BULLETIN
DE
L'ACADÉMIE POLONAISE DES SCIENCES

SÉRIE DES SCIENCES TECHNIQUES

Volume XIII, Numéro 4

VARSOVIE 1965

BULLETIN DE L'ACADÉMIE
8. POLONAISE DES SCIENCES
Série des sciences techniques
Volume XIII, No. 4 — 1965

APPLIED MECHANICS

Green Functions for a Thermoelastic Medium. III

by

W. NOWACKI

Presented on January 2, 1965

In our previously papers [1] and [2] the Green functions for the dynamic thermoelastic problem were derived. In this Note we are concerned with the determination of displacements and temperature accompanying the deformations in an unbounded thermoelastic semi-space for the case of a concentrated force and heat source. We assume that this force as well as the heat source are slowly changing in time. In the quasi-static problem the inertial members in equations of motion can be disregarded and, consequently, the fundamental equations of thermoelasticity may be formulated as follows:

$$\nabla^2\theta - \frac{1}{\varkappa}\frac{\partial\theta}{\partial t} - \eta\frac{\partial}{\partial t}\operatorname{div}\vec{u} = -\frac{Q}{\varkappa}, \tag{1}$$

$$\mu\nabla^2\vec{u} + (\lambda+u)\operatorname{grad}\operatorname{div}\vec{u} + \vec{X} = \gamma\operatorname{grad}\theta. \tag{2}$$

Eq. (1) is an expanded equation of heat conduction, whereas Eq. (2) represents the displacement equation of the theory of elasticity. These equations are mutually coupled. The notations adopted in this paper are: $\vec{u}$ stands for the displacement vector, $\vec{X}$ — for the vector of body forces, $\theta = T - T_0$ denotes the difference between the absolute temperature T and the temperature characterizing the natural thermic state of the body, T_0; Q is the function describing the intensity of heat sources. μ and λ are Lame's coefficients with reference to the isothermic state: $\varkappa = \lambda_0/\varrho c_\varepsilon$ denotes a coefficient, wherein: λ_0 is the heat conductivity constant, ϱ — density and c_ε — specific heat of the body, the deformation being assumed constant. Further, $\eta = \gamma T_0/\lambda_0$ where: $\gamma = (3\lambda+2\mu)\,\alpha_t$, α_t being the coefficient of linear heat dilatation. Finally, $Q = W/\varrho c_\varepsilon$, where W represents the quantity of heat generated in a volume unit of the body in a time unit. Functions $\vec{u}$, $\vec{X}$, Q, θ are the functions of position and time.

Decomposition of the displacement and the body forces vectors into the potential and the solenoidal vectors respectively leads to:

$$\vec{u} = \operatorname{grad}\varphi + \operatorname{rot}\vec{\varphi}, \tag{3}$$

$$\vec{X} = \varrho\,(\operatorname{grad}\vartheta + \operatorname{rot}\vec{\chi}). \tag{4}$$

We reduce the system of Eqs. (1) and (2) to the system of the following three equations:

$$\left(\nabla^2 - \frac{1}{\varkappa}\partial_t\right)\theta - \eta\partial_t\nabla^2\varphi = -\frac{Q}{\varkappa}, \tag{5}$$

$$\nabla^2\varphi - m\theta = -\frac{1}{c_1^2}\vartheta, \tag{6}$$

$$\nabla^2\vec{\psi} = -\frac{1}{c_2^2}\vec{\chi}, \quad c_1^2 = \frac{\lambda+2\mu}{\varrho}, \quad m = \frac{\gamma}{c_1^2\varrho}, \quad c_2^2 = \frac{\mu}{\varrho}, \quad \partial_t = \frac{\partial}{\partial t}. \tag{7}$$

Let the concentrated force, varying in time and acting along the x_1 axis be applied at the origin of the coordinate system. It means that

$$X_j = \delta(x_1)\,\delta(x_2)\,\delta(x_3)\,\delta_{ij}\,f(t).$$

Now, let us assume that this force starts its action at the moment $t = 0^+$.

We perform the Laplace integral transform on Eqs. (5)—(7). Introducing the transformation of the function:

$$L[g(t)] = \tilde{g}(p) = \int_0^t g(t)\,e^{-pt}\,dt, \quad p > 0,$$

and assuming homogeneous initial conditions, we reduce Eqs. (5)—(7) to the forms:

$$\left(\nabla^2 - \frac{p}{\varkappa}\right)\tilde{\theta} - \eta p\,\nabla^2\tilde{\varphi} = 0, \tag{8}$$

$$\nabla^2\tilde{\varphi} - m\tilde{\theta} = -\frac{1}{c_1^2}\tilde{\vartheta}, \tag{9}$$

$$\nabla^2\tilde{\vec{\psi}} = -\frac{1}{c_2^2}\tilde{\vec{\chi}}. \tag{10}$$

The quantities $\tilde{\vartheta}$ and $\tilde{\vec{\chi}}$ will be determined from the formulae:

$$\tilde{\vartheta}(\vec{x}, p) = -\frac{1}{4\pi\varrho}\int_B \tilde{\vec{X}}(\vec{x}^1, p)\cdot\operatorname{grad}\left(\frac{1}{R(\vec{x}, \vec{x}')}\right)dV(\vec{x}^1), \tag{11}$$

$$\tilde{\vec{\chi}}(\vec{x}, p) = -\frac{1}{4\pi\varrho}\int_B \tilde{\vec{X}}(\vec{x}^1, p)\times\operatorname{grad}\left(\frac{1}{R(\vec{x}, \vec{x}')}\right)dV(\vec{x}^1), \tag{12}$$

whence

$$\tilde{\vartheta}(\vec{x}, p) = -\frac{1}{4\pi\varrho}\partial_1\left(\frac{1}{R}\right)\tilde{f}(p), \quad \tilde{\chi}_1 = 0, \quad \tilde{\chi}_2 = \frac{1}{4\pi\varrho}\partial_3\left(\frac{1}{R}\right)\tilde{f}(p),$$

$$\tilde{\chi}_3 = -\frac{1}{4\pi\varrho}\partial_2\left(\frac{1}{R}\right)\tilde{f}(p), \quad R = (x_1^2 + x_2^2 + x_3^2)^{1/2}. \tag{13}$$

After eliminating the function φ from Eqs. (8) and (9), we obtain

$$(14)\qquad (\nabla^2 - \beta^2)\,\tilde{\theta} = \frac{q\varepsilon}{4\pi\varrho c_1^2 m}\,\partial_1\left(\frac{1}{R}\right)\tilde{f}(p), \qquad \beta^2 = \frac{p}{\varkappa}(1+\varepsilon),\ \varepsilon = \eta m\varkappa .$$

The function expressed by Eq. (15) is the particular integral of Eq. (14)

$$(15)\qquad \tilde{\theta} = \varepsilon A \partial_1\left(\frac{e^{-\beta R}}{R} - \frac{1}{R}\right)\tilde{f}(p), \qquad A = \frac{1}{4\pi\varrho c_1^2 m(1+\varepsilon)}.$$

On performing the inverse Laplace's transform we arrive at

$$(16)\qquad \theta(\vec{x}, t) = -\varepsilon A \partial_1\left\{\frac{f(t)}{R} - \int_0^t f(\tau)\sqrt{\frac{1+\varepsilon}{4\pi\varkappa(t-\tau)^3}}\exp\left[-\frac{R^2(1+\varepsilon)}{4\varkappa(t-\tau)}\right]d\tau\right\}.$$

Inserting Eq. (15) into (9), we get the following differential equation:

$$(17)\qquad \nabla^2\tilde{\varphi} = B\partial_1\left(\varepsilon\frac{e^{-\beta R}}{R} + \frac{1}{R}\right)\tilde{f}(p), \qquad B = Am = \frac{1}{4\pi\varrho c_1^2(1+\varepsilon)}.$$

Taking into account that

$$\nabla^2\left(\frac{x_1}{R}\right) = -\frac{2x_1}{R^3}, \qquad \nabla^2\left(\frac{e^{-\beta R}}{R}\right) = \beta^2\frac{e^{-\beta R}}{R},$$

we obtain the particular integral of Eq. (17) in the form:

$$(18)\qquad \tilde{\varphi}(\vec{x}, p) = \frac{B}{2}\partial_1\left\{R + \frac{2\varepsilon\varkappa}{p(1+\varepsilon)R}\exp\left[-R\sqrt{\frac{(1+\varepsilon)p}{\varkappa}}\right]\right\}\tilde{f}(p).$$

After carrying out the inverse Laplace transform on function $\tilde{\varphi}$, we have

$$(19)\qquad \varphi(\vec{x}, t) = \frac{B}{2}\partial_1\left\{Rf(t) + \frac{2\varepsilon\varkappa}{(1+\varepsilon)R}\int_0^t f(\tau)\,\mathrm{erfc}\left(\frac{R}{2}\sqrt{\frac{1+\varepsilon}{\varkappa(t-\tau)}}\right)d\tau\right\},$$

where

$$\mathrm{erfc}(z) = \frac{2}{\sqrt{\pi}}\int_z^\infty e^{-u^2}\,du .$$

The solution of Eqs. (10) is well-known:

$$\psi_1 = 0,\ \psi_2 = \frac{1}{8\pi\mu}\,\partial_3(R)f(t), \qquad \psi_3 = -\frac{1}{8\pi\mu}\,\partial_2(R)f(t).$$

Since

$$\mathrm{rot}\,\vec{\psi} = \frac{1}{8\pi\mu}\left\{\frac{1}{R^2} - \partial_1^2(R),\ -\partial_1\partial_2(R),\ -\partial_1\partial_3(R)\right\},$$

we may represent the components of displacement, by the formula (3):

$$u_j^{(1)} = \left[\left(\frac{B}{2} - \frac{1}{8\pi\mu}\right)\partial_1\,\partial_j\,(R) + \frac{1}{8\pi\mu R}\,\delta_{1j}\right] f(t) + \tag{20}$$

$$+ \frac{\varepsilon B\varkappa}{1+\varepsilon}\,\partial_1\,\partial_j \int_0^t \frac{f(\tau)}{R}\,\mathrm{erfc}\left(\frac{R}{2}\sqrt{\frac{1+\varepsilon}{\varkappa\,(t-\tau)}}\right) d\tau\,.$$

It we now transfer the point of application of the concentrated force from the origin of the coordinate system to the point $(\vec{\xi})$, and direct this force along the x_k-axis, we will obtain formulae for displacements and the corresponding temperature

$$u_j^{(k)} = \left[\left(\frac{B}{2} - \frac{1}{8\pi\mu}\right)\partial_j\,\partial_k\,(R) + \frac{1}{8\pi\mu R}\,\delta_{jk}\right] f(t) + \tag{21}$$

$$+ \frac{\varepsilon\varkappa B}{1+\varepsilon}\,\partial_j\,\partial_k \int_0^t \frac{f(\tau)}{R}\,\mathrm{erfc}\left(\frac{R}{2}\sqrt{\frac{1+\varepsilon}{\varkappa\,(t-\tau)}}\right) d\tau\,,$$

$$\theta^{(k)} = -\,\varepsilon A\partial_k\left\{\frac{f(t)}{R} - \int_0^t f(\tau)\sqrt{\frac{1+\varepsilon}{4\pi\varkappa\,(t-\tau)}}\,\exp\left[-\,\frac{R^2\,(1+\varepsilon)}{4\varkappa\,(t-\tau)}\right] d\tau\right\}. \tag{22}$$

Let us remark that displacement $u_j^{(k)}$ consists of two terms, the first varying with time similarly as the function $f(t)$, and the other in the form of convolution, characterizing the conjugation of the field of deformation with that of temperature. The same holds true for the function $\theta^{(k)}$. Assuming $f(t) = \delta\,(t)$ we obtain in virtue of Eqs. (21) and (22) Green functions for our problem, i.e. the action of the concentrated and instantaneous force acting at the point $(\vec{\xi})$ directed along the axis x_k

$$U_j^{(k)}\,(\vec{x}, \vec{\xi}, t) = \left[\left(\frac{B}{2} - \frac{1}{8\pi\mu}\right)\partial_j\,\partial_k\,(R) + \frac{1}{8\pi\mu R}\,\delta_{jk}\right]\delta\,(t) + \tag{23}$$

$$+ \frac{\varepsilon\varkappa B}{1+\varepsilon}\,\partial_j\,\partial_k\left[\frac{1}{R}\,\mathrm{erfc}\left(\frac{R}{2}\sqrt{\frac{1+\varepsilon}{\varkappa t}}\right)\right],$$

$$\Theta^{(k)} = -\,\varepsilon A\partial_k\left\{\frac{1}{R}\,\delta\,(t) - \sqrt{\frac{1+\varepsilon}{4\varkappa\,(\pi t^3)}}\,\exp\left[-\,\frac{R^2\,(1+\varepsilon)}{4\varkappa t}\right]\right\}, \tag{24}$$

$$R^2 = (x_i - \xi_i)\,(x_i - \xi_i)\,.$$

In the concentrated force varies harmonically with time, i.e. for $X_j\,(x, t) = \delta\,(\vec{x} - \vec{\xi})\,\delta_{jk}\,e^{-i\omega t}$, we obtain the equations of amplitudes, provided we replace in the Laplace transforms the parameter p by the parameter $-i\omega$, wherein ω stands for vibration frequency. Thus, function φ, for instance, will take the form:

$$\varphi\,(\vec{x}, t) = \frac{B}{2}\,\partial_1\left\{R - \frac{2\varepsilon\varkappa}{i\omega\,(1+\varepsilon)\,R}\,\exp\left(-\,R\sqrt{\frac{-i\omega\,(1+\varepsilon)}{\varkappa}}\right)\right\} e^{-i\omega t}. \tag{25}$$

The displacements and the temperature pertinent to harmonic vibrations are given by the formulae

$$(26)\qquad u_j^{(k)} = \left[\left(\frac{B}{2} - \frac{1}{8\pi\mu}\right)\partial_j\partial_k(R) + \frac{1}{8\pi\mu R}\delta_{jk}\right]e^{-i\omega t} + \\ + \frac{i\varkappa\varepsilon B}{\omega(1+\varepsilon)}\partial_j\partial_k\left\{\frac{1}{R}\exp\left[-i\omega\left(t-\frac{R}{c}\right)-\gamma R\right]\right\},$$

$$(27)\qquad \theta^{(k)} = -\varepsilon A\partial_k\left\{\frac{1}{R}\exp(-i\omega t) - \frac{1}{R}\exp\left[-i\omega\left(t-\frac{R}{c}\right)-\gamma_0 R\right]\right\},$$

where:

$$c = \frac{\omega}{\operatorname{Re}(k)} = \left(\frac{2\omega\varkappa}{1+\varepsilon}\right)^{1/2}, \quad \gamma_0 = \operatorname{Im}(k) = \left(\frac{\omega(1+\varepsilon)}{2\varkappa}\right)^{1/2}, \quad k = \left(\frac{i\omega(1+\varepsilon)}{\varkappa}\right)^{1/2}.$$

Let us now examine the non coupled problem. We apply the approximate classical equation of heat conductivity:

$$(28)\qquad \Delta^2\theta - \frac{1}{\varkappa}\frac{\partial\theta}{\partial t} = 0,$$

instead of Eq. (1).

Passing from Eq. (1) to Eq. (28), we introduce into the results obtained the quantities $\eta = 0$, or $\varepsilon = 0$ respectively.

In this way Eqs. (21) and (22) can be expressed as:

$$(29)\qquad u_j^{(k)} = \frac{\lambda+\mu}{8\pi\mu(\lambda+2\mu)}\left[\frac{x_i x_j}{R^3} + \frac{\lambda+3\mu}{\lambda+\mu}\delta_{jk}\left(\frac{1}{R}\right)\right]f(t), \quad \theta^{(k)} = 0.$$

It is to be noted that Eq. (29) can be written in this way:

$$(30)\qquad u_j^{(k)}(\vec{x}, t) = \hat{u}_j^{(k)}(\vec{x})\, f(t),$$

wherein $\hat{u}_j^{(k)}(\vec{x})$ is the Green function for the static problem, given by the known Kelvin's formula.

Basing on Green functions as expressed by Eqs. (29) and (24), we may formulate further singularities. If we apply at point $\left((\xi_1 + \frac{\Delta\xi_1}{2}, \xi_2, \xi_3\right)$ the concentrated force $\frac{P}{\Delta\xi_1}f(t)$ acting along the x_1-axis, and, on the other hand, at point $\left(\xi_1 - \frac{\Delta\xi_1}{2}\,\xi_2, \xi_3\right)$ the force $\frac{P}{\Delta\xi_1}f(t)$ acting along the negative axis x_1, then after passing $\Delta\xi_1 \to 0$ we obtain the following displacements, v_j, and temperature, θ:

$$(31)\qquad v_j = -P\frac{\partial u_j^{(1)}}{\partial\xi_1}, \quad \Theta = -P\frac{\partial\theta^{(1)}}{\partial\xi_1}.$$

For $P = 1$ we get Green functions for the double force without a moment. Assume at the point $(\vec{\xi})$ a compression centre is acting, that is three dual forces, the first of them acting along the x_1-axis, the second — along the x_2-axis, and the third — along the x_3-axis. Denoting the displacement by the symbol $\hat{U}_j$ and the temperature by $\hat{\theta}$. both due to the action of the compression centre, we have

$$\hat{U}_j = -(\partial_{1'} U_j^{(1)} + \partial_{2'} U_j^{(2)} + \partial_{3'} U_j^{(3)}) = -\left(B - \frac{1}{8\pi\mu}\right) f(t)\, \partial_j \left(\frac{1}{R}\right) + \tag{32}$$
$$+ \frac{\varepsilon B}{2} \sqrt{\frac{1+\varepsilon}{\varkappa}}\, \partial_j \left\{ \int_0^t \frac{f(\tau)}{\sqrt{\pi (t-\tau)^3}} \exp\left[-\frac{R^2(1+\varepsilon)}{4\varkappa(t-\tau)}\right] d\tau \right\},$$

$$\hat{\theta} = -(\partial_{1'} \theta^{(1)} + \partial_{2'} \theta^{(2)} + \partial_{3'} \theta^{(3)}) = -\frac{\varepsilon(1+\varepsilon) A}{\varkappa R} \int_0^t \frac{\partial f(\tau)}{\partial \tau} \times \tag{33}$$
$$\times \frac{1}{\sqrt{4\pi\varkappa(t-\tau)}} \exp\left[-\frac{R^2(1+\varepsilon)}{4\varkappa(t-\tau)}\right] d\tau, \qquad \partial_{j'} = \frac{\partial}{\partial \xi_j}.$$

If the compression centre varies harmonically with time variable, we obtain

$$\hat{U}_j = -\left(B - \frac{1}{8\pi\mu}\right) \partial_j \left(\frac{1}{R}\right) e^{-i\omega t} - \varepsilon B \partial_j \left\{\frac{1}{R} \exp\left[-i\omega\left(t - \frac{R}{c}\right) - \gamma_0 R\right]\right\}, \tag{33'}$$

$$\hat{\theta} = \frac{\varepsilon(1+\varepsilon) A i\omega}{\varkappa R} \exp\left[-i\omega\left(t - \frac{R}{c}\right) - \gamma_0 R\right]. \tag{34}$$

For the non-coupled problem ($\varepsilon = 0$) there is:

$$\hat{U}_j = \frac{\lambda + \mu}{8\pi\mu(\lambda + 2\mu)} \partial_j \left(\frac{1}{R}\right) f(t), \qquad \hat{\theta} = 0. \tag{35}$$

Assume now that a concentrated, varying with time a heat source is acting at the origin of the coordinate system

$$Q(R, t) = \delta(R) f(t).$$

Elimination of function φ from Eqs. (5) and (6) and performance of Laplace transform yields

$$(\nabla^2 - \beta^2)\hat{\Theta} = -\frac{\delta(R)}{\varkappa} \bar{f}(p). \tag{36}$$

Particular integral of this equation constitutes the function

$$\tilde{\theta} = \frac{1}{4\pi\varkappa R} e^{-\beta R} \tilde{f}(p), \qquad R = (x_1^2 + x_2^2 + x_3^2)^{1/2}. \tag{37}$$

After performation of inverse Laplace transform the temperature will be expressed by the formula

$$\theta(R, t) = \frac{1}{4\pi\varkappa} \int_0^t f(\tau) \sqrt{\frac{1+\varepsilon}{4\pi\varkappa(t-\tau)}} \exp\left[-\frac{R^2(1+\varepsilon)}{4\varkappa(t-\tau)}\right] d\tau. \tag{38}$$

Function $\tilde{\varphi}$ will be determined from the equation:

(39) $$\nabla^2 \tilde{\varphi} = m\tilde{\theta}\,.$$

The following function is the particular integral of (39):

(40) $$\tilde{\varphi} = \frac{m}{4\pi\varkappa\beta^2}\left(\frac{e^{-\beta R}}{R} - \frac{1}{R}\right)\tilde{f}(p)\,.$$

When the inverse Laplace transform is performed, we get

(41) $$\varphi(R, t) = \frac{m}{4\pi R(1+\varepsilon)}\left\{\int\limits_0^t f(\tau)\,\mathrm{erfc}\left(R\sqrt{\frac{1+\varepsilon}{4\varkappa(t-\tau)}}\right)d\tau - \int\limits_0^t f(\tau)\,d\tau\right\}.$$

The displacement, being the effect of the action of the heat source, may be obtained from the equation

(42) $$u_j = \partial_j \varphi_j\,.$$

If now the heat source is transferred from the origin of the coordinate system to the point $(\vec{\xi})$ we have to put $R^2 = (x_j - \xi_j)(x_j - \xi_j)$ in Eqs. (38), (41) and (42).

Returning to the non-coupled problem, we insert $\varepsilon = 0$ in formulae (36)–(42). Thus, for the static problem there is

(43) $$\theta = \frac{1}{4\pi\varkappa R}\cdot$$

The equation:

(44) $$\nabla^2\varphi = m\theta\,,$$

yields the particular integral

(45) $$\varphi = \frac{m}{8\pi\varkappa}R + \text{const.}$$

Thus,

(46) $$u_j = \frac{m}{8\pi R}(x_j - \xi_j).$$

Let us apply the concentrated force varying with time at point $(\vec{\xi})$, directed along the x_k-axis.

The formula for the temperature at the point $(\vec{\xi})$, induced by the action of this force is given by the relation (15)

(47) $$\tilde{\theta}_X(\vec{\xi}', \vec{\xi}, p) = \varepsilon A \partial_k\left(\frac{e^{-\beta R}}{R} - \frac{1}{R}\right)\tilde{f}(p), \qquad R^2 = (\xi_j' - \xi_j)(\xi_j' - \xi_j)\,.$$

Let us now apply the heat source at point $(\vec{\xi}')$ and determine the displacements U_k^Q at point $(\vec{\xi})$. Eq. (42) yields:

(48) $$\tilde{u}_k^Q(\vec{\xi}, \vec{\xi}', p) = \frac{m}{4\pi\beta^2}\left(\frac{e^{-\beta R}}{R} - \frac{1}{R}\right)\tilde{f}(p), \qquad R^2 = (\xi_j - \xi_j')(\xi_j - \xi_j')\,.$$

Comparing Eqs. (47) and (48), we arrive at

$$\tilde{\theta}_X(\vec{\xi}', \vec{\xi}, p) = \frac{\eta\varkappa p}{\gamma} \tilde{u}_k^Q(\vec{\xi}, \vec{\xi}^1, p), \quad \gamma = \frac{m}{c_1^2 \varrho}, \tag{49}$$

This relation may be derived from the theorem on the reciprocity [3].

In the transformed form, the theorem on the reciprocity, for an unbounded region, takes the following form

$$\eta\varkappa p \int_B (\tilde{X}_i \tilde{u}_i' - \tilde{X}_i' \tilde{u}_i)\, dV + \gamma \int_B (\tilde{Q}\tilde{\theta}' - \tilde{Q}'\tilde{\theta})\, dV = 0. \tag{50}$$

If we assume that

$$\tilde{X}_i = \delta(\vec{x} - \vec{\xi})\, \delta_{ik} \tilde{f}(p), \quad \tilde{X}_i' = 0, \quad \tilde{Q}_i' = \delta(\vec{x} - \vec{\xi}')\tilde{f}(p),$$
$$Q_i = 0, \quad \tilde{u}_k' = \tilde{u}_k^Q, \tilde{\theta} = \tilde{\theta}_X,$$

then from Eq. (50) we obtain

$$\eta\varkappa p \int_B \delta(\vec{x} - \vec{\xi})\, \delta_{ik} \tilde{f}(p) \tilde{u}_i^Q(\vec{x}, \vec{\xi}, p)\, dV(\vec{x}) -$$
$$- \gamma \int_B \delta(\vec{x} - \vec{\xi}')\tilde{f}(p)\, \tilde{\theta}_x(\vec{x}, \vec{\xi}, p)\, dV(\vec{x}) = 0,$$

whence

$$\theta_X(\vec{\xi}', \vec{\xi}, t) = \frac{\eta\varkappa}{\gamma} \frac{\partial u_k^Q(\vec{\xi}, \vec{\xi}', t)}{\partial t}, \tag{51}$$

in conformity with (49).

If the concentrated force and the heat source vary harmonically with time, then

$$\theta_X(\vec{\xi}', \vec{\xi}, t) = -\frac{i\omega\eta\varkappa}{\gamma} u_k^Q(\vec{\xi}, \vec{\xi}', t). \tag{52}$$

DEPARTMENT OF MECHANICS OF CONTINUOUS MEDIA, INSTITUTE OF FUNDAMENTAL TECHNICAL PROBLEMS, POLISH ACADEMY OF SCIENCES

(ZAKŁAD MECHANIKI OŚRODKÓW CIĄGŁYCH, INSTYTUT PODSTAWOWYCH PROBLEMÓW TECHNIKI, PAN)

REFERENCES

[1] W. Nowacki, Bull. Acad. Polon. Sci., Sér. sci. techn., **12** (1964), 315 [391].

[2] — , ibid., **12** (1964), 465 [651].

[3] V. Ionescu-Cazimir, **12** (1964), 473 [659].

В. НОВАЦКИЙ, **ФУНКЦИИ ГРИНА ДЛЯ ТЕРМОУПРУГОЙ СРЕДЫ.** III.

В сообщении приводятся в замкнутом виде функции Грина для действия сосредоточенных сил, а также источник температуры в бесконечной термоупругой среде. Решения справедливы для квазистатического состояния, в котором изменения сосредоточенных сил и источника являются замедленными во времени, так что можно в основных уравнениях термоупругости пренебречь инерциальными членами.