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APPLIED MECHANICS
(ELASTICITY)

On the Completeness of Stress Functions in Asymmetric Elasticity

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1. Introduction

In this paper we shall be concerned with an elastic, homogeneous, isotropic and centro-symmetric body. Under the effect of external loads the body will suffer deformation. A displacement field $\mathbf{u}(\mathbf{x}, t)$ and a rotation field $\boldsymbol{\omega}(\mathbf{x}, t)$ will form in the body changing with the position of the point $\mathbf{x}$ and with time t. The deformation state is described by two asymmetric tensors: asymmetric deformation tensor γ_{ji} and curvature-twist tensor $\varkappa_{ji}$. As known [1]—[3], there is:

(1.1)
$$\gamma_{ji} = u_{i,j} - \varepsilon_{kij}\,\omega_k, \quad \varkappa_{ji} = \omega_{i,j}.$$

The state of stress is characterized by two asymmetric tensors: force-stress tensor σ_{ji} and couple-stress tensor μ_{ji}.

The state of stress and that of deformation are connected by the constitutive relations

(1.2)
$$\sigma_{ji} = (\mu+\alpha)\,\gamma_{ji} + (\mu-\alpha)\,\gamma_{ij} + \lambda\gamma_{kk}\,\delta_{ij},$$

(1.3)
$$\mu_{ji} = (\gamma+\varepsilon)\,\varkappa_{ji} + (\gamma-\varepsilon)\,\varkappa_{ij} + \beta\varkappa_{kk}\,\delta_{ij},$$

where μ, λ are Lame' constants while α, β, γ and ε denote new material constants.

The system of equations of motion consists of the equation of balance of linear momentum and the equation of angular momentum

(1.4)
$$\sigma_{ji,j} + X_i - \varrho \ddot{u}_i = 0,$$

(1.5)
$$\varepsilon_{ijk}\,\sigma_{jk} + \mu_{ji,j} + Y_i - J\ddot{\omega}_i = 0.$$

In the above equations the symbols X_i and Y_i denote the volume density of body forces and body couples, respectively, ε_{ijk} stands for the alternator used, ϱ — for the density and I represents the characteristic of the body.

Introducing (1.2), (1.3) and (1.1) into (1.4) and (1.5), we obtain a system of two coupled vector equations describing the displacement and rotation fields [1]—[3]

(1.6)
$$\square_2\,\mathbf{u} + (\lambda+\mu-\alpha)\,\mathrm{grad\,div}\,\mathbf{u} + 2\alpha\,\mathrm{rot}\,\boldsymbol{\omega} + \mathbf{X} = 0,$$

(1.7)
$$\square_4\,\boldsymbol{\omega} + (\beta+\gamma-\varepsilon)\,\mathrm{grad\,div}\,\boldsymbol{\omega} + 2\alpha\,\mathrm{rot}\,\mathbf{u} + \mathbf{Y} = 0.$$

The following notations were introduced in the above system

$$\Box_2 = (\mu+\alpha)\,\nabla^2 - \varrho\partial_t^2, \quad \Box_4 = (\gamma+\varepsilon)\,\nabla^2 - 4\alpha - J\partial_t^2,$$

$$\nabla^2 = \frac{\partial}{\partial x_i}\,\frac{\partial}{\partial x_i}, \quad \partial_t^2 = \frac{\partial^2}{\partial t^2}.$$

Eqs. (1.6) and (1.7) are equations of elastokinetics; they are derived under the assumption of adiabatic process. Material constants appearing in these equations are measured in adiabatic state of the body.

The system of Eqs. (1.6) and (1.7) may be disjoined by decomposing the vectors $\mathbf{u}$ and $\boldsymbol{\omega}$ into potential and solenoidal parts [4].

Putting

$$\mathbf{u} = \operatorname{grad}\Phi + \operatorname{rot}\boldsymbol{\Psi}, \quad \operatorname{div}\boldsymbol{\Psi} = 0, \tag{1.8}$$

$$\boldsymbol{\omega} = \operatorname{grad}\Sigma + \operatorname{rot}\mathbf{H}, \quad \operatorname{div}\mathbf{H} = 0, \tag{1.9}$$

we get from Eqs. (1.6) and (1.7) — assuming the absence of body forces and moments — the following system of wave equations

$$\Box_1\,\Phi = 0, \tag{1.10}$$

$$\Box_3\,\Sigma = 0, \tag{1.11}$$

$$\Omega\boldsymbol{\Psi} = 0, \tag{1.12}$$

$$\Omega\mathbf{H} = 0, \tag{1.13}$$

where:

$$\Box_1 = (\lambda+2\mu)\,\nabla^2 - \varrho\partial_t^2, \quad \Box_3 = (\beta+2\gamma)\,\nabla^2 - 4\alpha - J\partial_t^2,$$

$$\Omega = \Box_2\,\Box_4 + 4\alpha^2\,\nabla^2.$$

Eq. (1.10) describes the longitudinal wave, Eq. (1.11) — the rotational wave, while Eqs. (1.12) and (1.13) the modified (as compared with that of classical elastokinetics) transversal wave.

2. Stress functions. Theorem on the completeness of solutions

In static problems of the classical theory of elasticity an important role was played by Galerkin's functions [5], in dynamical problems the same role was devolved upon Iacovache's functions [6], particularly when deriving basic solutions. N. Sandru [7] elaborated the functions of this type for the asymmetric theory of elasticity making use of the general algorithm constructed by Gr. C. Moisil [8].

In what follows we advance a different—and we hope it to be simpler—method of getting the stress functions which permits to avoid the cumbersome solutions of determinants of 6th order. Finally we shall demonstrate the theorem on the completeness of stress functions.

We take as the starting point of our considerations the differential equations (1.6) and (1.7). Eliminating from these equations first the quantity $\boldsymbol{\omega}$ and then $\mathbf{u}$, we obtain the following system of equations

$$\Omega\mathbf{u}+\operatorname{grad}\operatorname{div}\Gamma\mathbf{u}+\Box_4\mathbf{X}-2\alpha\operatorname{rot}\mathbf{Y}=0, \tag{2.1}$$

$$\Omega\boldsymbol{\omega}+\operatorname{grad}\operatorname{div}\Theta\boldsymbol{\omega}+\Box_2\mathbf{Y}-2\alpha\operatorname{rot}\mathbf{X}=0, \tag{2.2}$$

where

$$\Omega=\Box_2\Box_4+4\alpha^2\nabla^2,\quad \Gamma=(\lambda+\mu-\alpha)\Box_4-4\alpha^2,\quad \Theta=(\beta+\gamma-\varepsilon)\Box_2-4\alpha^2. \tag{2.3}$$

Let us consider first the system of Eqs. (2.1); we rewrite it in the operator form

$$L_{ij}(u_j)+\Box_4X_i-2\alpha\,\varepsilon_{ijk}Y_{k,j}=0,\quad i,j,k=1,2,3, \tag{2.4}$$

where

$$L_{ij}=\Omega\delta_{ij}+\partial_i\partial_j\Gamma.$$

We introduce the vector stress function $\boldsymbol{\zeta}$ connected with the components of the displacement $\mathbf{u}$ by the following relations

$$u_1=\begin{vmatrix}\zeta_1 & L_{12} & L_{13}\\ \zeta_2 & L_{22} & L_{23}\\ \zeta_3 & L_{32} & L_{33}\end{vmatrix},\quad u_2=\begin{vmatrix}L_{11} & \zeta_1 & L_{13}\\ L_{21} & \zeta_2 & L_{23}\\ L_{31} & \zeta_3 & L_{33}\end{vmatrix},\quad u_3=\begin{vmatrix}L_{11} & L_{12} & \zeta_1\\ L_{21} & L_{22} & \zeta_2\\ L_{31} & L_{32} & \zeta_3\end{vmatrix}. \tag{2.5}$$

Now, after some simple transformations it is seen that the vector $\mathbf{u}$ is connected with the vector $\boldsymbol{\zeta}$ by the following relation

$$\mathbf{u}=\Box_1\Box_4\boldsymbol{\zeta}-\operatorname{grad}\operatorname{div}\Gamma\boldsymbol{\zeta}. \tag{2.6}$$

The procedure to be applied to Eq. (2.2) is quite similar; we get the dependence

$$\boldsymbol{\omega}=\Box_2\Box_3\boldsymbol{\eta}-\operatorname{grad}\operatorname{div}\Theta\boldsymbol{\eta}, \tag{2.7}$$

where $\boldsymbol{\eta}$ is the second vector stress function.

Introducing (2.6) and (2.7) into (2.1) and (2.2), we obtain the following system of equations for determining the functions $\boldsymbol{\zeta}$ and $\boldsymbol{\eta}$

$$\Omega\Box_1\Box_4\boldsymbol{\zeta}=-\Box_4\mathbf{X}+2\alpha\operatorname{rot}\mathbf{Y}, \tag{2.8}$$

$$\Omega\Box_2\Box_3\boldsymbol{\eta}=-\Box_2\mathbf{Y}+2\alpha\operatorname{rot}\mathbf{X}. \tag{2.9}$$

Thus we get two independent of each other systems of equations. These equations proved, however, to be inconvenient since on the right-hand side of the equations there appear operations of differentiation upon the body forces and moments. If, however, instead of the representations (2.6) and (2.7) we take the relations

$$\mathbf{u}=\Box_1\Box_4\boldsymbol{\varphi}-\operatorname{grad}\operatorname{div}\Gamma\boldsymbol{\varphi}-2\alpha\operatorname{rot}\Box_3\boldsymbol{\psi}, \tag{2.10}$$

$$\boldsymbol{\omega}=\Box_2\Box_3\boldsymbol{\psi}-\operatorname{grad}\operatorname{div}\Theta\boldsymbol{\psi}-2\alpha\operatorname{rot}\Box_1\boldsymbol{\varphi}, \tag{2.11}$$

where $\boldsymbol{\varphi}$ and $\boldsymbol{\psi}$ denote new stress functions, then, introducing (2.10) and (2.11) into the system of Eqs. (2.1) and (2.2), we get the following relations

$$\Box_4(\Omega\Box_1\boldsymbol{\varphi}+\mathbf{X})-2\alpha\,\mathrm{rot}\,(\Omega\Box_3\boldsymbol{\psi}+\mathbf{Y})=0,$$

$$\Box_2(\Omega\Box_3\boldsymbol{\psi}+\mathbf{Y})-2\alpha\,\mathrm{rot}\,(\Omega\Box_1\boldsymbol{\varphi}+\mathbf{X})=0,$$

wherefrom we derive the following equations

$$\Box_1\Omega\boldsymbol{\varphi}+\mathbf{X}=0,\quad \Box_3\Omega\boldsymbol{\psi}+\mathbf{Y}=0, \tag{2.12}$$

suitable for the determination of the stress functions $\boldsymbol{\varphi}$ and $\boldsymbol{\psi}$. Eqs. (2.12) coincide with those derived by N. Sandru in a different way.

The theorem on the completeness may be formulated as follows:

THEOREM. *Let* $\mathbf{u}(\mathbf{x},t)$ *and* $\boldsymbol{\omega}(\mathbf{x},t)$ *be the solutions of the system of Eqs.* (1.6) *and* (1.7) *within the interval* $-\infty<t<\infty$.

We assert that there exist vector functions $\boldsymbol{\zeta}$ and $\boldsymbol{\eta}$ such that the displacements $\mathbf{u}$ and the rotations $\boldsymbol{\omega}$ are expressed by the representation (2.6) and (2.9), the functions satisfying the wave equations (2.8) and (2.9).

The starting point for the demonstration of this Theorem will be the representation of the vectors $\mathbf{u}$ and $\boldsymbol{\omega}$ in the form of Stokes-Helmholtz solution, (1.8) (1.9). Introducing (1.8) and (1.9) into Eqs. (2.1) and (2.2), respectively, and taking into account that

$$\Omega=\Box_2\Box_4+4\alpha^2\nabla^2=\Box_1\Box_4-\nabla^2\Gamma=\Box_2\Box_3-\nabla^2\Theta. \tag{2.13}$$

we obtain the following system of equations

$$\mathrm{grad}\,(\Box_1\Box_4\Phi)+\mathrm{rot}\,(\Omega\boldsymbol{\Psi})+\Box_4\mathbf{X}-2\alpha\,\mathrm{rot}\,\mathbf{Y}=0, \tag{2.14}$$

$$\mathrm{grad}\,(\Box_2\Box_3\Sigma)+\mathrm{rot}\,(\Omega\mathbf{H})+\Box_2\mathbf{Y}-2\alpha\,\mathrm{rot}\,\mathbf{X}=0. \tag{2.15}$$

Now we shall make use of Eqs. (2.8) and (2.9) and express the vectors $\boldsymbol{\zeta}$ and $\boldsymbol{\eta}$ by their potential and solenoidal parts

$$\boldsymbol{\zeta}=\mathrm{grad}\,\vartheta+\mathrm{rot}\,\boldsymbol{\lambda},\quad \mathrm{div}\,\boldsymbol{\lambda}=0, \tag{2.16}$$

$$\boldsymbol{\eta}=\mathrm{grad}\,\sigma+\mathrm{rot}\,\boldsymbol{\chi},\quad \mathrm{div}\,\boldsymbol{\chi}=0. \tag{2.17}$$

Introducing (2.16) into (2.8) and (2.17) into (2.9), we get the system of equations

$$\mathrm{grad}\,(\Omega\Box_1\Box_4\vartheta)+\mathrm{rot}\,(\Omega\Box_1\Box_4\boldsymbol{\lambda})+\Box_4\mathbf{X}-2\alpha\,\mathrm{rot}\,\mathbf{Y}=0, \tag{2.18}$$

$$\mathrm{grad}\,(\Omega\Box_2\Box_3\sigma)+\mathrm{rot}\,(\Omega\Box_2\Box_3\boldsymbol{\chi})+\Box_2\mathbf{Y}-2\alpha\,\mathrm{rot}\,\mathbf{X}=0. \tag{2.19}$$

Comparing, on the one hand Eq. (2.14) and Eq. (2.18) and, on the other hand, (2.15) and (2.19), we get

$$\Omega\vartheta=\Phi,\quad \Omega\sigma=\Sigma, \tag{2.20}$$

$$\Box_1\Box_4\boldsymbol{\lambda}=\boldsymbol{\Psi},\quad \Box_2\Box_3\boldsymbol{\chi}=\mathbf{H}. \tag{2.21}$$

Performing on Eq. (2.16) the operation $\Box_1 \Box_4$, and taking into account $(2.21)_1$, we have

$$\Box_1 \Box_4 \boldsymbol{\zeta} = \operatorname{grad}(\Box_1 \Box_4 \vartheta) + \operatorname{rot} \boldsymbol{\Psi}. \tag{2.22}$$

Eliminating the term $\operatorname{rot} \boldsymbol{\Psi}$ from (2.22) and (1.8), we arrive at

$$\mathbf{u} = \operatorname{grad} \Phi + \Box_1 \Box_4 \boldsymbol{\zeta} - \operatorname{grad} \Box_1 \Box_4 \vartheta. \tag{2.23}$$

Taking into consideration Eq. $(2.20)_1$, the dependence (2.13) and the relation $\operatorname{div} \boldsymbol{\zeta} = \nabla^2 \vartheta$ resulting from (2.16), we reduce Eq. (2.23) to the form

$$\mathbf{u} = \Box_1 \Box_4 \boldsymbol{\zeta} - \operatorname{grad} \operatorname{div} \Gamma \boldsymbol{\zeta}. \tag{2.24}$$

Thus we obtained the relation identical with that derived previously, (2.6). Performing on (2.17) the operation $\Box_2 \Box_3$, and taking into account the relation $(2.21)_2$, we arrive at

$$\Box_2 \Box_2 \boldsymbol{\eta} = \operatorname{grad}(\Box_2 \Box_3 \sigma) + \operatorname{rot} \mathbf{H}. \tag{2.25}$$

The elimination of the term $\operatorname{rot} \mathbf{H}$ from (2.25) and (1.9), making allowance for Eqs. $(2.20)_2$, (2.13) and the relation $\operatorname{div} \boldsymbol{\eta} = \nabla^2 \sigma$, leads to the representation

$$\boldsymbol{\omega} = \Box_2 \Box_3 \boldsymbol{\eta} - \operatorname{grad} \operatorname{div} \Theta \boldsymbol{\eta}, \tag{2.26}$$

which is in accordance with (2.7).

Thus, the Theorem on the completeness of solutions arrived at with the use of the stress functions $\boldsymbol{\zeta}$ and $\boldsymbol{\eta}$ is demonstrated.

It may be shown in a similar way that there exist vector functions $\boldsymbol{\varphi}$, $\boldsymbol{\psi}$ such that the displacements $\mathbf{u}$ and the rotations $\boldsymbol{\omega}$ are expressed by the relations (2.10) and (2.11) and the functions $\boldsymbol{\varphi}$ and $\boldsymbol{\psi}$ satisfy the wave equations (2.12).

3. Dependences between the potentials Φ, Σ, Ψ, $\mathbf{H}$ and the stress functions $\boldsymbol{\varphi}$, $\boldsymbol{\psi}$

Let us consider the homogeneous wave equation (2.12) — the absence of body forces and moments being assumed

$$\Box_1 \Omega \boldsymbol{\varphi} = 0, \quad \Box_3 \Omega \boldsymbol{\psi} = 0. \tag{3.1}$$

The solution of these equations — in accordance with a theorem of T. Boggio [9] may consist of two parts

$$\boldsymbol{\varphi} = \boldsymbol{\varphi}' + \boldsymbol{\varphi}'', \quad \boldsymbol{\psi} = \boldsymbol{\psi}' + \boldsymbol{\psi}''. \tag{3.2}$$

The functions $\boldsymbol{\varphi}', \boldsymbol{\varphi}'', \boldsymbol{\psi}', \boldsymbol{\psi}''$ verify the following equations

$$\Box_1 \boldsymbol{\varphi}' = 0, \quad \Omega \boldsymbol{\varphi}'' = 0, \tag{3.3}$$

$$\Box_3 \boldsymbol{\psi}' = 0; \quad \Omega \boldsymbol{\psi}'' = 0. \tag{3.4}$$

Introducing (3.2) into (2.10) and (2.11) and making use of Eqs. (3.3) and (3.4), we obtain the following representation

$$\mathbf{u} = \Box_1 \Box_4 \boldsymbol{\varphi}'' - \operatorname{grad} \operatorname{div} \Gamma(\boldsymbol{\varphi}'+\boldsymbol{\varphi}'') - 2\alpha \operatorname{rot} \Box_3 \boldsymbol{\psi}'', \tag{3.5}$$

$$\boldsymbol{\omega} = \Box_2 \Box_3 \boldsymbol{\psi}'' - \operatorname{grad} \operatorname{div} \Theta(\boldsymbol{\psi}'+\boldsymbol{\psi}'') - 2\alpha \operatorname{rot} \Box_1 \boldsymbol{\varphi}''. \tag{3.6}$$

Taking advantage of the known dependence

$$\operatorname{rot} \operatorname{rot} \mathbf{U} = \operatorname{grad} \operatorname{div} \mathbf{U} - \nabla^2 U,$$

and bearing in mind the relations (2.13), we reduce the representation (3.5) and (3.6) to the form

$$\mathbf{u} = -\operatorname{grad} \operatorname{div} \Gamma \boldsymbol{\varphi}' - 2\alpha \operatorname{rot} \Box_3 \boldsymbol{\psi}'', \tag{3.7}$$

$$\boldsymbol{\omega} = -\operatorname{grad} \operatorname{div} \Theta \boldsymbol{\psi}' - 2\alpha \operatorname{rot} \Box_1 \boldsymbol{\varphi}''. \tag{3.8}$$

Comparing the Stokes-Helmholtz representation (1.8), the representation (3.7) and, finally, the representations (1.9) and (3.8) we obtain the following relations:

$$\Phi = -\operatorname{div} \Gamma \boldsymbol{\varphi}', \quad \boldsymbol{\Psi} = -2\alpha \Box_3 \boldsymbol{\psi}'', \tag{3.9}$$

$$\Sigma = -\operatorname{div} \Theta \psi', \quad \mathbf{H} = -2\alpha \Box_1 \boldsymbol{\varphi}''. \tag{3.10}$$

There are the sought for dependences between the potentials Φ, Σ, $\boldsymbol{\Psi}$, $\mathbf{H}$ and the stress functions $\boldsymbol{\varphi}, \boldsymbol{\psi}$. Yet we have to verify whether the relations (3.9), (3.10) satisfy the wave equations (1.10)–(1.13). It may be easily shown that, indeed, they do.

4. Stress functions for the asymmetric thermoelasticity

Basic differential equations for coupled thermoelasticity have the form [10]:

$$\Box_2 \mathbf{u} + (\lambda+\mu-\alpha) \operatorname{grad} \operatorname{div} \mathbf{u} + 2\alpha \operatorname{rot} \boldsymbol{\omega} + \mathbf{X} = \nu \operatorname{grad} \theta, \tag{4.1}$$

$$\Box_4 \boldsymbol{\omega} + (\beta+\gamma-\varepsilon) \operatorname{grad} \operatorname{div} \boldsymbol{\omega} + 2\alpha \operatorname{rot} \mathbf{u} + \mathbf{Y} = 0, \tag{4.2}$$

$$D\theta - \eta_0 \operatorname{div} \dot{\mathbf{u}} = -\frac{Q}{\varkappa}, \quad D = \nabla^2 - \frac{1}{\varkappa}\partial_t, \tag{4.3}$$

where T denotes the absolute temperature, T_0 stands for the temperature of the body in natural state with no stress and deformations. $\theta = T - T_0$ expresses the increase of temperature. Q is related with the intensity of the heat source. The quantities $\varkappa, \eta_0$, ν are material constants. Eq. (4.3) is the equation of heat conductivity in an expanded form.

Applying the same procedure as in Sec. **3**, we assume for the vectors $\mathbf{u}$ and $\boldsymbol{\omega}$ and for the function θ the following representation constructed with the help of two vector functions $\boldsymbol{\varphi}$, $\boldsymbol{\psi}$ and the scalar function ϑ

$$\mathbf{u} = M \Box_2 \boldsymbol{\varphi} - \operatorname{grad} \operatorname{div} \mathbf{N} \boldsymbol{\varphi} - 2\alpha \operatorname{rot} \Box_3 \boldsymbol{\psi} + \nu \operatorname{grad} \vartheta, \tag{4.4}$$

$$\boldsymbol{\omega} = \Box_2 \Box_3 \boldsymbol{\psi} - \operatorname{grad} \operatorname{div} \Theta \boldsymbol{\psi} - 2\alpha \operatorname{rot} M \boldsymbol{\varphi}, \tag{4.5}$$

$$\theta = \eta_0 \partial_t \operatorname{div} \Omega \boldsymbol{\varphi} + \Box_1 \vartheta, \tag{4.6}$$

where

$$M = D\Box_1 - \nu\eta_0\, \partial_t\, \nabla^2,$$

$$N = D\Gamma - \nu\eta_0\, \partial_t\, \Box_4.$$

Introducing Eqs. (4.4)—(4.6) into Eqs. (4.1)—(4.3), we obtain the following system of wave equations

(4.7) $$\Omega M \boldsymbol{\varphi} + \mathbf{X} = 0,$$

(4.8) $$\Omega \Box_3 \boldsymbol{\psi} + \mathbf{Y} = 0$$

(4.9) $$M\vartheta + \frac{Q}{\varkappa} = 0.$$

In a similar way as in Sec. **3** may be shown the completeness of the functions $\boldsymbol{\varphi}, \boldsymbol{\psi}, \vartheta$.

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В. НОВАЦКИЙ, **О ПОЛНОТЕ ФУНКЦИЙ НАПРЯЖЕНИЙ В НЕСИММЕТРИЧНОЙ УПРУГОСТИ**

В настоящей заметке выведены по новому методу формулы для функций напряжений в теории несимметричной упругости. Доказана полнота функций напряжений. Показаны также соотношения между потенциалами Стокса-Гельмгольца и функциями напряжений. Наконец, дано представление перемещений $\mathbf{u}$, вращений $\boldsymbol{\omega}$ и температуры θ при помощи двух векторных функций $\boldsymbol{\varphi}$ и $\boldsymbol{\psi}$ и скалярной функции ϑ. Выведено волновое уравнение, которому должны удовлетворять упомянутые функции.