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APPLIED MECHANICS

Combined Elastic and Electromagnetic Waves Produced by Thermal Shock in the Case of a Medium of Finite Electric Conductivity

by

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1. Introduction

The object of the present paper is to obtain the solution of the thermo-magnetic -elasticity problem in the one-dimensional case of an elastic semi-space adjacent to a vacuum with a thermal shock acting on the bounding plane. It is assumed that the original magnetic field in the body and in the vacuum is parallel to the bounding plane. The latter is assumed to be suddenly heated to a temperature remaining constant. The electric conductivity of the body is assumed to be constant. The essential physical feature is the existence of thermal, elastic and electromagnetic waves and the radiation of an electromagnetic discontinuity wave into the vacuum.

A similar problem was solved, with qualitatively similar results, by the present authors in [2], where a perfect conductor was considered. This problem has been solved in an accurate manner. The problem of finite conductivity requires the solution of the complete set of equations obtained in [1], and is much more involved owing, among other things, to the fact that equations of much higher order are concerned. Some additional effects resulting from the finite conductivity of the body are certain discontinuity waves of mechanical and electrical nature propagating with the velocity of light in the medium (in addition to discontinuity waves propagating with a velocity of the order of the velocity of sound). If we reject, in the case of a good conductor, the displacement currents, the discontinuity waves propagating with light velocity become solutions of a diffusion character, corresponding to an infinite propagation velocity of perturbations. This results from the fact that the set of equations of the combined mechanical and electromagnetic field (thermal influences being rejected) changes, with such an assumption, from the hyperbolic to the parabolic type. We shall disregard the displacement currents, what leads to insignificant quantitative changes. What more, in view of mathematical difficulties, we shall confine ourselves to approximate solutions in two cases, for which a small parameter can be introduced. In the first case the original magnetic field is weak and the electric conductivity is arbitrary, in the second the original field is arbitrary and the conductivity is very high. No solutions of thermo-magneto-elastic problems are found in the literature except the solutions of a broad class of thermoelastic problems [3], in particular the one-parameter problem of the semi-space [4].

Sec. 2 of the present paper is devoted to a general formulation of equations and boundary conditions of the problem. Sec. 3 brings an approximate solution for finite electric conductivity in the case of conductivity of arbitrary magnitude and a weak primary magnetic field. In Sec. 4 the same problem is solved for any original magnetic field and high conductivity. In the conclusion, further problems connected with the present one are mentioned.

2. General equations

According to [1] the general form of the linearized thermo-magneto-elastic equations for a homoge- isotropic body is:

$$
(2.1)\quad \begin{cases}
\operatorname{rot} \boldsymbol{E} - = \dfrac{\mu}{c}\dfrac{\partial h}{\partial t}, \\[2mm]
\operatorname{rot} \boldsymbol{h} = \dfrac{4\pi}{c}j + \dfrac{\varepsilon}{c}\dfrac{\partial \boldsymbol{E}}{\partial t} + \dfrac{\varepsilon\mu - 1}{c^2}\dfrac{\partial}{\partial t}\left[\dfrac{\partial \boldsymbol{u}}{\partial t}\times \boldsymbol{H}\right], \\[2mm]
j = \eta\left[E + \dfrac{\mu}{c}\left(\dfrac{\partial \boldsymbol{u}}{\partial t}\times H\right)\right] - \varkappa_0 \operatorname{grad} T, \\[2mm]
\operatorname{div} \boldsymbol{h} = 0, \qquad \operatorname{div} \boldsymbol{D} = 0, \qquad \boldsymbol{D} = \varepsilon\left[\boldsymbol{E} + \dfrac{\mu\varepsilon - 1}{c\varepsilon}\left(\dfrac{\partial \boldsymbol{u}}{\partial t}\times \boldsymbol{H}\right)\right],
\end{cases}
$$

$$
(2.2)\quad \begin{cases}
\varrho \dfrac{\partial^2 \boldsymbol{u}}{\partial t^2} = G\nabla^2 \boldsymbol{u} + (\lambda + 2G) \operatorname{grad} \operatorname{div} \boldsymbol{u} + \dfrac{\mu}{c}(\boldsymbol{j}\times \boldsymbol{H}) - 3\alpha_t K \operatorname{grad} T + \boldsymbol{P}, \\[2mm]
K = \lambda + \frac{2}{3} G
\end{cases}
$$

$$
(2.3)\quad c_v \frac{\partial T}{\partial t} + \frac{c_p - c_v}{3\alpha_t}\frac{\partial}{\partial t}\operatorname{div} \boldsymbol{u} + \pi_0 \operatorname{div} \boldsymbol{j} - \lambda_1 \nabla^2 T = Q.
$$

The set of Eqs. (2.1) is a set of electrodynamic equations of slowly moving media, the Eqs. (2.2) constituting the equations of motion of an elastic body. Eq. (2.3) is the heat equation. The equations of electrodynamics involve mechanical and thermal couplings. The equations of motion and heat conduction include terms of electromagnetic coupling. The symbols $\boldsymbol{h}$, $\boldsymbol{E}$ denote vectors of perturbed intensities of magnetic and electric fields, and $\boldsymbol{j}$—the vector of current density. $\boldsymbol{H}$ is the vector of the original, constant electromagnetic field, $\boldsymbol{u}$ — displacement vector, T — temperature of the unstressed body, $\boldsymbol{P}$ — vector of the mass forces, Q — intensity of the heat sources c — light velocity, μ, ε — magnetic and electric permeability, η — electric conductivity, $\varkappa_0$ coefficient linking the electric field with the temperature gradient, π_0 — coefficient relating the current velocity vector with that of heat flow. The symbols c_v, c_p denote the specific heat with constant volume and constant pressure, respectively, λ_1 — the coefficient of heat conduction, α_t — the coefficient of thermal dilatation, λ, G — are elastic constants and ϱ — is the density of the medium.

Assuming that μ approaches unity ($\mu \approx 1$ will be assumed for simplicity) and disregarding the displacement currents, the set of Eqs. (2.1)—(2.3) takes, on eliminating $\boldsymbol{E}$ and making use of the condition $\operatorname{div}\boldsymbol{h}=0$ the form

$$(2.4)\qquad \begin{cases} \dfrac{\partial \boldsymbol{h}}{\partial t} - \operatorname{rot}\left(\dfrac{\partial \boldsymbol{u}}{\partial t}\times \boldsymbol{H}\right) - \dfrac{c^2}{4\pi\mu\eta}\nabla^2 \boldsymbol{h} = 0,\\[2mm] \varrho\dfrac{\partial^2 \boldsymbol{u}}{\partial t^2} = G\nabla^2\boldsymbol{u} + (\lambda+G)\operatorname{grad}\operatorname{div}\boldsymbol{u} + \dfrac{\mu}{4\pi}(\operatorname{rot}\boldsymbol{h}\times\boldsymbol{H}) - 3a_t K \operatorname{grad} T,\\[2mm] c_v\dfrac{\partial T}{\partial t} - \lambda_1 \nabla^2 T = Q, \end{cases}$$

where

$$(2.5)\qquad \boldsymbol{E} = \frac{c}{4\pi\eta}\operatorname{rot}\boldsymbol{h} - \frac{\mu}{c}\left(\frac{\partial \boldsymbol{u}}{\partial t}\times\boldsymbol{H}\right) + \frac{\varkappa_0}{\eta}\operatorname{grad} T.$$

In the heat equation the small term expressing the thermo-mechanical coupling, $\dfrac{c_p-c_v}{3a_t}\dfrac{\partial}{\partial t}\operatorname{div}\boldsymbol{u}$ has been omitted. The equations in vacuum are

$$(2.6)\qquad \begin{cases} \left(\dfrac{\partial^2}{\partial t^2} - c^2\Delta^2\right)\boldsymbol{E}^* = 0,\\[2mm] \left(\dfrac{\partial^2}{\partial t^2} - c^2\Delta^2\right)\boldsymbol{h}^* = 0,\\[2mm] \operatorname{rot}\boldsymbol{E}^* = -\dfrac{1}{c}\dfrac{\partial \boldsymbol{h}^*}{\partial t}, \qquad \operatorname{rot}\boldsymbol{h}^* = \dfrac{1}{c}\dfrac{\partial \boldsymbol{E}^*}{\partial t}. \end{cases}$$

In the case of an elastic semi-space in contact with vacuum and with a prescribed temperature on the boundary, and assuming $\mu_{\text{medium}}=1$ and finite electric conductivity, the boundary conditions are:

$$(2.7)\qquad \begin{cases} T = T_0 H(t),\\ \sigma_{3i} = T_{3i} - T^*_{3i} = 0, \qquad i=1,2,3,\\ E_1 = E^*_1, \quad E_2 = E^*_2, \quad h_1 = h^*_1, \quad h_2 = h^*_2, \quad h_3 = h^*_3, \end{cases}$$

where $H(t)$ is the Heaviside step function. The mechanical stresses and the components of Maxwell's tensors in the body and in vacuum are

$$(2.8)\qquad \begin{cases} \sigma_{3i} = \lambda\Theta\delta_{3i} + 2G\varepsilon_{3i} - 3Ka_t T\delta_{3i}, \qquad \Theta = \varepsilon_{ii}\\[2mm] T_{3i} = \dfrac{1}{4\pi}[H_3 h_i + H_i h_3 - \delta_{3i}\boldsymbol{Hh}],\\[2mm] T^*_{3i} = \dfrac{1}{4\pi}[H_3 h^*_i + H_i h^*_3 - \delta_{3i}\boldsymbol{Hh}^*], \qquad i=1,2,3. \end{cases}$$

The set of Eqs. (2.4)—(2.6) with the boundary conditions (2.7) constitutes the general formulation of our problem which, in the one-dimensional case and with the field H directed along the axis $x_1 = x$ takes the form

$$\tag{2.9} \begin{cases} \dfrac{\partial^2 u}{\partial t^2} = a_0^2 \dfrac{\partial^2 u}{\partial z^2} - \dfrac{\mu H}{4\pi\varrho} \dfrac{\partial h}{\partial z} - \dfrac{3\alpha_t K}{\varrho} \dfrac{\partial T}{\partial z}, \\ \dfrac{\partial h}{\partial t} + H \dfrac{\partial^2 u}{\partial t \partial z} - \dfrac{c^2}{4\pi\mu\eta} \dfrac{\partial^2 h}{\partial z^2} = 0, \\ c_v \dfrac{\partial T}{\partial t} - \lambda_1 \dfrac{\partial^2 T}{\partial z^2} = 0, \end{cases}$$

where

$$\tag{2.10} \begin{cases} a_0^2 = \dfrac{\lambda + 2G}{\varrho}, \quad h = h_1, \quad u = u_3, \\ E = E_2 = \dfrac{c}{4\pi\eta} \dfrac{\partial h}{\partial z} - \dfrac{H}{c} \dfrac{\partial u}{\partial t}. \end{cases}$$

The equations for vacuum are

$$\tag{2.11} \begin{cases} \left(\dfrac{\partial^2}{\partial t^2} - c^2 \dfrac{\partial^2}{\partial z^2}\right) E^* = 0, \quad \left(\dfrac{\partial^2}{\partial t^2} - c^2 \dfrac{\partial^2}{\partial z^2}\right) h^* = 0, \\ (E^* = E_2^*, \quad h^* = h_1^*). \end{cases}$$

For $\mu = 1$ the boundary conditions become for $z = 0$:

$$\tag{2.12} \begin{cases} T = T_0 H(t), \\ a_0^2 \dfrac{\partial u}{\partial z} - \dfrac{3K\alpha_t}{\varrho} T = 0, \quad h = h_1^*, \\ \dfrac{c}{4\pi\eta} \dfrac{\partial^2 h}{\partial z \partial t} - \dfrac{H}{c} \dfrac{\partial^2 u}{\partial t^2} = c \dfrac{\partial h^*}{\partial z}. \end{cases}$$

3. The case of finite, arbitrary electric conductivity assuming a weak original magnetic field

Let us consider the set of Eqs. (2.9) to (2.11) with the boundary conditions (2.12), assuming that the original magnetic field is bounded in such a manner that $\dfrac{\varkappa}{a_0} \ll 1$.

This assumption enables us to apply the perturbation method for approximate solution of the problem. In the present work we shall confine ourselves to the first approximation.

Let us introduce the following variables and notations:

$$\tag{3.1} \zeta = \frac{a_0 z}{\varphi}, \quad \tau = \frac{a_0^2 t}{\varphi}, \quad \varphi = \lambda_1 / c_v.$$

On introducing the variables (3.1) and performing the Laplace transformation Eqs. (2.9) to (2.11) take, assuming homogeneous initial conditions, the form

$$
(3.2)\qquad \begin{cases} \bar{u}_{,\zeta\zeta} - p^2 \bar{u} - \beta \bar{h}_{,\zeta} = m\bar{T}_{,\zeta} \\ \bar{T}_{,\zeta\zeta} - p\bar{T} = 0, \\ s\bar{h}_{,\zeta\zeta} - p\bar{h} - rp\bar{u}_{,\zeta} = 0, \\ \bar{h}^*_{,\zeta\zeta} - b^2 p^2 \bar{h}^* = 0, \end{cases}
$$

where

$$
s = r/\varphi, \quad r = Ha_0/\varphi, \quad m = \frac{3K\alpha_t \varphi}{\varrho a_0^3}, \quad \beta = \frac{\varkappa\varphi}{Ha_0^3}, \quad b = \frac{\varphi}{a_0 c}.
$$

The function $\bar{E}$ is expressed thus

$$
(3.3)\qquad \bar{E} = \gamma(s\bar{h}_{,\zeta} - rp\bar{u}), \quad \gamma = a_0/c.
$$

The boundary conditions (2.12) become, for $\zeta = 0$:

$$
(3.4)\qquad \begin{cases} \bar{T}(p) = T_0/p, \\ \bar{u}_{,\zeta} - m\bar{T} = 0, \\ \bar{h}^*_{,\zeta} = p\gamma^2 (s\bar{h}_{,\zeta} - rp\bar{u}), \quad \bar{h}^* = \bar{h}. \end{cases}
$$

Our approximate solution of the problem will be obtained as follows. The quantity β being small, the first of Eqs. (3.2) are solved for $\beta = 0$. From the first two of Eqs. (3.2) we obtain, with the first two boundary conditions (3.4), the functions $\bar{u}, \bar{T}$

$$
(3.5)\qquad \bar{u} = \frac{mT_0}{p(p-1)}\left(\frac{e^{-\zeta\sqrt{p}}}{\sqrt{p}} - e^{-p\zeta}\right), \quad \bar{T} = \frac{T_0}{p} e^{-\zeta\sqrt{p}}.
$$

The transform of $\bar{\sigma}_{33}$ yields

$$
(3.6)\qquad \bar{\sigma}_{33} = \frac{\varrho a_0^2 m T_0}{\varphi(p-1)} (e^{-p\zeta} - e^{-\zeta\sqrt{p}}).
$$

The results obtained are identical with those for no coupling between the temperature and strain with the electromagnetic field.

Next, the $\bar{u}$ function thus obtained is introduced into the third of Eqs. (3.2). From this equation we obtain the function $\bar{h}$ in the first approximation, which is essential for practical application. Knowing $\bar{h}$, we can obtain, by solving the first of Eqs. (3.2) a correction of the perturbation for $\bar{u}$. However, the determination of the correction for $\bar{u}$, which is of secondary importance will be omitted.

Proceeding, according to the above scheme, we obtain the following solutions for the first approximations for $\bar{h}$ and $\bar{h}^*$.

$$
(3.7)\qquad \bar{h} = B e^{-\zeta\sqrt{ps_1}} - \frac{rps_1}{p-s_1} A e^{-p\zeta} - \frac{rs_1 mT_0}{p(p-1)(1-s_1)} e^{-\zeta\sqrt{p}},
$$

$$
\bar{h}^* = \left(B + \frac{rps_1}{s_1 - p} A + \frac{rs_1 mT_0}{p(p-1)(s_1-1)}\right) e^{pb\zeta},
$$

where

$$A = \frac{mT_0}{p(1-p)}, \quad B = \frac{mT_0 s_1 r}{(1+\gamma_1\sqrt{p})(p-1)}\left[\frac{\gamma_2+1}{p-s_1} + \frac{1+\gamma_2\sqrt{p}}{p(1-s_1)}\right],$$

and

$$s_1 = 1/s, \quad \gamma_1 = \frac{\gamma^2}{b}\sqrt{s}, \quad \gamma_2 = \gamma^2/b.$$

On performing the inverse Laplace transformation, we obtain the following functions

$$(3.8) \quad h(\zeta, \tau) = \frac{mT_0 rs_1}{1-s_1}\{(2+\gamma_2) g_1(\zeta, \tau; 1) - (1+\gamma_2) g_1(\zeta, \tau; s_1) - g_1(\zeta, \tau; 0) + \gamma_2[g_2(\zeta, \tau; 1) - g_2(\zeta, \tau; 0)]\} - \frac{rmT_0}{1-s_1}[g_3(\zeta, \tau; 1) - g_3(\zeta, \tau; 0)] + \frac{rs_1 mT_0}{1-s_1}[f_1(\zeta, \tau; 1) - f_1(\zeta, \tau; s_1)],$$

$$h^*(\zeta, \tau) = \frac{mT_0 rs_1}{1-s_1}[(2+\gamma_2) f_3(\zeta, \tau; 1) - (1+\gamma_2) f_3(\zeta, \tau; s_1) - f_3(\zeta, \tau; 0) + \gamma_2[f_4(\zeta, \tau; 1) - f_4(\zeta, \tau; 0)] + \frac{mT_0 rs_1}{s_1-1}[f_5(\zeta, \tau; s_1) - f_5(\zeta, \tau; 0)],$$

where

$$(3.9) \quad g_1(\zeta, \tau; \delta) = \frac{e^{\delta\tau}}{2}\left\{\frac{\exp(-\zeta\sqrt{s_1\delta})}{1+\gamma_1\delta}\operatorname{erfc}\left(\frac{\zeta}{2}\sqrt{\frac{s_1}{\tau}} - \sqrt{\delta\tau}\right) + \exp\left(\frac{\zeta\sqrt{s_1\delta}}{1-\gamma_1\sqrt{\delta}}\right) \times \operatorname{erfc}\left(\frac{\zeta}{2}\sqrt{\frac{s_1}{\tau}} + \sqrt{\delta\tau}\right)\right\} - \exp\left(\frac{\zeta}{\gamma_1}\sqrt{s_1} + \frac{\tau}{\gamma_1^2}\right)\operatorname{erfc}\left(\frac{\zeta}{2}\sqrt{\frac{s_1}{\tau}} + \frac{\sqrt{\tau}}{\gamma_1}\right),$$

$$g_2(\zeta, \tau; \delta) = \frac{e^{\delta\tau}\sqrt{\delta}}{2}\left[\frac{\exp(-\zeta\sqrt{\delta s_1})}{1+\gamma_1\sqrt{s_1}}\operatorname{erfc}\left(\frac{\zeta}{2}\sqrt{\frac{s_1}{\tau}} - \sqrt{\delta\tau}\right) - \frac{\exp(\zeta\sqrt{\delta s_1})}{1-\gamma_1\sqrt{\delta}}\operatorname{erfc}\left(\frac{\zeta}{2}\sqrt{\frac{s_1}{\tau}} + \sqrt{\tau\delta}\right)\right] + \frac{\exp\left(\frac{\zeta\sqrt{s_1}}{\gamma_1} + \frac{\tau}{\gamma_1^2}\right)}{\gamma_1(1-\gamma_1^2\delta)}\operatorname{erfc}\left(\frac{\zeta}{2}\sqrt{\frac{s_1}{\tau}} + \frac{\sqrt{\tau}}{\gamma_1}\right),$$

$$f_1(\zeta, \tau; \delta) = e^{\delta(\tau-\zeta)} H(\tau-\zeta),$$

$$g_3(\zeta, \tau; \delta) = \frac{e^{\delta\tau}}{2}\left[e^{\zeta\sqrt{\delta}}\operatorname{erfc}\left(\frac{\zeta}{2\sqrt{\tau}} + \sqrt{\delta\tau}\right) + e^{-\zeta\sqrt{\delta}}\operatorname{erfc}\left(\frac{\zeta}{2\sqrt{\tau}} - \sqrt{\delta\tau}\right)\right],$$

and

$$f_3(\zeta, \tau; \delta) = \int_0^\tau g_1(0, \tau - v; \delta) H(v+\zeta b)\, dv,$$

$$f_4(\zeta, \tau; \delta) = \int_0^\tau g_2(0, \tau - v; \delta) H(v+\zeta b)\, dv,$$

$$f_5(\zeta, \tau; 0) = e^{\delta(\tau+b\zeta)} H(\tau+b\zeta).$$

From (3.8) it is inferred that $h(\zeta, \tau)$ is composed of two functions, one of a diffusion character, the other characterizing the propagation of the discontinuity wave with the velocity a_0, which is that of the elastic wave. The function $h^*(\zeta, \tau)$ is a discontinuity wave moving with the velocity of light c. It should be observed that in contrast to the solution for a perfect conductor the diffusion members are influenced not only by the temperature but also by the coupling with the magnetic field. If displacement currents were taken into consideration these diffusion perturbations would constitute a discontinuity wave moving with light velocity in the medium.

4. The case of finite electric conductivity of high magnitude assuming arbitrary original magnetic field

Let us consider the set of Eqs. (2.9) to (2.11) with the boundary conditions (2.12) assuming that the original magnetic field is arbitrary and the conductivity is very high so that $\nu = \dfrac{c^2}{4\pi\eta}$ is small.

Similarly to the foregoing section we shall confine ourselves to the first approximation. We shall determine the zero approximation to u and the first approximation to the functions h and h^*.

For the approximate solution procedure it will be convenient to introduce new variables different from those of Sec. **3**.

Let us assume

$$\zeta = \frac{az}{\varphi}, \quad \tau = \frac{a^2 t}{\varphi}, \quad \varphi = \lambda_1 / c_v. \tag{4.1}$$

On applying the Laplace transformation and assuming homogeneous initial conditions Eqs. (2.9) to (2.11), take the form

$$\begin{cases} \dfrac{a_0^2}{a^2}\bar{u},_{\zeta\zeta} - p^2 \bar{u} - \beta \bar{h},_{\zeta} = m\bar{T},_{\zeta}, \\ \bar{T},_{\zeta\zeta} - p\bar{T} = 0, \\ s\bar{h},_{\zeta\zeta} - p\bar{h} - rp\bar{u},_{\zeta} = 0. \end{cases} \tag{4.2}$$

Let us observe in addition that

$$\bar{E} = \gamma (s\bar{h},_{\zeta} - rp\bar{u}), \tag{4.3}$$

with the notations

$$m = \frac{3K\alpha_t \varphi}{\varrho a^3}, \quad \beta = \frac{\varkappa\varphi}{Ha^3}, \quad \nu = \frac{c^2}{4\pi\eta}, \quad s = \nu/\varphi,$$

$$r = \frac{Ha}{\varphi}, \quad a^2 = a_0^2 + \varkappa, \quad \gamma = a/c.$$

The boundary conditions of the problems are

$$
(4.4) \qquad \begin{cases} \overline{T} = T_0/p, \\ \overline{u},_{\zeta} - m_0 \overline{T} = 0, \\ \overline{h}^*,_{\zeta} = p\gamma^2 (s\,\overline{h},_{\zeta} - rp\overline{u}), \quad \overline{h} = \overline{h}^*, \end{cases}
$$

where

$$
m_0 = m\,a^2/a_0^2.
$$

For the zero approximation we assume that $\nu = 0$. Then Eqs. (4.2) become

$$
(4.5) \qquad \begin{cases} \overline{u},_{\zeta\zeta} - p^2\,\overline{u} = m\overline{T},_{\zeta}, \\ \overline{h} = -r\,\overline{u},_{\zeta}, \\ \overline{T},_{\zeta\zeta} - p\overline{T} = 0. \end{cases}
$$

Finding $\overline{u}$ from (4.5), we determine the corrected function $\overline{h}$ from the equation

$$
(4.6) \qquad p\overline{h} - s\,\overline{h},_{\zeta\zeta} + rp\overline{u},_{\zeta} = 0.
$$

The solution thus obtained should satisfy the boundary conditions (4.4). The number of constants corresponds in this case to the number of boundary conditions. For further approximations the corresponding integration constants should be determined from the homogeneous boundary conditions, the boundary conditions 4.4) being satisfied by the set of Eqs. (4.5) and (4.6).

Let us observe that the solution is not built up by the formal perturbation procedure. Practical reasons require that the correction for the perturbed electromagnetic field be found above all and the calculation with the zero approximation for $\overline{u}$ be finished. Hence, the necessity of preserving certain small quantities, the remaining ones of the same order, being rejected.

Solving according to the above procedure the set of Eqs. (4.5) and satisfying the boundary conditions (4.4), we obtain the following Laplace transforms of the functions

$$
(4.7) \qquad \begin{cases} \overline{u} = \dfrac{mT_0\, e^{-\zeta\sqrt{p}}}{p\sqrt{p}(p-1)} - \dfrac{T_0}{p^2}\left(m_0 + \dfrac{m}{p-1}\right) e^{-p\zeta}, \\[2ex] \overline{\sigma}_{33} = \dfrac{\varrho a_0^2\, a T_0}{\varphi p}\left(m_0 + \dfrac{m}{p-1}\right)[e^{-p\zeta} - e^{-\zeta\sqrt{p}}], \\[2ex] \overline{h} = B e^{-\zeta\sqrt{ps_1}} + rs_1\left[\dfrac{T_0\, e^{-p\zeta}}{p(s_1-p)}\left(m_0 + \dfrac{m}{p-1}\right) - \dfrac{mT_0\, e^{-\zeta\sqrt{p}}}{p(p-1)(1-s_1)}\right], \\[2ex] \overline{h}^* = \left[B - rs_1\left(\dfrac{pA}{p-s_1} + \dfrac{mT_0}{p(p-1)(1-s_1)}\right)\right] e^{-b\zeta p}, \end{cases}
$$

where

$$
(4.8)\quad \begin{cases} A = -\dfrac{T_0}{p^2}\left(m_0 + \dfrac{m}{p-1}\right), \\ B = \dfrac{s_1}{1+\gamma_1\sqrt{p}}\left[\dfrac{r\gamma_2 pA}{p-s_1} + \dfrac{rpA}{p-s_1} + \dfrac{\gamma_2 rmT_0\sqrt{p}}{(1-s_1)\,p\,(1-p)} + \dfrac{mT_0 r}{p\,(p-1)\,(1-s_1)}\right], \end{cases}
$$

$$
s_1 = 1/s, \quad \gamma_1 = \frac{\gamma^2}{b}\sqrt{s}, \quad \gamma_2 = \gamma^2/b.
$$

Making use of (4.7), we perform on $\bar{\sigma}_{33}$, $\bar{h}$, $\bar{h}^*$ the inverse Laplace transformation. Similarly as in Sec. **3**, we introduce the functions $f_1, f_2, f_3, f_4, f_5, g_1, g_2$. There is a difference, however, consisting in that they refer to the variables ζ, τ determined by Eqs. (4.1).

Thus, we have successively

$$
(4.9)\quad \begin{cases} \sigma_{33}(\zeta,\tau) = \dfrac{\varrho a_0 a T_0}{\varphi p}\Big\{(m_0-m)\,[f_1(\zeta,\tau;0) - g_3(\zeta,\tau;0) + m\,[f_1(\zeta,\tau;1) - \\ \qquad\qquad - g_3(\zeta,\tau;1)]\Big\}, \\ h(\zeta,\tau) = \dfrac{mT_0 rs_1}{1-s_1}\Big\{g_1(\zeta,\tau;1) - g_1(\zeta,\tau;0) + \gamma_2\,[g_2(\zeta,\tau;1) - \\ -g_2(\zeta,\tau;0)] - (1+\gamma_2)\,[g_1(\zeta,\tau;s_1) - g_1(\zeta,\tau;0)\Big\} - r(1+\gamma_2)(m-m_0)T_0\times \\ \times[g_1(\zeta,\tau;s_1) - g_1(\zeta,\tau;0)] + rT_0\Big\{m_0\,[f_1(\zeta,\tau;s_1) - f_1(\zeta,\tau;0)] + \dfrac{m}{s_1-1}\times \\ \qquad \times\Big[f_1(\zeta,\tau;s_1) - \dfrac{s_1}{s_1-1}f_1(\zeta,\tau;1) + f_1(\zeta,\tau;0)\Big]\Big\} + \\ \qquad + \dfrac{mT_0}{1-s_1}[g_3(\zeta,\tau;1) - g_3(\zeta,\tau;0)], \\ h^*(\zeta,\tau) = \dfrac{mT_0 rs_1}{1-s_1}\Big\{(1+\gamma_2)f_3(\zeta,\tau;s_1) - \gamma_2 f_3(\zeta,\tau;1) - f_3(\zeta,\tau;0) + \\ \quad + \gamma_2\,[f_4(\zeta,\tau;1) - f_4(\zeta,\tau;0)] - (1+\gamma_2)\,rT_0\,(m_0-m)\,[f_3(\zeta,\tau;s_1) - \\ \quad - f_3(\zeta,\tau;0)]\Big\} + \dfrac{mT_0}{1-s_1}[f_5(\zeta,\tau;1) - f_5(\zeta,\tau;0)] + rT_0\Big\{\Big(m_0 + \dfrac{m}{s_1-1}\Big)\times \\ \qquad \times f_5(\zeta,\tau;s_1) - \dfrac{ms_1}{s_1-1}f_5(\zeta,\tau;1) + (m-m_0)\,f_5(\zeta,\tau;0)\Big\}. \end{cases}
$$

Let us observe that, assuming $a_0/a_1 = 1$, $m_0 = m$ the expressions for the transforms $\bar{u}$ and $\bar{T}$ become the corresponding expressions of Sec. **3**. In the stress σ_{33} there are two terms, one of the character of a diffusion member, the other of the character of a discontinuity wave propagating with the velocity a.

In the expression $h(\zeta, \tau)$ there are diffusion terms and discontinuity wave terms for a discontinuity wave moving with the velocity a in the medium. The diffusion terms in these solution originate not only (as was the case in [2]) from the thermal shock but also from the magnetic coupling. Although the qualitative character of the solutions of Secs. **3** and **4** in the first approximation is similar quantitatively, these solutions differ in an essential manner.

5. Conclusion

From the above considerations it follows that the essential difference between the solution for finite electric conductivity and perfect electric conductors [2], in addition to the quantitative side, consists in a different structure of the diffusion members in the body, appearing as a consequence of the parabolic character of the equation of the magnetic field, e. g for a given u_0. If, as mentioned above, displacement currents were taken into consideration then, in addition to discontinuities propagating with sound velocity, there would also appear discontinuity waves propagating with the velocity of the order of the velocity of light in the medium. Also the electromagnetic wave radiated into the vacuum being important for practical measurement problems undergoes considerable quantitative changes. The obtention of a solution for an arbitrary conductivity and arbitrary magnetic field, taking into consideration the displacement currents what is not done in the present paper, seems to be of interest. The same applies to more general three-dimensional initial and boundary-value problem of thermoelasticity and plasticity in view of their practical and theoretical interest. Their solution may create a basis for investigation of analogous problems in more complicated cases, those of ferromagnetics, ferrites etc., which are of fundamental practical importance.

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С. КАЛИСКИЙ и В. НОВАЦКИЙ, **КОМБИНИРОВАННЫЕ УПРУГИЕ И ЭЛЕКТРОМАГНИТНЫЕ ВОЛНЫ, ВЫЗВАННЫЕ ТЕРМИЧЕСКИМ УДАРОМ ДЛЯ СЛУЧАЯ СРЕДЫ КОНЕЧНОЙ ЭЛЕКТРОПРОВОДНОСТИ**

В работе рассматривается проблема возбуждения механоэлектромагнитных волн в упругом полупространстве при воздействии термического удара на поверхности полупространства. Принимается, что над полупространством

имеется вакуум. Рассмотрена конечная электропроводность среды при пренебрежении токами перемещения.

Настоящая работа является продолжением предыдущей работы [2], в которой обсуждалась аналогичная одномерная проблема полупространства для идеального проводника, что позволило снизить порядок исследуемых уравнений и получить решение в замкнутом виде.

Для действительного проводника в настоящей работе рассмотрены два случая: произвольного предварительного магнитного поля и большой электропроводности, а также произвольной электропроводности и малого предварительного поля. Это дало возможность ввести малые параметры и получить практически достаточные точные решения в первом приближении. Определены возникающие под влиянием воздействия термического удара механические и электромагнитные волны в среде и излучающиеся в вакуум электромагнитные волны. Такое решение отличается от решения для идеального проводника добавочными членами, имеющими диффузионный характер. В случае учета токов перемещения, кроме модифицированных волн разрыва со скоростью пропагации порядка скорости звука, образовались бы также волны разрыва со скоростью пропагации порядка скорости света в среде.