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APPLIED MECHANICS
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On Certain Thermoelastic Problems in Micropolar Elasticity

by

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1. Introduction

The aim of this paper is to derive integral formulae similar to those of Maysel in the classical theory of thermoelasticity [1]. We shall also be concerned with problems, the solutions of which are identical with those of classical thermoelasticity.

Under the effect of external loads and heating the body will suffer deformations. Displacement $\mathbf{u}(\mathbf{x}, t)$, rotation $\boldsymbol{\varphi}(\mathbf{x}, t)$ and temperature $\theta(\mathbf{x}, t)$ fields will form in the body changing with the position of the point $\mathbf{x}$ and time t.

The state of deformation is described by two tensors: the asymmetric deformation tensor γ_{ji} and the curvature-twist tensor $\varkappa_{ji}$. As known [2], there is

$$\gamma_{ji} = u_{i,j} - \epsilon_{kij}\varphi_k, \qquad \varkappa_{ji} = \varphi_{i,j}. \tag{1.1}$$

The state of stress is characterized by two asymmetric tensors: the force stress tensor σ_{ji} and couple-stress tensor μ_{ji}. The state of stress and that of deformation are connected by the constitutive equations [3]:

$$\begin{aligned} \sigma_{ji} &= (\mu+\alpha)\gamma_{ji} + (\mu-\alpha)\gamma_{ij} + (\lambda\gamma_{kk} - \nu\theta)\delta_{ij}, \\ \mu_{ji} &= (\gamma+\varepsilon)\varkappa_{ji} + (\gamma-\varepsilon)\varkappa_{ij} + \beta\varkappa_{kk}\delta_{ij}, \end{aligned} \tag{1.2}$$

where μ, λ are Lamé constants, while $\alpha, \beta, \gamma, \varepsilon$ denote new material constants. $\gamma = (3\lambda+2\mu)\alpha_t$ is the coefficient of linear thermal dilatation.

Introducing Eqs. (1.2) into the equations of motion

$$\sigma_{ji,j} + X_i - \varrho\ddot{u}_i = 0, \qquad \epsilon_{ijk}\sigma_{jk} + \mu_{ji,j} + Y_i - I\ddot{\varphi}_i = 0. \tag{1.3}$$

and then making use of the relations (1.1), we obtain the following system of equations

$$\begin{aligned} &\square_2 \mathbf{u} + (\lambda+\mu-\alpha)\,\mathrm{grad\,div}\,\mathbf{u} + 2\alpha\,\mathrm{rot}\,\boldsymbol{\varphi} + \boldsymbol{X} = \nu\,\mathrm{grad}\,\theta, \\ &\square_4 \boldsymbol{\varphi} + (\beta+\gamma-\varepsilon)\,\mathrm{grad\,div}\,\boldsymbol{\varphi} - 4\alpha\boldsymbol{\varphi} + 2\alpha\,\mathrm{rot}\,\mathbf{u} + \boldsymbol{Y} = 0, \end{aligned} \tag{1.4}$$

where

$$\Box_2 = (\mu+\alpha)\,\nabla^2 - \varrho\partial_t^2, \quad \Box_4 = (\gamma+\varepsilon)\,\nabla^2 - 4\alpha - I\partial_t^2,$$
$$\nabla^2 = \frac{\partial}{\partial x_i}\,\frac{\partial}{\partial x_i}, \quad \partial_t^2 = \frac{\partial^2}{\partial t^2}.$$

These equations should be supplemented by the equation of heat conductivity

$$D\theta = -\frac{Q}{\varkappa}, \quad D = \nabla^2 - \frac{1}{\varkappa}\,\partial_t. \tag{1.5}$$

In the above equations the symbols X_i and Y_i denote the volume densities of body forces and body couples, respectively, ϵ_{ijk} stands for the alternator used, ϱ — for the density of the body and I — for the rotational inertia. $\theta = T - T_0$ is the difference between the absolute temperature T and the temperature of the body T_0 in natural state; $\varkappa = \lambda_0/\varrho c_\varepsilon$ is a coefficient, wherein λ_0 denotes the heat conductivity and c_ε the specific heat, the deformation being assumed constant. Finally, $Q = W/\varrho c_\varepsilon$, where W stands for the quantity of heat generated in a volume unit of the body within a time unit.

In the sequel we shall make use of the theorem on the reciprocity of works. Considering two systems of causes and effects (the second one will be marked with "primes"), we obtain for the problem of non-coupled thermoelasticity the following equation [1], the initial conditions of the functions $\mathbf{u}, \mathbf{u}', \boldsymbol{\varphi}, \boldsymbol{\varphi}'$ being assumed homogeneous

$$\begin{aligned} \int_V (X_i * u_i' + Y_i * \varphi_i')\,dV + \int_A (p_i * u_i' + m_i * \varphi_i')\,dA + \nu \int_V \theta * \gamma_{kk}'\,dV = \\ = \int_V (X_i' * u_i + Y_i' * \varphi_i)\,dV + \int_A (p_i' * u_i + m_i' * \varphi_i)\,dA + \nu \int_V \theta' * \gamma_{kk}\,dV, \end{aligned} \tag{1.6}$$

where

$$X_i * u_i' = \int_0^t X_i(\mathbf{x}, t-\tau)\cdot u_i'(\mathbf{x}, \tau)\,d\tau = \int_0^t X_i(\mathbf{x},\tau)\,u_i'(\mathbf{x}, t-\tau)\,d\tau,$$

and so on.

2. Generalized Maysel's formulae

Let us consider a micropolar elastic body subjected to heating. The displacements $\mathbf{u}(\mathbf{x}, t)$, rotations $\boldsymbol{\varphi}(\mathbf{x}, t)$ and the temperature as well have to verify the following system of differential equations

$$\begin{aligned} L(\mathbf{u}) + M(\boldsymbol{\varphi}) + N(\theta) &= 0, \\ M(\mathbf{u}) + K(\boldsymbol{\varphi}) &= 0, \\ D(\theta) + \frac{Q}{\varkappa} &= 0. \end{aligned} \tag{2.1}$$

The following notations have been introduced in Eq. (2.1)

$$L(\mathbf{u}) = (\mu+\alpha)\,\Box_2\,\mathbf{u}+(\lambda+\mu-\alpha)\,\mathrm{grad\,div}\,\mathbf{u}, \qquad M(\mathbf{u}) = 2\alpha\,\mathrm{rot}\,\mathbf{u},$$

$$K(\boldsymbol{\varphi}) = (\gamma+\varepsilon)\,\Box_4\,\boldsymbol{\varphi}+(\beta+\gamma-\varepsilon)\,\mathrm{grad\,div}\,\boldsymbol{\varphi}-4\alpha\boldsymbol{\varphi},$$

$$N(\theta) = -\nu\,\mathrm{grad}\,\theta, \qquad D(\theta) = \left(\nabla^2-\frac{1}{\varkappa}\,\partial_t\right)\theta.$$

Let us assume that on the surface A, bounding the body, the following homogeneous mixed boundary conditions are prescribed:

$$\begin{aligned} &\mathbf{u}=0, \quad \boldsymbol{\varphi}=0, \quad \theta_{,n}=0, \quad \mathbf{x}\in A_u, \\ &\mathbf{p}=0, \quad \mathbf{m}=0, \quad \theta=0, \quad \mathbf{x}\in A_\sigma, \quad A_u+A_\sigma=A. \end{aligned} \tag{2.2}$$

We assume the initial conditions to be homogeneous.

Now let us assume that an instantaneous concentrated unary force $\mathbf{X}=\delta(\mathbf{x}-\boldsymbol{\xi})\,\delta(t)\,\mathbf{e}_j$ — directed in parallel to the x_j-axis — is acting at the point $\boldsymbol{\xi}\in V$ of the body in isothermal state ($\theta'=0$). The action of this force will induce in the body displacements $\mathbf{u}'=\mathbf{U}^{(j)}(\mathbf{x},\boldsymbol{\xi},t)$ and rotations $\boldsymbol{\varphi}'=\boldsymbol{\Phi}^{(j)}(\mathbf{x},\boldsymbol{\xi},t)$. These functions have to verify the following differential equations

$$\begin{aligned} &L(\mathbf{U}^{(j)})+M(\boldsymbol{\Phi}^{(j)})+\delta(\mathbf{x}-\boldsymbol{\xi})\,\delta(t)\,\mathbf{e}_j=0, \\ &M(\mathbf{U}^{(j)})+K(\boldsymbol{\Phi}^{(j)})=0. \end{aligned} \tag{2.3}$$

The boundary conditions are assumed to be homogeneous, i.e. there is

$$\begin{aligned} &\mathbf{U}^{(j)}=0, \quad \boldsymbol{\Phi}^{(j)}=0, \quad \mathbf{x}\in A_u, \\ &\mathbf{p}^{(j)}=0, \quad \mathbf{m}^{(j)}=0, \quad \mathbf{x}\in A_\sigma. \end{aligned} \tag{2.4}$$

Here, $\mathbf{p}^{(j)}$ denotes the main stress vector, while $\mathbf{m}^{(j)}$ stands for the main couple-stress vector on the surface A_σ.

We shall apply the theorem on the reciprocity of works, Eq. (1.6), to the two systems of causes and effects considered in this paper. As a result we obtain the following formula

$$u_j(\boldsymbol{\xi},t) = \nu\int_V dV(\mathbf{x})\int_0^t \theta(\mathbf{x},t-\tau)\,U^{(j)}_{k,k}(\mathbf{x},\boldsymbol{\xi},\tau)\,d\tau. \tag{2.5}$$

Consider now another system of loadings, that marked by "primes". An instantaneous concentrated unary body couple $\mathbf{Y}=\delta(\mathbf{x}-\boldsymbol{\xi})\,\delta(t)\,\mathbf{e}_j$ is supposed to act in parallel to the x_j-axis at the point $\boldsymbol{\xi}\in V$ of the body in isothermal state ($\theta'=0$). We denote the displacements and rotations induced by this action by $\mathbf{u}'=\mathbf{V}^{(j)}(\mathbf{x},\boldsymbol{\xi},t)$ and $\boldsymbol{\varphi}'=\boldsymbol{\Gamma}^{(j)}(\mathbf{x},\boldsymbol{\xi},t)$, respectively. These functions have to verify the differential equations of micropolar thermoelasticity

$$\begin{aligned} &L(\mathbf{V}^{(j)})+M(\boldsymbol{\Gamma}^{(j)})=0, \\ &M(\mathbf{V}^{(j)})+K(\boldsymbol{\Gamma}^{(j)})+\delta(\mathbf{x}-\boldsymbol{\xi})\,\delta(t)\,\mathbf{e}_j=0, \end{aligned} \tag{2.6}$$

assuming the homogeneity of initial and boundary (on the surfaces A_u and A_σ) conditions.

Making use of the theorem on reciprocity, Eq. (1.6), we obtain

$$\varphi_j(\boldsymbol{\xi}, t) = \nu \int_V dV(\mathbf{x}) \int_0^t \theta(\mathbf{x}, t-\tau)\, V^{(j)}_{k,k}(\mathbf{x}, \boldsymbol{\xi}, \tau)\, d\tau. \tag{2.7}$$

Formulae (2.5) and (2.7) represent the generalization of Maysel's formulae on the problems of micropolar thermoelasticity.

We may rewrite the expressions (2.5) and (2.7) in a somewhat modified form

$$\begin{aligned} u_j(\boldsymbol{\xi}, t) &= \nu \int_V dV(\mathbf{x}) \int_0^t \theta(\mathbf{x}, t-\tau)\, W_j(\boldsymbol{\xi}, \mathbf{x}, \tau)\, d\tau, \\ \varphi_j(\boldsymbol{\xi}, t) &= \nu \int_V dV(\mathbf{x}) \int_0^t \theta(\mathbf{x}, t-\tau)\, \Omega_j(\boldsymbol{\xi}, \mathbf{x}, \tau)\, d\tau. \end{aligned} \tag{2.8}$$

The term $W_j(\mathbf{x}, \boldsymbol{\xi}, t)$ stands here for the displacement, while $\Omega_j(\mathbf{x}, \boldsymbol{\xi}, t)$ for the rotation induced at the point $\mathbf{x}$ owing to the action of the pressure center situated at the point $\boldsymbol{\xi}$. The relations

$$W_j(\boldsymbol{\xi}, \mathbf{x}, t) = \mathbf{U}^{(j)}_{k,k}(\mathbf{x}, \boldsymbol{\xi}, t), \qquad \Omega_j(\boldsymbol{\xi}, \mathbf{x}, t) = V^{(j)}_{k,k}(\mathbf{x}, \boldsymbol{\xi}, t), \tag{2.9}$$

are also a consequence of the theorem on reciprocity, (1.6). For the steady-state problems of thermoelasticity the theorem on reciprocity takes the following form [3].

$$\begin{aligned} &\int_V (X_i u_i' + Y_i \varphi_i')\, dV + \int_A (p_i u_i' + m_i \varphi_i')\, dA + \nu \int_V \theta \gamma_{kk}'\, dV = \\ &= \int_V (X_i' u_i + Y_i' \varphi_i)\, dV + \int_A (p_i' u_i + m_i' \varphi_i)\, dA + \nu \int_V \theta' \gamma_{kk}\, dV. \end{aligned} \tag{2.10}$$

Proceeding similarly as in the dynamic problem, we get

$$\begin{aligned} u_j(\boldsymbol{\xi}) &= \int_V \theta(\mathbf{x})\, U^{(j)}_{k,k}(\mathbf{x}, \boldsymbol{\xi})\, dV(\mathbf{x}), \\ \varphi_j(\boldsymbol{\xi}) &= \int_V \theta(\mathbf{x})\, V^{(j)}_{k,k}(\mathbf{x}, \boldsymbol{\xi})\, dV(\mathbf{x}). \end{aligned} \tag{2.11}$$

The expressions (2.5) and (2.7) may be written in a particularly simple form in the case of an infinite space. Solving the systems of Eqs. (2.3) and (2.6), we obtain

$$\begin{aligned} u_j(\boldsymbol{\xi}, t) &= -\frac{1}{4\pi(\lambda+2\mu)} \frac{\partial}{\partial x_i} \int_V \frac{\theta\left(\mathbf{x}, t - \dfrac{R}{c_1}\right) dV(\mathbf{x})}{R(\mathbf{x}, \boldsymbol{\xi})}, \quad c_1 = \left(\frac{\lambda+2\mu}{\varrho}\right)^{1/2}, \\ \varphi_j(\boldsymbol{\xi}, t) &= 0. \end{aligned} \tag{2.12}$$

The latter result is due to the fact that $V^{(j)}_{k,k} = 0$. For the steady-state problem there is

$$\begin{aligned} u_j(\boldsymbol{\xi}) &= -\frac{1}{4\pi(\lambda+2\mu)} \frac{\partial}{\partial x_i} \int_V \frac{\theta(\mathbf{x})\, dV(\mathbf{x})}{R(\mathbf{x}, \boldsymbol{\xi})}, \\ \varphi_j(\boldsymbol{\xi}) &= 0. \end{aligned} \tag{2.13}$$

The above solutions are identical with those obtained in classical thermoelasticity.

3. Problem of thermoelasticity with axial symmetry and symmetry with respect to the point

Let us consider the axi-symmetric problem, the temperature being assumed to depend on the variables r, z and t. Within the system of cylindrical coordinates (r, ϑ, z) — under the assumption that all the causes of effects are independent of the angle ϑ — we obtain the following equations of thermoelasticity [5]:

$$\begin{aligned}
&(\mu+\alpha)\left(\nabla^2-\frac{1}{r^2}\right)u_r+(\lambda+\mu-\alpha)\frac{\partial e}{\partial r}-2e\frac{\partial \varphi_\vartheta}{\partial z}=\varrho\ddot{u}_r+\nu\frac{\partial \theta}{\partial z},\\
(3.1)\qquad &(\mu+\alpha)\,\nabla^2 u_z+(\lambda+\mu-\alpha)\frac{\partial e}{\partial z}+2\alpha\frac{\partial}{\partial r}(r\varphi_\vartheta)=\varrho\ddot{u}_z+\nu\frac{\partial \theta}{\partial z},\\
&(\gamma+\varepsilon)\left(\nabla^2-\frac{1}{r^2}\right)\varphi_\vartheta-4\alpha\varphi_\vartheta+2\alpha\left(\frac{\partial u_r}{\partial z}-\frac{\partial u_z}{\partial r}\right)=I\ddot{\varphi}_\vartheta,
\end{aligned}$$

where

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2},\qquad e=\frac{1}{r}\frac{\partial}{\partial r}(ru_r)+\frac{\partial u_z}{\partial z},\qquad r=(x_1^2+x_2^2)^{1/2}.$$

The case of temperature $\theta(r, t)$ distribution, where $u_z=0$ and $\varphi_\vartheta=0$, is of particular interest. Neglecting the derivatives with respect to z we obtain the following form of Eq. $(3.1)_1$

$$\begin{aligned}
&(\lambda+2\mu)\left(\nabla_r^2-\frac{1}{r^2}\right)u_r=\varrho\ddot{u}_r+\nu\frac{\partial\theta}{\partial r},\\
(3.2)\qquad &\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}.
\end{aligned}$$

The above equation is identical with that of classical thermoelasticity. Maysel's formulae for this case may be presented in the form

$$(3.3)\quad u_r(r,t)=\frac{\nu}{r}\int_a^b \varrho d\varrho\int_0^t \theta(\varrho, t-\tau)\,e(\varrho, r, \tau)\,d\tau,\qquad \varphi_\vartheta=0,\qquad u_z=0,$$

or else

$$(3.4)\quad u_r(r,t)=\frac{\nu}{r}\int_a^b \varrho d\varrho\int_0^t \theta(\varrho, t-\tau)\,W_r(r, \varrho, \tau)\,d\tau,\qquad \varphi_\vartheta=0,\qquad u_z=0.$$

Formulae (3.3) and (3.4) refer to an infinite hollow cylinder with inner radius a and outer radius b. Here, the term $U_r(\varrho, r, t)$ represents the displacement on the surface of the cylinder $\varrho=\text{const}$, due to the action of instantaneous radial forces distributed uniformly on the surface of the cylinder $r=\text{const}$. The quantity $e=\dfrac{U_r}{\varrho}+\dfrac{\partial U_r}{\partial\varrho}$ is the dilatation due to the action of radial forces on the surface $r=\text{const}$. The function $W_r(r, \varrho, t)$ should be regarded as the displacement on the surface $r=\text{const}$, induced by the action of instantaneous pressure centers distributed uniformly on the surface of the cylinder $\varrho=\text{const}$.

Formulae (3.3) and (3.4) are identical with those obtained in classical thermoelasticity [4].

We shall now consider the axi-symmetric thermoelastic problem. To begin with, we shall rewrite Eq. (2.1) so as to fit the spherical coordinate system (R, ϑ, η). Assuming axial symmetry with respect to x_3, we obtain the following system of equations describing the displacements $\mathbf{u} \equiv (u_R, u_\vartheta, 0)$ and the rotations $\boldsymbol{\varphi} = (0, 0, \varphi_\eta)$.

$$
\begin{aligned}
&(\mu+\alpha)\left\{\nabla^2 u_R - \frac{2}{R^2}\left[u_R + \frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}(u_\vartheta \sin\vartheta)\right]\right\} + \\
&\qquad +(\lambda+\mu-\alpha)\frac{\partial e}{\partial R} + \frac{2\alpha}{R\sin\vartheta}\frac{\partial}{\partial\vartheta}(\varphi_\eta \sin\vartheta) - \varrho\ddot{u}_R = \nu\frac{\partial\theta}{\partial R}, \\
&(\mu+\alpha)\left\{\nabla^2 u_\vartheta - \frac{2}{R^2}\left[\frac{\partial u_R}{\partial\vartheta} - \frac{u_\vartheta}{2\sin^2\vartheta}\right]\right\} + (\lambda+\mu-\alpha)\frac{1}{R}\frac{\partial e}{\partial R} - \\
&\qquad - \frac{2\alpha}{R}\frac{\partial}{\partial R}(R\varphi_\eta) - \varrho\ddot{u}_\vartheta = \frac{\nu}{R}\frac{\partial\theta}{\partial\vartheta}, \\
&(\gamma+\varepsilon)\left\{\nabla^2 \varphi_\eta - \frac{\varphi_\eta}{R^2\sin^2\vartheta}\right\} - 4\alpha\varphi_\eta - \\
&\qquad - 2\alpha\left(\frac{1}{R}\frac{\partial u_R}{\partial\vartheta} - \frac{1}{R}\frac{\partial}{\partial R}(Ru_\vartheta)\right) - I\ddot{\varphi}_\eta = 0.
\end{aligned}
\tag{3.5}
$$

In the above equations the following notations have been used

$$
\nabla^2 u_R = \frac{\partial^2 u_R}{\partial R^2} + \frac{2}{R^2}\frac{\partial u_R}{\partial R} + \frac{1}{R^2\sin^2\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial u_R}{\partial\vartheta}\right),
$$

$$
e = \frac{1}{R^2}\frac{\partial}{\partial R}(u_R R^2) + \frac{1}{R\sin\vartheta}\frac{\partial}{\partial\vartheta}(u_\vartheta \sin\vartheta), \qquad R = (x_1^2+x_2^2+x_3^2)^{1/2}.
$$

In the case of symmetry with respect to the point, all the derivatives with respect to ϑ should be dropped and, moreover, $u_\vartheta = 0$ and $\varphi_\eta = 0$. Thus, what remains from the system of Eqs. (3.5) is the equation $(3.5)_1$ which now reads as follows

$$
\begin{aligned}
&(\lambda+2\mu)\left(\nabla_R^2 - \frac{1}{R^2}\right)u_R - \varrho\ddot{u}_R = \nu\frac{\partial\theta}{\partial R}, \\
&\nabla_R^2 = \frac{\partial^2}{\partial R^2} + \frac{2}{R}\frac{\partial}{\partial R}.
\end{aligned}
\tag{3.6}
$$

The above equation is identical with that of classical thermoelasticity. The displacement $u_R(R, t)$ may be described by the formula

$$
u_R(R, t) = \frac{\nu}{R^2}\int_a^b \varrho^2\, d\varrho \int_0^t \theta(\varrho, t-\tau)\, e(\varrho, R, \tau)\, d\tau, \qquad u_\vartheta = 0, \qquad \varphi_\eta = 0 \tag{3.7}
$$

or

$$(3.8) \qquad u_R(R, t) = \frac{\nu}{R^2} \int_a^b \varrho^2 \, d\varrho \int_a^t \theta(\varrho, t-\tau) \, W_R(R, \varrho, \tau) \, d\tau, \quad u_\theta = 0, \quad \varphi_\eta = 0.$$

These formulae refer to a hollow sphere with inner radius a and outer radius b. The quantity e represents the dilatation induced on the surface $\varrho = \text{const}$ by the action of instantaneous radial forces distributed uniformly on the surface $R = \text{const}$. The function $W_R(R, \varrho, \tau)$ expresses the radial displacement on the surface $R = \text{const}$, due to the action of instantaneous unary pressure centers distributed uniformly on the surface $\varrho = \text{const}$.

Our considerations presented in this note lead to a general conclusion: all problems — be they static, quasi-static or dynamic — characterized by the symmetry with respect to the point and referring to a hollow sphere have identical solutions for micropolar and Hooke's media [4]. The same holds true for a full sphere with $a \to 0$, for an infinite space with a spherical cavity with $b \to \infty$ and for an infinite space with $a \to 0$, $b \to \infty$.

Let us remark that we can argue quite similarly in unidimensional problems (those of infinite space, half-space, elastic layer) where the temperature depends solely on the variables x_1 and t.

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В. НОВАЦКИЙ, **НЕКОТОРЫЕ ПРОБЛЕМЫ ТЕРМОУПРУГОСТИ В МИКРОПОЛЯРНОЙ УПРУГОСТИ**

В настоящей заметке рассматриваются две проблемы термоупругости. Итак:

Выведены интегральные формулы пригодные для определения перемещений и оборотов, вызванных воздействием поля температуры в микрополярной среде Коссера. Это является обобщением формул Майзеля для среды Гука на среду Коссера.

Далее рассмотрен класс осе-симметрических проблем. Показано, что для случая осе-симметрической проблемы независящей от переменной z, а также для проблемы с симметрией по отношению к точке, решения идентичны так для среды Гука, как и для микрополярной среды Коссера.