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APPLIED MECHANICS

Excitation of Mechanical-electromagnetic Waves Induced by a Thermal Shock

by

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1. Introduction

The purpose of this paper is to give a solution of the thermo-magnetic elasticity problem, with respect to the one-dimensional case of elastic half-space being in contact with vacuum under the action of a thermal shock on the surface of this half-space. It is assumed that in the elastic medium and in the vacuum as well an initial magnetic field exists, parallel to the plane, limiting the half-space. The plane limiting the elastic half-space was suddenly heated to a constant temperature.

The essential physical effect of the solution given consists in the fact that a coupled mechanical and electromagnetic wave in the medium exists, and that there exists a radiation of the electromagnetic wave of discontinuity into the vacuum.

Basing on general equations of thermo-magnetic elasticity given in [1] and [2], an equation is derived by the present authors for an ideal conductor. Using this simplification, it was possible to solve the problem in a closed form and to express the electromagnetic perturbations explicitly by means of mechanical and thermal effects. Moreover, this simplification enabled a simple demonstration of the effect of the excitation of electromagnetic waves in the medium and in the vacuum, and made possible the discussion of the solutions obtained. Supposing the existence of a real conductor, initial equations become more complicated. The solution of the problem of excitation of mechanical-electromagnetic waves for real conductor induced by a thermal shock will be considered in a separate paper.

A large and general class of boundary problems is already solved in dynamical thermo-elasticity [3]. The one-dimensional problem of propagation of the elastic wave due to the action of thermal shock was solved in [4]. An analogical problem of thermal-magnetic elasticity remains, so far we know, as yet unsolved. In the next paragraph general equations of thermal-magnetic elasticity are given, as well as the transition from a real conductor to an ideal one.

In the subsequent paragraph the boundary problem of the half-space is formulated, and its solution in a closed form is given. Finally, we point to the possibilities of a further development of the problem.

2. General equations

For the case of an isotropic and homogeneous body the coupled mechanical-thermo-electromagnetic problem (after being linearized) is described by the following system of equations (cf. [1], [2]) *)

$$(2.1)\quad \begin{cases} \operatorname{rot} \boldsymbol{h} = \dfrac{4\pi}{c}\boldsymbol{j} + \dfrac{\varepsilon}{c}\dfrac{\partial \boldsymbol{E}}{\partial t} + \dfrac{\varepsilon\mu-1}{c^2}\left[\dfrac{\partial \boldsymbol{u}}{\partial t}\times \boldsymbol{H}\right] \\ \operatorname{rot} \boldsymbol{E} = \dfrac{\mu}{c}\dfrac{\partial \boldsymbol{h}}{\partial t}, \qquad \boldsymbol{j} = \lambda_0\left\{\boldsymbol{E} + \dfrac{\mu}{c}\left[\dfrac{\partial \boldsymbol{u}}{\partial t}\times \boldsymbol{H}\right]\right\} - \varkappa_0 \operatorname{grad} T \end{cases}$$

$$\operatorname{div} \boldsymbol{h} = 0, \qquad \operatorname{div} \boldsymbol{D} = 4\pi\varrho_e,$$

where

$$\boldsymbol{D} = \varepsilon\left\{\boldsymbol{E} + \frac{1}{c}\,\frac{\mu\varepsilon-1}{\varepsilon}\left[\frac{\partial \boldsymbol{u}}{\partial t}\times \boldsymbol{H}\right]\right\};$$

$$(2.2)\quad \varrho\frac{\partial^2 \boldsymbol{u}}{\partial t^2} = G\nabla^2 \boldsymbol{u} + (\lambda+G)\operatorname{grad}\operatorname{div}\boldsymbol{u} + \frac{\mu}{c}[\boldsymbol{j}\times\boldsymbol{H}] + \frac{\mu\varepsilon-1}{4\pi c}\left[\frac{\partial \boldsymbol{E}}{\partial t}\times\boldsymbol{H}\right] +$$
$$+ \frac{\mu(\mu\varepsilon-1)}{4\pi c^2}\,\frac{\partial}{\partial t}\left[\frac{\partial \boldsymbol{u}}{\partial t}\times\boldsymbol{H}\right]\times\boldsymbol{H} + \boldsymbol{P} - 3\alpha_t K \operatorname{grad} T, \qquad K = \lambda + \frac{2}{3}G,$$

$$(2.3)\quad c_v\frac{\partial T}{\partial t} + \frac{c_p - c_v}{3\alpha_t}\,\frac{\partial}{\partial t}\operatorname{div}\boldsymbol{u} + \pi_0\operatorname{div}\boldsymbol{j} - \lambda_1\nabla^2 T = Q.$$

The system of Eqs. (2.1) represents the equations of electrodynamics of slowly moving media, the system (2.2) — the equations of displacement of the motion of an elastic body and, finally, Eq. (2.3) represents the equation of thermal conductivity. In the equations of electrodynamics we have thermal and mechanical couplings, whereas in the equations of displacement and of thermal conductivity we have terms representing electromagnetic couplings.

We denote by $\boldsymbol{h}$ and $\boldsymbol{E}$ vectors of perturbed strength of magnetic and electric fields, respectively; by $\boldsymbol{j}$ — the vector of current density. $\boldsymbol{H}$ stands for the vector of initial and constant magnetic field. Further, $\boldsymbol{u}$ denotes the vector of displacement of the medium, T — the temperature as referred to the natural (strainless) state of an elastic body. $\boldsymbol{P}$ stands for the vector of the volume force and Q — for the intensity of heat sources.

The light velocity is denoted by c; μ and ε are magnetic and electric constants, λ_0 — electric conductivity, $\varkappa_0$ — the coefficient connecting the electric field with the temperature gradient, π_0 — the coefficient connecting the current density vector with the vector of the density of heat flow**). Further, c_p, c_v denote the specific

*)The momentum of the electromagnetic field is defined as in [5]. By another definition some terms in Eq. (2.2) disappear.

**) In a particular case this coefficient transforms into the Peltier's coefficient.

heat at a constant pressure and constant volume, λ_1 — the coefficient of heat conductivity, a_t — the coefficient of the linear thermal expansion, λ, G — Lamé constants of an elastic body. Finally, we denote by ϱ the density of the elastic medium and by ϱ_e — the density of compound electric charges.

The system of Eqs. (2.1)—(2.3) may be considerably simplified, assuming for an elastic medium $\varepsilon\mu \approx 1$. Then

$$\text{rot}\,\boldsymbol{h} = \frac{4\pi}{c}\boldsymbol{j} + \frac{\varepsilon}{c}\frac{\partial \boldsymbol{E}}{\partial t}, \qquad \text{rot}\,\boldsymbol{E} = -\frac{\mu}{c}\frac{\partial \boldsymbol{h}}{\partial t}, \tag{2.4}$$

$$\boldsymbol{j} = \lambda_0 \boldsymbol{E} - \varkappa_0 \,\text{grad}\, T + \frac{\lambda_0 \mu}{c}\left[\frac{\partial \boldsymbol{u}}{\partial t} \times \boldsymbol{H}\right],$$

$$\varrho \frac{\partial^2 \boldsymbol{u}}{\partial t^2} = G\nabla^2 \boldsymbol{u} + (\lambda + G)\,\text{grad div}\,\boldsymbol{u} + \frac{\mu}{c}[\boldsymbol{j} \times \boldsymbol{H}] + \boldsymbol{P} - 3a_0 K \text{grad}\, T, \tag{2.5}$$

$$c_v \frac{\partial T}{\partial t} + \frac{c_p - c_v}{3a_t}\frac{\partial}{\partial t}\text{div}\,\boldsymbol{u} + \pi_0\,\text{div}\,\boldsymbol{j} - \lambda_1 \nabla^2 T = Q\,. \tag{2.6}$$

Let us assume now that the medium is characterized by an ideal electric conductivity, i.e. $\lambda_0 = \infty$. This condition may be expressed by the coefficient $\varkappa_0$ tending simultaneously with λ_0 to infinity or not. Quite generally, let us assume $\varkappa_0 = \varkappa_1 \lambda_0$, where $\varkappa_1$ — constant.

From the third equation of the (2.4) group we have

$$\boldsymbol{E} = \varkappa_0 \,\text{grad}\, T - \frac{\mu}{c}\left[\frac{\partial \boldsymbol{u}}{\partial t} \times \boldsymbol{H}\right]. \tag{2.7}$$

Neglecting in Maxwell's equations the displacement current $\dfrac{\varepsilon}{c}\dfrac{\partial E}{\partial E}$ and subsequently eliminating from equations (2.4)—(2.7) the vectors $\boldsymbol{E}$, $\boldsymbol{j}$, we obtain the following system of equations

$$\varrho \frac{\partial^2 \boldsymbol{v}}{\partial t^2} = G\nabla^2 \boldsymbol{v} + (\lambda + G)\,\text{grad div}\,\boldsymbol{v} + \frac{\mu}{4\pi}\left[\text{rot rot}\,[\boldsymbol{v} \times \boldsymbol{H}]\right] \times \boldsymbol{H} - \tag{2.8}$$

$$- \frac{c\varkappa_1}{4\pi}(\text{rot rot grad}\, T) \times \boldsymbol{H} - 3a_t\,\text{grad}\frac{\partial T}{\partial t} + \frac{\partial \boldsymbol{P}}{\partial t}, \qquad \boldsymbol{v} = \frac{\partial \boldsymbol{u}}{\partial t},$$

$$c_v \frac{\partial T}{\partial t} + \frac{c_p - c_v}{3a_t}\,\text{div}\frac{\partial \boldsymbol{v}}{\partial t} + \pi_0\left[\frac{c}{4\pi}\text{rot rot}\,[\boldsymbol{v} \times \boldsymbol{H}]\right] - \frac{c^2 \varkappa_0}{4\pi\mu}\text{rot rot}\, T - \tag{2.9}$$

$$- \lambda_1 \nabla^2 \frac{\partial T}{\partial t} = \frac{\partial Q}{\partial t}.$$

Let us observe that the terms $\varkappa_0$ and π_0, in view of rot grad $\varphi = 0$ and div rot $\psi = 0$, are dropped from the equations, Thus, we can put $\dfrac{\partial^*}{\partial t}$ before brackets*).

*) We assume the operator in brackets $\neq$ const with respect to t.

So we have

$$\varrho\frac{\partial^2 \boldsymbol{u}}{\partial t^2} = G\nabla^2\boldsymbol{u}+(\lambda+G)\operatorname{grad}\operatorname{div}\boldsymbol{u}+\frac{\mu}{4\pi}[\operatorname{rot}\operatorname{rot}[\boldsymbol{u}\times\boldsymbol{H}]]\times\boldsymbol{H}-3\alpha_t K\operatorname{grad}T+\boldsymbol{P}, \tag{2.10}$$

$$c_v\frac{\partial T}{\partial t}+\frac{c_p-c_v}{3\alpha_t}\operatorname{div}\frac{\partial \boldsymbol{u}}{\partial t}-\lambda_1\nabla^2 T = Q. \tag{2.11}$$

In what follows we will neglect in Eq. (2.11) the term containing the velocity of dilatation. This term, characterizing the thermo-mechanical coupling, is of a very low value and therefore may be neglected.

3. One-dimensional half-space problem

We will now consider an elastic half-space being in contact with vacuum (Fig. 1). The plane $x_3 = 0$ is assumed to be heated suddenly to the temperature T_0, this temperature being maintained constant. Let the vector of the initial magnetic field

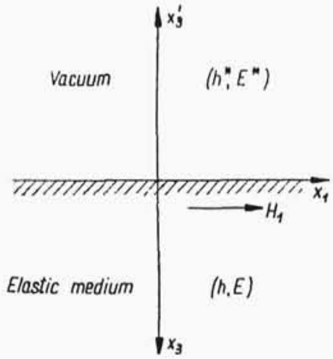

Fig. 1

$\boldsymbol{H} = (H_1, 0, 0)$ be paralled to the axis x_1. We assume further $\boldsymbol{P} = 0$, $Q = 0$. Eqs. (2.10) and (2.11) will become then of particularly simple form, namely:

$$a^2\frac{\partial^2 u_3}{\partial x_3^2}-\frac{\partial^2 u_3}{\partial t^2} = \frac{3K\alpha_t}{\varrho}\frac{\partial T}{\partial x_3}, \tag{3.1}$$

$$\frac{\partial^2 T}{\partial x_3^2}-\frac{1}{\eta}\frac{\partial T}{\partial t} = 0, \tag{3.2}$$

where

$$a^2 = a_0^2+\varkappa, \quad \varkappa = \frac{\mu H_1^2}{4\pi\varrho}, \quad a_0^2\varrho = 1+2G, \quad \eta = \lambda_1/c_v.$$

Equations of electromagnetic field in vacuum are of the following form*)

$$c^2\frac{\partial^2 h_1^*}{\partial x_3'^2}-\frac{\partial^2 h_1^*}{\partial t^2} = 0, \quad c^2\frac{\partial^2 E_2^*}{\partial x_3'^2}-\frac{\partial^2 E_2^*}{\partial t^2} = 0. \tag{3.3}$$

*) For the x_3' axis we retain the positive direction of vectors $\vec{E}^*\,\vec{h}^*$ as for the axis of the system without 's.

The strength of perturbed electric and magnetic fields in an elastic medium may be expressed by means of the following equations

$$h_1 = -H_1 \frac{\partial u_3}{\partial x_3}, \qquad E_2 = -\frac{\mu}{c} H_1 \frac{\partial u_3}{\partial t}. \tag{3.4}$$

Boundary conditions of the problem assume the following form

$$\sigma_{33} + T_{33} - T_{33}^* = 0, \quad E_2^* = E_2, \quad T = T_0 H(t), \tag{3.5}$$

where, generally, σ_{ij} denotes the tensor of mechanical strains, T_{ij} — tensor of Maxwell's stresses in an elastic body, and T_{ij}^* — that in vacuum, $H(t)$ stands for the Heaviside's function.

Expanding conditions (3.5) into an explicit form, we obtain the following relations for $x_3 = 0$

$$\begin{cases} a_0^2 \dfrac{\partial u_3}{\partial x_3} - \dfrac{3K\alpha_t T}{\varrho} = -\dfrac{\mu H_1}{4\pi\varrho} h_1^*, \\ \dfrac{\partial h_1^*}{\partial x_3} + \dfrac{\mu H_1}{c^2} \dfrac{\partial^2 u_3}{\partial t^2} = 0, \qquad T = T_0 H(t). \end{cases} \tag{3.6}$$

We shall introduce new notations and variables

$$u_3 = u, \quad h_1^* = h^*, \quad h_1 = h, \quad \zeta = \frac{a x_3}{\eta}, \quad \tau = \frac{a^2 t}{\eta}, \quad \zeta' = \frac{a x_3'}{\eta}.$$

In this way Eqs. (3.1)—(3.3) will get the following form

$$\begin{cases} u_{,\zeta\zeta} - u_{,\tau\tau} = mT_{,\zeta}, \quad T_{,\zeta\zeta} - T_{,\tau} = 0, \\ h^*_{,\zeta'\zeta'} - \dfrac{a^2}{c^2} h^*_{,\tau\tau} = 0, \quad E^*_{,\zeta'\zeta'} - \dfrac{a^2}{c^2} E^*_{,\tau\tau} = 0, \quad m = \dfrac{3\alpha_t K \eta}{\varrho a^3}. \end{cases} \tag{3.7}$$

For $\zeta = 0$ we have the following form of boundary conditions

$$u_{,\zeta} - m_0 T = -\beta_0 h^*, \quad h^*_{,\zeta} + n u_{,\tau\tau} = 0, \quad T = T_0 H(\tau), \tag{3.8}$$

where

$$m_0 = m \frac{a^2}{a_0^2}, \quad n = \frac{\mu H_1 a^3}{c^2 \eta}, \quad \beta_0 = \frac{\mu H_1 \eta}{4\pi \varrho^2 a_0^2 a}.$$

Assuming, for $t < 0$, the elastic medium to be in a stressless state, we shall use — in order to solve Eqs. (3.7) — the Laplace transformation. Denoting the Laplace transform of the function $f(\zeta, \tau)$ by

$$\bar{f}(\zeta, p) = \int_0^\infty f(\zeta, \tau) e^{-p\tau} d\tau$$

we may write Eqs. (3.7) and the boundary conditions (3.8) in the following form

$$\bar{u}_{,\zeta\zeta} - p^2 \bar{u} = m\bar{T}_{,\zeta}, \quad \bar{T}_{,\zeta\zeta} - p\bar{T} = 0, \quad \bar{h}^*_{,\zeta'\zeta'} - \frac{a^2 p^2}{c^2} \bar{h}^* = 0, \tag{3.9}$$

$$|\bar{u}_{,\zeta\zeta} - m_0 \bar{T} + \beta_0 \bar{h}^*|_{\zeta=0} = 0, \quad |\bar{h}^*_{,\zeta} + np^2 \bar{u}|_{\zeta=0} = 0, \quad \bar{T} = T_0/p. \tag{3.10}$$

Solving the system of Eqs. (3.9) and taking into account (3.10), we obtain

$$(3.11)\quad \bar{u}=\frac{mT_0}{p^2(1-p)(1+\beta)}\{e^{-p\zeta}[1-\vartheta(1-p)+\beta\sqrt{p}]-(1+\beta)\sqrt{p}\,e^{-\zeta\sqrt{p}}\},$$

$$\vartheta=\frac{m_0}{m}=\frac{a^2}{a_0^2}>1,\qquad \beta=\beta_0\frac{\mu_1 H_1 a^2 a_0}{c\varkappa},$$

$$(3.12)\quad \bar{T}=\frac{T_0}{p}e^{-\zeta\sqrt{p}},\qquad \bar{h}^*=-\frac{npc}{a}\bar{u}(0,p)\,e^{-\frac{pa}{c}\zeta'}.$$

Mechanical strain may be calculated from formula:

$$(3.13)\quad \bar{\sigma}_{33}=(\lambda+2G)\frac{d\bar{u}_3}{dx_3}-3K\alpha_t\bar{T}=s\left[\frac{\partial\bar{u}}{\partial\zeta}-m_0\bar{T},\right],\qquad s=\frac{aa_0^2\varrho}{\eta}.$$

Substituting into the above expression for $\bar{u}$ the term from (3.11) and for $\bar{T}$— the term from (3.12), we get the following formula

$$(3.14)\quad \bar{\sigma}_{33}=-\frac{M}{(1+\beta)\vartheta^2(1-p)}\left\{e^{-p\zeta}\left[\frac{1-\vartheta}{p}+\vartheta+\frac{\beta}{\sqrt{p}}\right]-\right.$$
$$\left.-e^{-\zeta\sqrt{p}}(1+\beta)\left[\frac{1-\vartheta}{p}+\vartheta\right]\right\},$$
$$M=\frac{E\alpha_t T_0}{1-2\nu}.$$

The transform of $\bar{h}^*$ may be obtained from (3.12) and (3.11)

$$(3.15)\quad \bar{h}^*=-\frac{N}{1-p}\left[\frac{1-\vartheta}{p}+\vartheta-\frac{1}{\sqrt{p}}\right],\qquad N=\frac{H_1\mu a^2 mT_0}{\eta c(1+\beta)},\qquad \mu=1$$

and the transform of $\bar{h}$ from the formula (3.4)

$$(3.16)\quad \bar{h}=-\frac{P}{1-p}\left\{e^{-p\zeta}\left[\frac{1-\vartheta}{p}+\vartheta+\frac{\beta}{\sqrt{p}}\right]-(1+\beta)\frac{e^{-\zeta\sqrt{p}}}{p}\right\},\qquad P=\frac{amT_0H_1}{\eta(1+\beta)}.$$

Let us perform an inverse transformation on the formulae (3.14)—(3.16). As a result we get

$$(3.17)\quad \sigma_{33}(\zeta,\tau)=-\frac{M}{(1+\beta)\vartheta^2}\{(1-\vartheta)f_1(\zeta,\tau)-f_2(\zeta,\tau)-\beta f_2(\zeta,\tau)-$$
$$-(1+\beta)[(1-\vartheta)g_1(\zeta,\tau)-g_2(\zeta,\tau)]\},$$

$$(3.18)\quad h^*(\zeta',\tau)=N[\vartheta-1+f_2^*(\zeta',\tau)-f_3^*(\zeta',\tau)],$$

$$(3.19)\quad h(\zeta,\tau)=-P\{(1-\vartheta)f_1(\zeta,\tau)-f_2(\zeta,\tau)-\beta f_3(\zeta,\tau)-$$
$$-(1+\beta)[g_1(\zeta,\tau)-g_2(\zeta,\tau)]\},$$

where

$$
(3.20)\quad \begin{cases}
f_1(\zeta,\tau) = H(\tau-\zeta), \quad f_2(\zeta,\tau) = e^{\tau-\zeta} H(\tau-\zeta), \\
f_2^*(\zeta',\tau) = e^{\tau-\zeta'\frac{a}{c}} H\left(\tau-\zeta'\frac{a}{c}\right), \\
f_3(\zeta,\tau) = e^{\tau-\zeta} \operatorname{erf}\sqrt{\tau-\zeta}\, H(\tau-\zeta), \\
f_3^*(\zeta',\tau) = e^{\tau-\zeta'\frac{a}{c}} \operatorname{erf}\sqrt{\tau-\zeta'\frac{a}{c}}\, H\left(\tau-\zeta'\frac{a}{c}\right), \quad g_1(\zeta,\tau) = \operatorname{erfc}\frac{\zeta}{\sqrt{4\tau}}, \\
g_2(\zeta,\tau) = \frac{e^{\tau}}{2}\left[e^{\zeta}\operatorname{erfc}\left(\frac{\zeta}{\sqrt{4\tau}}+\sqrt{\tau}\right) + e^{-\zeta}\operatorname{erfc}\left(\frac{\zeta}{\sqrt{4\tau}}-\sqrt{\tau}\right)\right].
\end{cases}
$$

In a particular case when the thermo-elastic field is not coupled with the mechanical-electromagnetic one ($\beta=0,\ v=1$), we obtain

$$
(3.21)\qquad \sigma_{33}(\zeta,\tau) = -M\,[g_2(\zeta,\tau) - f_2(\zeta,\tau)], \quad h^* = h = 0.
$$

The stress σ_{33} from the above formula represents the known solution of W. I. Danilovskaya [4] for a non-coupled problem. We will now analyze the stress $\sigma_{33}(\zeta,\tau)$ in formula (3.17). Let us observe, first, that the functions g_1 and g_2 are of diffusional character, whereas the functions f_1, f_2 and f_3 correspond to a propagating wave. For τ finite and $\zeta\to\infty$ the stress vanishes. For τ finite and $\zeta=0$, we obtain

$$
(3.22)\qquad \sigma_{33}(0,\tau) = -\frac{M\beta}{(1+\beta)\vartheta^2}\,[e^{\tau}(1-\operatorname{erf}\sqrt{\tau}) - 1 + \vartheta].
$$

The mechanical part of the stress σ_{33} — for $\zeta=0$ — differs from 0. Only after adjoining the term $T_{33}-T_{33}^*$ (conformly to (3.5)) reduces it to 0. The stress vanishes if $\beta\to 0$, i.e. if the coupling does not exist.

Let us consider an arbitrary cross-section $\zeta=\text{const}$ inside the elastic medium. In the elastic half-space a term of stress σ_{33} will appear in this cross-section, related with functions g_1 and g_2. At the moment $\tau=\zeta$ (it means for $t=x_3/a$) at the cross--section considered the front of a modified elastic wave arrives, propagating with velocity a, i.e. faster than the sound velocity. This wave is characterized by the functions f_1, f_2 and f_3. The transition of the modified elastic wave across the cross--section is accompanied by a jump in the stress τ_{33}, the value of this jump being $M/(1+\beta)\vartheta$. Let us note that this value is constant. For $\zeta>\tau$ the stress decreases and for $\zeta\to\infty$ it attains the asymptotic value $M(1-\vartheta)/(1+\beta)\vartheta^2$.

In the case of non-coupled problem ($\vartheta=1$) $\beta=0$ the wave propagates with a velocity a_0 and the value of the stress jump will be M. For $\zeta>\tau$ the stress rapidly decreases and for $\zeta\to\infty$ it tends to 0. The course of the function $h(\zeta,\tau)$ in the elastic medium is analogous to the course of the stress σ_{33}, as it appears from the juxtaposition of the formulae (3.17) and (3.19). The modified electromagnetic wave propagates with the velocity $a=\sqrt{a_0^2+\varkappa}$, i.e. with the same velocity as the modified elastic wave. For $\tau=\zeta$ the function $h(\zeta,\tau)$ shows a discontinuity, the value of which is of $P\vartheta$.

The function $h^*(\zeta', \tau)$ may be expressed by the following formula

$$(3.23) \quad h^*(\zeta', \tau) = N[\vartheta - 1 + e^{\tau - a\zeta'} \operatorname{erfc}\sqrt{\tau - a\zeta'}]\, H(\tau - a\zeta'), \qquad a = \frac{a}{c}.$$

This wave propagates in vacuum with the velocity of light. On the front wave of discontinuity, it means for $\tau = a\zeta'$ $(t = x_3'/c)$ the value of the function h^* is constant, $N\vartheta = N\sqrt{1+\varkappa/a_0^2}$.

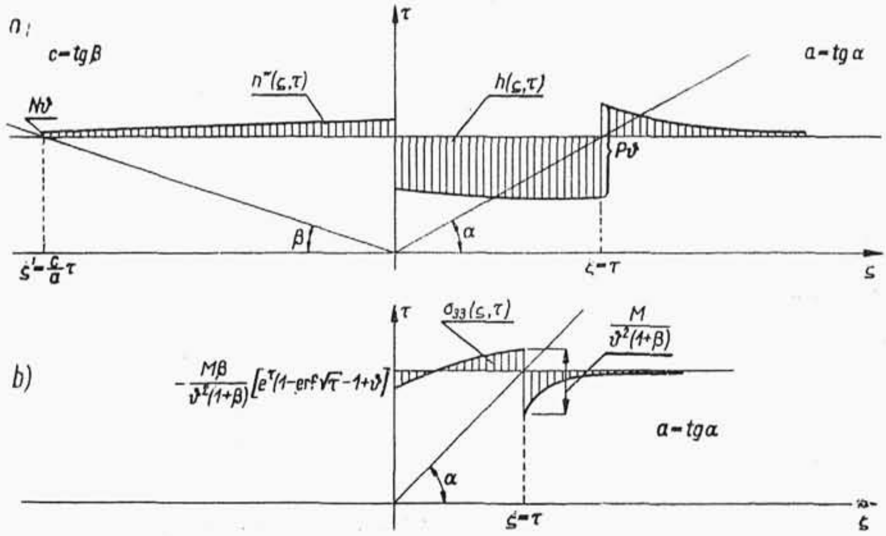

Fig. 2

In the cross-section $\zeta' = 0$ we have

$$h^*(0, \tau) = N[\vartheta + (e^{\tau} \operatorname{erfc}\sqrt{\tau} - 1)] > \vartheta N.$$

The solutions for the functions h, h^* and σ_{33} on the phase plane are shown in Fig. 2.

Conclusions

From the above considerations it appears that as a result of the action of the thermal shock a modified elastic wave and an electromagnetic wave h propagate in an elastic medium; there occurs also the radiation of the electromagnetic wave into the vacuum.

Besides a manifest theoretical interest in describing the coupled phenomena occurring in an elastic body, the solution obtained is of essential value for the measurement technique.

An ideal conductor being assumed, we obtained only one reflected wave of discontinuity jointly for the mechanical and electromagnetic waves. Nevertheless, this assumption enabled us to express the thermo-elastic and electromagnetic effects in a closed fo1m.

A more accurate approach to the problem — taking into account the finite conductivity and the displacement currents — should result in obtaining for both thermo-elastic and electromagnetic fields two reflected waves of discontinuity propagating with the velocity of the order of velocity of sound and light in the medium. The amplitude of waves propagating, with a velocity of the order of velocity of light *n* the medium is insignificant and, generally speaking, may be neglected — as it was done in the present paper.

In some problems, however, a more accurate approach may be also of practical value. The present authors intend to consider the problem in a separate paper, taking into account the finite conductivity.

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С. КАЛИСКИЙ и В. НОВАЦКИЙ, **ВОЗБУЖДЕНИЕ МЕХАНИКО-ЭЛЕКТРОМАГНИТНЫХ ВОЛН ПРИ ВОЗДЕЙСТВИИ ТЕРМИЧЕСКИМ ШОКОМ**

В работе представлено решение проблемы термо-магнито-упругости, касающееся краевой задачи упругого полупространства, сохраняющего контакт с вакуумом.

При предположении, что в упругой среде и в вакууме существует первоначальное магнитное поле с вектором, параллельным к плоскости, ограничивающей упругое полупространство, ограничивающая плоскость нагревается в момент $t = 0$ от нулевой температуры, отвечающей состоянию тела, лишенному напряжений, до температуры T_0.

В § 2 приводится общее уравнение термо-магнито-упругости одновременно с переходом от действительного проводника к идеальному проводнику, в § 3 дается формулировка и решение краевой задачи, а в § 4 приводятся возможности дальнейшего развития проблемы.

Физически существенным эффектом решения краевой задачи является доказательство существования в среде модифицированных волн — механической и электромагнитной, а также констатирование излучения электромагнитной разрывной волны в вакуум при воздействии термического шока на поверхности полупространства.