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Thermal Stresses in Elastic and Viscoelastic Shells

by

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The purpose of the present paper is to determine the displacements produced in shells by a field of initial deformations (distortions), in particular by the field of temperature. The starting point for our considerations is the reciprocal theorem for a three-dimensional elastic body, distortions being taken into account. This theorem will be specialized to the plane state of stresses in shells. The theorem in three-dimensional state of stresses assumes the form [1], [2]:

$$(1.1)\qquad \int_V X'_i u''_i dV + \int_S p'_i u''_i dS + \int_V \varepsilon^{0'}_{ij} \sigma''_{ij} dV = \int_V X''_i u'_i dV + \int_S p''_i u'_i dS + \int_V \varepsilon^{0''}_{ij} \sigma'_{ij} dV, \quad i, j = 1, 2, 3.$$

In the above equation there appear two states denoted correspondingly by "prime" and "double prime". Thus, in the first state u'_i denote the components of displacement vector $\vec{u}'$; X'_i are the components of the body forces vector $\vec{X}'$, while p'_i are the components of the vector of surface loadings $\vec{p}'$; σ'_{ij} denote the stress tensor components and $\varepsilon^{0'}_{ij}$ are the distortion components. λ and μ are the Lamé constants. The integration is performed over volume V and surface S bounding an elastic body.

Let us consider an elastic body clamped on the part S_1 of the region S and on the remaining part S_2 free from loadings. Suppose the required displacements u'_i to be produced by distortions $\varepsilon^{0'}_{ij}$. We assume that $X'_i = 0$. As the second state of loading we assume the concentrated unit force acting at point (ξ) and directed along the x_k axis. Moreover, let $\varepsilon^{0''}_{ij} = 0$. Substituting in (1.1) $X'_i = 0$, $\varepsilon^{0''}_{ij} = 0$, and

$$X''_i = \delta(x_1 - \xi_1)\, \delta(x_2 - \xi_2)\, \delta(x_3 - \xi_3)\, \delta_{ik},$$

and taking into account that the surface integrals vanish, we obtain:

$$(1.2)\qquad 1''_k u'_k(\xi) = \int [2\mu \varepsilon^{0'}_{ij}(x)\, \varepsilon''^{(k)}_{ij}(x, \xi) + \lambda e^{0'}(x)\, e''^{(k)}(x, \xi)]\, dV(x), \quad i, j, k = 1, 2, 3.$$

Deformations $\varepsilon''^{(k)}_{ij}(x, \xi)$ and dilatation $e''^{(k)}(x, \xi)$ are Green functions; they denote the deformations and the dilatation at point (x) produced by the action of the concentrated force applied at (ξ) and directed along the x_k axis. Eq. (1.2) may be written also in the following form:

$$u'_k(\xi) = \int_V \varepsilon^0_{ij}(x)\, \sigma''^{(k)}_{ij}(x, \xi)\, dV(x), \quad i, j, k = 1, ,2\, 3, \tag{1.3}$$

where the components of the stress state produced by the action of the concentrated force at point (ξ) and directed along the x_k axis are denoted by $\sigma''^{(k)}_{ij}(x, \xi)$.
The stresses $\sigma''^{(k)}_{ij}$ are given by the formulae

$$\begin{gathered} \sigma''^{(k)}_{ij} = 2\mu(\gamma^{(k)}_{ij} + x_3\, \varkappa^{(k)}_{ij}) + \eta\delta_{ij}(\gamma^{(k)} + x_3\, \varkappa^{(k)}), \\ \gamma^{(k)} = \gamma^{(k)}_{jj}, \quad \varkappa^{(k)} = \varkappa^{(k)}_{jj}, \quad \eta = \frac{2\mu\lambda}{\lambda + 2\mu}. \end{gathered} \tag{1.4}$$

Suppose that distortions ε^0_{ij} are linear functions of x_3

$$\varepsilon^0_{ij} = \psi^0_{ij} + x_3\, \varphi^0_{ij}, \quad \psi^0_{ij} \equiv \psi^0_{ij}(x_1, x_2), \quad \varphi^0_{ij} \equiv \varphi^0_{ij}(x_1, x_2). \tag{1.5}$$

If we substitute (1.5) into (1.3) and make use of relations (1.4), then performing the integration with respect to x_3, we obtain the following formulae for, displacement:

$$\begin{aligned} u_k(\xi_1, \xi_2) = \iint_{(\Gamma)} [\psi^0_{ij}(x_1, x_2)\, N^{(k)}_{ij}(x_1, x_2, \xi_1, \xi_2) + \\ + \varphi^0_{ij}(x_1, x_2)\, M^{(k)}_{ij}(x_1, x_2, \xi_1, \xi_2)]\, dx_1, dx_2 \end{aligned} \tag{1.6}$$

Here, $N^{(k)}_{ji}$ denote the normal and shear forces, $M^{(k)}_{ij}$ — the bending moments and torques produced in the shell by the action of the concentrated force at point (ξ) and directed along the x_k axis.

This method of solution briefly outlined above may be extended on the problems of shells and even — making use of the elastic-viscoelastic analogy — on the viscoelastic shells.

In the final part an example ls given relating to the thermal stresses in an open cylindrical shell.

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