring{\square}_1-\Gamma\,(\partial_1^2\,\mathring{\square}_1+\partial_3^2\,\square_1)]\,M_2+Y_2=0,\\
[\square_1\,\square_4\,\mathring{\square}_1-\Gamma\,(\partial_1^2\,\mathring{\square}_1+\partial_3^2\,\square_1)]\,F_3+X_3=0.
\end{cases}
$$

The above equations are a generalization of the Galerkin's vector on the problem of micropolar magneto-elasticity.

Eqs. (3.7) may be used for finding out singular solutions of Eqs. (3.1) in the infinite elastic space.

The considerations set forth in this paper will be dealt with more amply in Proceedings of Vibration Problems.

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В. Новацки, **Двухмерная проблема микрополярной магнито-упругой среды**

Содержание. Рассмотрена микрополярная упругая, изотропная, однородная и центросимметрическая среда. Упомянутая среда находится в постоянном, первоначальном, магнитном поле **H**. Действие сил и массовых моментов вызывает деформацию среды, а также возбуждает электромагнитное поле. В первой части работы приводятся основные уравнения магнитоупругости при предположении идеально проводящей среды. Во второй части работы рассмотрена двухмерная проблема, независимая от переменной x_3, при предположении, что $\mathbf{H} \equiv (0, 0, H_3)$. Путем разложения вектора перемещения на потенциальную и соленоидальную части, получены волновые уравнения. В третьей части работы рассмотрено двухмерное поле деформации зависимое от x_1 и x_3 при предположении, что $\mathbf{H} \equiv (0, 0, H_3)$. Уравнения движения здесь разложены на две независимые системы уравнений. Одна из них сводится к обыкновенным волновым уравнениям путем декомпозиции векторов оборота (вращения) на потенциальную и соленоидальную части, другая система сводится к волновым уравнениям при использовании функции напряжений.