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APPLIED MECHANICS

The Reciprocity Theorem of Magneto-thermo-elasticity. II. Real Conductors

by

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In [1] is derived by the present authors the reciprocity theorem cf magneto--thermo-elasticity of isotropic bodies in the case of perfect electric conductivity of the body. The present paper contains a generalization of heat theorem to isotropic bodies having finite electric conductivity taking into consideration thermoelectric effects.

1. Fundamental equations

We start out from the set of elastic equations of magneto-thermo-elasticity [2] for slowly moving bodies, taking into consideration thermoelastic effects.

The thermodynamic discussion and the symmetry relations for these equations have been given in [3].

Our point of departure will be the full set of linearized equations of magneto--thermo-elasticity, bearing in mind thermoelastic effects [2] and disregarding coupled terms of secondary character only (cf. [4], [5]) which is admissible if we assume that μ_0 differ but little from unity. In addition, it will be assumed for simplicity that in the case of a finite body in vacuum the density of the spatial charge is, in vacuum, equal to zero.

The equations of magneto-thermo-elasticity have, for a homogeneous isotropic body, the form.

1. The equations of electrodynamics

$$\begin{aligned} &\operatorname{rot} \boldsymbol{h} = \frac{4\pi}{c} \boldsymbol{j}_0 + \frac{\varepsilon}{c} \frac{\partial \boldsymbol{E}}{\partial t}, \\ (1.1) \qquad &\operatorname{rot} \boldsymbol{E} = -\frac{\mu_0}{c} \frac{\partial \boldsymbol{h}}{\partial t}, \\ &\operatorname{div} \boldsymbol{h} = 0, \quad \operatorname{div} \boldsymbol{D} \approx \varepsilon \operatorname{div} \boldsymbol{E} = 4\pi\varrho_c, \quad \operatorname{div} \boldsymbol{j}_c = -\frac{\partial \varrho_c}{\partial t}, \end{aligned}$$

$$j = \eta\left[E + \frac{\mu_0}{c}\left(\frac{\partial u}{\partial t}\, x H\right)\right] - \varkappa_0 \operatorname{grad}\theta, \tag{1.1}$$

$$j_c = j + j_z.$$

the term $\frac{1}{c}\left[\varepsilon\left(\frac{\partial u}{\partial t}\times B\right) - \left(\frac{\partial u}{\partial t}\times H\right)\right]$ in the expression for $D \approx \varepsilon E$ being rejected, in agreement with the assumption that $\varepsilon \approx 1$.

2. Equation of elasticity

$$\mu\nabla^2 u + (\lambda+\mu)\operatorname{grad}\operatorname{div} u - \gamma \operatorname{grad}\theta + \frac{\mu_0}{c}(j_c\times H) - \frac{\varrho\partial^2 u}{\partial t^2} + X = 0, \tag{1.2}$$

3. Heat equation

$$\nabla^2\theta - \frac{1}{\varkappa}\frac{\partial\theta}{\partial t} - \nu\frac{\partial e}{\partial t} - \pi_0 \operatorname{div} j_c = -\frac{\theta}{\varkappa}, \tag{1.3}$$

$$e = \operatorname{div} u.$$

Eqs. (1.1)—(1.3) are coupled. The symbols E, h denote the perturbed electric and magnetic field, j — vector of current density, j_2 — vector of external current density, H — vector of primary constant magnetic field, u — vector of displacement, c — velocity of light in vacuum, ε_{μ_0} — dielectric constant and magnetic permeability of the body, η — electric conductivity, θ — temperature, ϱ — density, λ, μ — elastic constants, $\gamma = (3\lambda+2\mu)\,\alpha_t$, where α_t is the coefficient of linear thermal dilatation. Furthermore, $\varkappa_0$ is the coupling constant between the temperature gradient and the density of current and $\lambda_0\,\pi_0 = \pi$ a constant of the Peltier type relating div j with the heat emitted.

In addition, the symbol $\varkappa_0 = \frac{\lambda_0}{\varrho c_\varepsilon}$ where λ_0 is the coefficient of heat conduction, c_ε — specific heat with constant strain has been introduced in (1.3). We have also $\nu = \frac{\gamma T_0}{\lambda_0}$, $Q = \frac{W}{\varrho c_\varepsilon}$ where T_0 is the absolute temperature in the natural state and W — density of internal heat sources.

The states π and $\varkappa_0$ are interrelated by an equation following from the Onsager symmetry relation (cf. [3]), $\pi\eta = \varkappa_0 T_0$. In what follows these constants will be expressed by independent symbols, bearing in mind the above relation, however.

The above equations are completed with the Duhamel—Neuman relations between the stresses, the strains and the temperature field

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + (\lambda e - \gamma\theta)\,\delta_{ij}. \tag{1.4}$$

Eqs. (1.2) can also be written in the form

$$\sigma_{ij,j} + T_{ij,j} - \varrho\ddot{u}_i + X_i = 0, \tag{1.5}$$

where T_{ij} Maxwell's tensor which, after linearization is expressed in the form:

$$T_{ij} = \frac{\mu_0}{4\pi} [h_i H_j + h_j H_i - \delta_{ij} h_k H_k]. \tag{1.6}$$

In the case of a finity body in vacuum we have, in addition to the above equations for an elastic body, the field equations in vacuum. They have the form

$$\left(\nabla^2 - \frac{1}{c^2} \partial_t^2\right) (\boldsymbol{h}^0, \boldsymbol{E}^0) = 0. \tag{1.7}$$

The above set of equations must be completed with the boundary conditions on the surface of the body A. The quantities prescribed on that surface may be the stresses or strains, the temperature or its normal derivatives and the continuity conditions of the field [5].

The set of Eqs. (1.1)—(1.7) will now be subjected to the Laplace transform defined thus

$$\tilde{\boldsymbol{u}} (x_i, p) = L\{\boldsymbol{u} (x_i, t)\} = \int_0^\infty \boldsymbol{u} (x_i, t) e^{-pt} dt, \quad \text{itd.} \tag{1.8}$$

The initial conditions will be homogeneous, it being also assumed that any source producing motion will start action at the instant of time $t = 0^+$.

The reciprocity theorem will be discussed for the integral transforms.

2. Reciprocity theorem

Let us consider two sets of causes and effects. One of these sets will be denoted by primes the other being left unchanged. We start out from the identity (1.4) after subjecting it first to an integral transformation. We find easily the relation

$$(\tilde{\sigma}_{ij} + \gamma \tilde{\theta} \delta_{ij}) \tilde{\varepsilon}'_{ij} = (\tilde{\sigma}'_{ij} + \gamma \tilde{\theta}' \delta_{ij}) \tilde{\varepsilon}_{ij}. \tag{2.1}$$

On integrating (2.1) over the region B we obtain

$$\int_B (\tilde{\sigma}_{ij} \tilde{\varepsilon}'_{ij} - \tilde{\sigma}'_{ij} \tilde{\varepsilon}_{ij}) \, dV + \gamma \int_B (\tilde{\theta} \tilde{e}' - \tilde{\theta}' \tilde{e}) \, dV = 0. \tag{2.2}$$

Bearing in mind that $\tilde{\sigma}_{ij} \tilde{\varepsilon}_{ij} = \tilde{\sigma}_{ij} u_{i,j}$ and making use of the divergence theorem and Eq. (1.5), we find the relation

$$\int_B (\tilde{X}_i \tilde{u}'_i - \tilde{X}'_i \tilde{u}_i) \, dV + \int_A (\tilde{p}^*_i \tilde{u}'_i - \tilde{p}'^*_i \tilde{u}_i) \, dA + \\ + \gamma \int_B (\tilde{\theta} \tilde{e}' - \tilde{\theta}' \tilde{e}) \, dV = \int_B (\tilde{T}_{ij} \tilde{\varepsilon}'_{ij} - \tilde{T}'_{ij} \tilde{\varepsilon}_{ij}) \, dV, \tag{2.3}$$

where

$$\tilde{p}^*_i = (\tilde{\sigma}_{ij} + \tilde{T}_{ij}) n_j, \quad \tilde{p}'^*_i = (\tilde{\sigma}'_{ij} + \tilde{T}'_{ij}) n_j. \tag{2.4}$$

The relation (2.3) can be transformed to obtain

$$(2.5)\qquad \int_B (\tilde{X}_i \tilde{u}_i' - \tilde{X}_i' \tilde{u}_i)\, dV + \int_A (\tilde{p}_i \tilde{u}_i' - \tilde{p}_i' \tilde{u}_i)\, dA + \gamma \int_B (\tilde{\theta}\tilde{e}' - \tilde{\theta}'\tilde{e})\, dV =$$
$$= -\int_B (\tilde{T}_{ij,j}\tilde{u}_i' - \tilde{T}_{ij,j}'\tilde{u}_i)\, dV = -\frac{\mu_0}{c}\int_B [(\tilde{\boldsymbol{j}}_c \times \boldsymbol{H})_i \tilde{u}_i' - (\tilde{\boldsymbol{j}}_c' \times \boldsymbol{H})_i \tilde{u}_i]\, dV,$$

where

$$\tilde{p}_i = \tilde{\sigma}_{ij} n_j, \quad \tilde{p}_i' = \tilde{\sigma}_{ij}' n_j.$$

From the heat equation (1.3) we find now

$$(2.6)\qquad \int_B \tilde{\theta}' \nabla^2 \tilde{\theta} - \tilde{\theta}\nabla^2\tilde{\theta}')\, dV = \nu p \int_B (\tilde{e}\tilde{\theta}' - \tilde{e}'\tilde{\theta})\, dV -$$
$$-\frac{1}{\varkappa}\int_B (\tilde{Q}\tilde{\theta}' - \tilde{Q}'\tilde{\theta})\, dV + \pi_0 \int_B (\tilde{\theta}' \operatorname{div} \tilde{\boldsymbol{j}}_c - \tilde{\theta}\operatorname{div}\tilde{\boldsymbol{j}}_c')\, dV.$$

Making use of Green's identity and the relation (1.1), we transform (2.6) to obtain

$$(2.7)\qquad \int_B (\tilde{e}'\tilde{\theta} - \tilde{e}\tilde{\theta}')\, dV = \frac{1}{\varkappa\nu p}\int_B (\tilde{Q}'\tilde{\theta} - \tilde{Q}\tilde{\theta}')\, dV +$$
$$+\frac{1}{\nu p}\int_A (\tilde{\theta}\tilde{\theta}'_{,n} - \tilde{\theta}'\tilde{\theta}_{,n})\, dA - \frac{\pi_0}{\nu}\int_B (\tilde{\theta}'\varrho_e - \tilde{\theta}\varrho_e')\, dV\cdot$$

On substituting (2.7) into (2.5), we find

$$(2.8)\qquad \nu\varkappa p\left[\int_B (\tilde{X}_i \tilde{u}_i' - \tilde{X}_i' \tilde{u}_i)\, dV + \int_A (\tilde{p}_i \tilde{u}_i' - \tilde{p}_i'\tilde{u}_i)\, dA\right] - \gamma\int_B (\tilde{Q}\tilde{\theta}' - \tilde{Q}'\tilde{\theta})\, dV +$$
$$-\gamma\varkappa\int_A (\tilde{\theta}'\tilde{\theta}_{,n} - \tilde{\theta}\tilde{\theta}'_{,n})\, dA - \gamma\varkappa\pi_0 p\int_B (\tilde{\theta}'\tilde{\varrho}_c - \tilde{\theta}\tilde{\varrho}_c')\, dV =$$
$$= -\frac{\mu_0 \nu\varkappa p}{c}\int_B [(\tilde{\boldsymbol{j}}_c \times \boldsymbol{H})_i \tilde{u}_i' - (\tilde{\boldsymbol{j}}' \times \boldsymbol{H})_i \tilde{u}_i]\, dV.$$

Let us now consider the set of Eqs. (1.1). The sets with and without primes can be written thus

$$(2.9.1)\qquad \begin{cases} \operatorname{rot}\tilde{\boldsymbol{h}} = \dfrac{4\pi}{c}\tilde{\boldsymbol{j}}_c + \dfrac{\varepsilon p}{c}\tilde{\boldsymbol{E}} = \dfrac{4\pi}{c}\tilde{\boldsymbol{j}}_z + \dfrac{1}{c}(4\pi\eta + \varepsilon p)\tilde{\boldsymbol{E}} + \dfrac{4\pi\mu_0\eta p}{c^2}(\tilde{\boldsymbol{u}} \times \boldsymbol{H}) \\ -\dfrac{4\pi\varkappa_0}{c}\operatorname{grad}\tilde{\theta}, \\ \operatorname{rot}\tilde{\boldsymbol{E}} = -\dfrac{\mu_0 p}{c}\tilde{\boldsymbol{h}}, \end{cases}$$

$$(2.9.2)\quad \begin{cases} \operatorname{rot}\tilde{h}' = \dfrac{4\pi}{c}\tilde{j}'_c + \dfrac{\varepsilon p}{c}\tilde{E}' = \dfrac{4\pi}{c}\tilde{j}'_z + \dfrac{1}{c}(4\pi\eta + \varepsilon p)\tilde{E}' + \dfrac{4\pi\mu_0\eta p}{c^2}(\tilde{u}'\times H) \\ -\dfrac{4\pi\varkappa_0}{c}\operatorname{grad}\tilde{\theta}', \\ \operatorname{rot}\tilde{E}' = -\dfrac{\mu_0 p}{c}\tilde{h}'. \end{cases}$$

By performing scalar multiplication of the first of Eqs. (2.9.1) by $\tilde{E}'$, and the second of Eqs. (2.9.2) by $\tilde{h}$, subtracting and taking into account the known equation

$$(2.10)\qquad \operatorname{div}(A\times B) = B\operatorname{rot}A - A\operatorname{rot}B,$$

we find

$$(2.11)\qquad \frac{c}{4\pi}[\operatorname{div}(\tilde{E}'\times\tilde{h}) - \operatorname{div}(\tilde{E}\times\tilde{h}')] = (\tilde{j}'_c\tilde{E} - \tilde{j}_c E').$$

Substituting into (2.11) the following:

$$(2.12)\qquad \tilde{j}_c = \eta\left[\tilde{E} + \frac{\mu_0 p}{c}(\tilde{u}\times H)\right] - \varkappa_0\operatorname{grad}\tilde{\theta} + \tilde{j}_z,$$

and an analogous expression for $\tilde{j}_c$, we find

$$(2.13)\qquad \tilde{j}'_c\tilde{E} - \tilde{j}_c\tilde{E}' = \frac{\eta\mu_0 p}{c}[(\tilde{u}'\times H)\tilde{E} - (\tilde{u}\times H)\tilde{E}'] + \\ + \varkappa_0(\tilde{E}'\operatorname{grad}\tilde{\theta} - \tilde{E}\operatorname{grad}\tilde{\theta}') + \tilde{j}'_z\tilde{E} - \tilde{j}_z\tilde{E}'.$$

The integrand in the right-hand member of (2.8) will now be transformed thus:

$$(2.14)\qquad (\tilde{j}_c\times H)\tilde{u}' - (\tilde{j}'_c\times H)\tilde{u} = (\tilde{u}\times H)\tilde{j}'_c - (\tilde{u}'\times H)\tilde{j}_c = \\ = (\tilde{j}_z\times H)\tilde{u}' - (\tilde{j}'_z\times H)\tilde{u} + \eta[\tilde{E}'(\tilde{u}\times H) - \tilde{E}(\tilde{u}'\times H)] + \\ + \varkappa_0[(\tilde{u}'\times H)\operatorname{grad}\theta - (\tilde{u}\times H)\operatorname{grad}\theta'].$$

Taking advantage of relations (2.14) and (2.11), Eq. (2.8) can be expressed in the form:

$$(2.15)\qquad \nu\varkappa p\int_B(\tilde{X}_i\tilde{u}'_i - \tilde{X}'_i\tilde{u}_i)\,dV - \gamma\int_B(\tilde{Q}\theta' - \tilde{Q}'\tilde{\theta})\,dV + \varkappa\nu\varkappa_0\int_B(E'\operatorname{grad}\tilde{\theta} - \\ - \tilde{E}\operatorname{grad}\tilde{\theta}')\,dV - \gamma\varkappa\pi_0 p\int_B(\tilde{\theta}'\tilde{\varrho}_c - \tilde{\theta}\tilde{\varrho}'_c)\,dV + \frac{\varkappa_0\nu\varkappa\mu_0 p}{c}\int_B[(\tilde{u}'\times H)\operatorname{grad}\tilde{\theta} - \\ - (\tilde{u}\times H)\operatorname{grad}\tilde{\theta}']\,dV + \nu\varkappa\int_B(\tilde{j}'_z\tilde{E} - \tilde{j}_z\tilde{E}')\,dV + \frac{\nu\varkappa p\mu_0}{c}\int_B[(\tilde{j}_z\times H)\tilde{u}' - \\ - (\tilde{j}'_z\times H)\tilde{u}]\,dV = -\nu\varkappa p\int_A(\tilde{p}_i\tilde{u}'_i - \tilde{p}'_i\tilde{u}_i)\,dA + \\ + \gamma\varkappa\int_A(\tilde{\theta}'\tilde{\theta}_{,n} - \tilde{\theta}\tilde{\theta}'_{,n})\,dA + \frac{c\varkappa\nu}{4\pi}\int_A[(\tilde{E}'\times h) - (\tilde{E}\times\tilde{h}')]_i n_i\,dA.$$

which can be transformed thus:

$$(2.16)\qquad \nu\varkappa p\int_B (\tilde{X}_i\tilde{u}_i' - \tilde{X}_i'\tilde{u}_i)\,dV - \gamma\int_B (\tilde{Q}\tilde{\theta}' - \tilde{Q}'\tilde{\theta})\,dV + \varkappa\nu\varkappa_0\int_B [\tilde{E}_0'\,\mathrm{grad}\,\tilde{\theta} - \tilde{E}_0\,\mathrm{grad}\,\tilde{\theta}')\,dV + \nu\varkappa\int_B (\tilde{j}_z'\tilde{E}_0 - \tilde{j}_z E_0')\,dV - \gamma\varkappa\pi_0 p\int_B (\tilde{\theta}'\tilde{\varrho}_e - \tilde{\theta}\tilde{\varrho}_e')\,dV =$$

$$= -\nu\varkappa p\int_A (\tilde{p}_i\tilde{u}_i' - \tilde{p}_i'\tilde{u}_i)\,dA + \gamma\varkappa\int_A (\tilde{\theta}'\tilde{\theta}_{,n} - \tilde{\theta}\tilde{\theta}_{,n}')\,dA + \frac{c\varkappa\nu}{4\pi}\int_A [(\tilde{E}'\times\tilde{h}) - (\tilde{E}\times\tilde{h}')]_i n_i\,dA,$$

where $\tilde{E}_0 = \tilde{E} + \frac{\mu_0 p}{c}(\tilde{u}\times H)$ stands for the field in the system at rest. Eq. (2.16) expresses the reciprocity theorem of magneto-thermo-elasticity in its final form. This equation should be completed with the appropriate equation for the vacuum surrounding the body B.

Considerations analogous to the above show that with no currents and no electric charges in vacuum it takes the form

$$(2.17)\qquad \int_A [(\tilde{E}'^0\times\tilde{h}^0) - (\tilde{E}^0\times\tilde{h}'^0)]\,n\,dA = 0.$$

The two sets of Eqs. (2.16) and (2.17) are interrelated additionally by the continuity equations of the field, which for $\mu_0\approx 1$, $\varepsilon\approx 1$ for instance, have, with no surface charges, the form

$$(2.18)\qquad \tilde{h}\approx\tilde{h}^0;\quad \tilde{E}\approx\tilde{E}^0.$$

If μ_0, ε do not approach unity, the conditions (2.18) are valid for tangential fields only and will change for normal induction. If the field in vacuum is disregarded, that is if the boundary values of the field are given in an explicit manner, the corresponding Eq. (2.17) falls-off, and Eqs. (2.18) express assigned values at the boundary. If the boundary conditions are assumed in an appropriate manner to be homogeneous, the last surface integral in the right-hand member of (2.16) vanishes.

These integrals vanish also, if the region B is introduced to an infinite region.

Let us observe that for $\tilde{\varrho}_e = 0$, $\tilde{\varrho}_e' = 0$ and if the terms with coefficients composed of products of the small quantities ν and $\varkappa_0$, are rejected, Eq. (2.16) becomes

$$(2.19)\qquad \nu\varkappa p\int_B (\tilde{X}_i\tilde{u}_i' - \tilde{X}_i'\tilde{u}_i)\,dV - \gamma\int_B (\tilde{Q}\tilde{\theta}' - \tilde{Q}'\tilde{\theta})\,dV + \varkappa\nu\int_B (\tilde{j}_z'\tilde{E}_0 - \tilde{j}_z\tilde{E}_0')\,dV = -\nu\varkappa p\int_A (\tilde{p}_i\tilde{u}_i' - \tilde{p}_i'\tilde{u}_i)\,dA + \gamma\varkappa\int_A (\tilde{\theta}'\tilde{\theta}_{,n} - \tilde{\theta}\tilde{\theta}_{,n}')\,dA + \frac{c\nu\varkappa}{4\pi}\int_A [(\tilde{E}'\times\tilde{h}) - (\tilde{E}\times\tilde{h}')]_i n_i\,dA.$$

This equation represents that form of the reciprocity theorem of magneto-thermo--elasticity, in which the thermoelectric effects are disregarded. If the electric conductivity in (2.19) passes into infinity, and if the external currents are ignored, we obtain the reciprocity theorem for magneto-thermo-elasticity of perfect conductors as obtained in [1].

This equation follows directly from (2.3) and (2.7), if we set $\pi_0 = 0$ in (2.7), and if we express the coordinates of Maxwell's tensors in (2.3) in terms of equations corresponding to the case of perfect conductor.

The remains the inverse Laplace transform to be performed on the general reciprocity equation (2.16). Consequently, we obtain:

$$(2.20)\quad \nu\varkappa \int_B dV(x) \int_0^t \left[X_i(x, t-\tau) \frac{\partial u_i'(x,\tau)}{\partial \tau} - X_i'(x, t-\tau) \frac{\partial u_i(x,\tau)}{\partial \tau} \right] d\tau -$$

$$- \gamma \int_B dV(x) \int_0^t [Q(x, t-\tau)\,\theta'(x,\tau) - Q'(x, t-\tau)\,\theta(x,\tau)]\, d\tau +$$

$$+ \nu\varkappa \int_B dV(x) \int_0^t [\boldsymbol{j}_z'(x, t-\tau)\,\boldsymbol{E}_0(x,\tau) - \boldsymbol{j}_z(x, t-\tau)\,\boldsymbol{E}_0'(x,\tau)]\, d\tau +$$

$$+ \varkappa\nu\varkappa_0 \int_B dV(x) \int_0^t [\boldsymbol{E}_0'(x, t-\tau)\,\mathrm{grad}\,\theta(x,\tau) - \boldsymbol{E}_0(x, t-\tau)\,\mathrm{grad}\,\theta'(x,\tau)]\, d\tau -$$

$$+ \gamma\varkappa\pi_0 \int_B dV(x) \int_0^t \left[\varrho_e'(x, t-\tau) \frac{\partial \theta(x,\tau)}{\partial \tau} - \varrho(x, t-\tau) \frac{\partial \theta'(x,\tau)}{\partial \tau} \right] d\tau =$$

$$= -\nu\varkappa \int_A dA(x) \int_0^t \left[p_i(x, t-\tau) \frac{\partial u_i'(x,\tau)}{\partial \tau} - p_i'(x, t-\tau) \frac{\partial u_i(x,\tau)}{\partial \tau} \right] d\tau +$$

$$+ \gamma\varkappa \int_A dA(x) \int_0^t [\theta'(x, t-\tau)\,\theta_{,n}(x,\tau) - \theta(x, t-\tau)\,\theta'_{,n}(x,\tau)]\, d\tau +$$

$$+ \frac{c\varkappa\nu}{4\pi} \int_A dA(x) \int_0^t [(\boldsymbol{E}'(x, t-\tau) \times \boldsymbol{h}(x,\tau)) - (\boldsymbol{E}(x, t-\tau) \times \boldsymbol{h}'(x,\tau))]_i\, n_i\, d\tau.$$

3. Final remarks

The general reciprocity theorem derived in the present paper is applicable to many problems such as the problem of constructing integral equations for various boundary-value problems or the obtainment of the other type solutions. In view of their variety and the lack of space the applications of the reciprocity theorem will not be discussed here. Some examples of application of this theorem will be furnished in further papers devoted to the solution of various special problems of magneto-thermo-elasticity.

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С. КАЛИСКИЙ, В. НОВАЦКИЙ, **ТЕОРЕМА О ВЗАИМНОСТИ В МАГНИТО-ТЕРМО-УПРУГОСТИ. II. ЕСТЕСТВЕННЫЕ ПРОВОДНИКИ.**

В работе выведена теорема о взаимности в магнито-термо-упругости для естественных проводников, причем учитываются одновременно термоэлектрические эффекты. В особом случае выведенная теорема сводится к виду представленному в работе [1], где обсуждалась проблема взаимности в магнито-термо-упругости для идеальных упругих проводников.

Полученная в настоящей работе общая теорема пригодна для многих применений, в особенности для построения интегральных уравнений в различных проблемах магнито-термо-упругости.