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APPLIED MECHANICS

Two-dimensional Problem of Magnetothermoelasticity. I

by

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1. General equations

Let us consider a thermoelastic medium with a primary constant magnetic field $\vec{H}$. The body forces and heat sources give rise not only to the temperature field, accompanying the deformations, but also to the induced electromagnetic field. We shall confine our considerations to the infinite thermoelastic medium. We assume that the medium is isotropic and homogeneous and exhibits a perfect electric conductivity.

We proceed from three groups of equations. The first group consists of the equations of electrodynamics of slow-moving media [1]

$$\operatorname{rot}\vec{h} = \frac{4\pi}{c}\vec{j}, \tag{1.1}$$

$$\operatorname{rot}\vec{E} = -\frac{\mu_0}{c}\dot{\vec{h}}, \tag{1.2}$$

$$\vec{E} = -\frac{\mu_0}{c}(\dot{\vec{u}}\times\vec{H}), \tag{1.3}$$

$$\operatorname{div}\vec{h} = 0. \tag{1.4}$$

In these equations $\vec{h}$, $\vec{E}$ are the vectors of the magnetic and electric fields intensities, respectively, $\vec{j}$ is the vector of the current density, $\vec{H}$ — the vector of the primary, constant magnetic field, $\vec{u}$ — the displacement vector, μ_0 — the magnetic permeability factor, and c — the velocity of light. By a dot we denote the partial derivative with respect to time t.

The second group consists of the equations of motion of the thermoelastic medium subject to the primary constant magnetic field [2], [3],

$$\mu\nabla^2\vec{u} + (\lambda+\mu)\operatorname{grad}\operatorname{div}\vec{u} + \frac{\mu_0}{c}(\vec{j}\times\vec{H}) + \vec{F} - \gamma\operatorname{grad}\theta = \varrho\ddot{\vec{u}}. \tag{1.5}$$

In the latter equations $\vec{F}$ denotes the vector of the body force, and θ — the temperature referred to the natural, unstressed state of the body. The quantities μ, λ are Lamé's isothermic constants, and $\gamma = (3\lambda+2\mu)\alpha_t$, where α_t is the coefficient of linear expansion.

The set of fundamental equations is to be supplemented by the generalized equation of conduction of heat [4],

$$\nabla^2\theta - \frac{1}{\varkappa}\dot{\theta} - \eta \operatorname{div}\dot{\vec{u}} = -Q/\varkappa. \tag{1.6}$$

In this equation $\varkappa$ is the thermal diffusivity, $Q = W/c_\varepsilon$, where W is the quantity of heat produced per unit time and volume, and c_ε — the specific heat for constant deformation, referred to unit volume. Finally, $\eta = \gamma T_0/k$ is a coefficient describing the coupling of the temperature field with the field of deformations where T_0 denotes the absolute temperature of the body in its natural state ($\theta = 0$), and k is the coefficient of thermal conductivity.

In our further considerations we shall assume (with no loss of generality) that the primary magnetic field is reduced to the component $\vec{H} = (0, 0, H_3)$ in the direction of axis x_3.

According to this assumption we obtain from Eqs. (1.1)÷(1.3) the following relations

$$E_1 = -\frac{\mu_0 H_3}{c}\dot{u}_2, \qquad E_2 = \frac{\mu_0 H_3}{c}\dot{u}_1, \qquad E_3 = 0, \tag{1.7}$$

$$\dot{h}_1 = \frac{c}{\mu_0}\partial_3 E_2, \qquad \dot{h}_2 = -\frac{c}{\mu_0}\partial_3 E_1, \qquad \dot{h}_3 = -\frac{c}{\mu_0}(\partial_1 E_2 - \partial_2 E_1), \tag{1.8}$$

$$j_1 = \frac{c}{4\pi}(\partial_2 h_3 - \partial_3 h_2), \qquad j_2 = \frac{c}{4\pi}(\partial_3 h_1 - \partial_1 h_3), \qquad \dot{j}_3 = \frac{c}{4\pi}(\partial_1 h_2 - \partial_2 h_1), \tag{1.9}$$

$$\partial_i = \partial/\partial x_i.$$

Expressing in the above equations the components of vector $\vec{j}$ by the components of vector $\vec{u}$ and then inserting them into Eqs. (1.5) we arrive at the following system of equations in terms of displacements [5],

$$\begin{aligned}
&\mu\nabla_1^2 u_1 + (\lambda+\mu+a_0^2\varrho)\partial_1 e + a_0^2\varrho\partial_3(\partial_3 u_1 - \partial_1 u_3) + F_1 - \gamma\partial_1\theta = \varrho\ddot{u}_1,\\
&\mu\nabla_1^2 u_2 + (\lambda+\mu+a_0^2\varrho)\partial_2 e + a_0^2\varrho\partial_3(\partial_3 u_2 - \partial_2 u_3) + F_2 - \gamma\partial_2\theta = \varrho\ddot{u}_2,\\
&\mu\nabla_1^2 u_3 + (\lambda+\mu)\partial_3 e + F_3 - \gamma\partial_3\theta = \varrho\ddot{u}_3,
\end{aligned} \tag{1.10}$$

where

$$e = \partial_1 u_1 + \partial_2 u_2 + \partial_3 u_3, \qquad a_0^2 = \frac{\mu_0 H_3^2}{4\pi\varrho}.$$

Here e is the dilatation and a_0 Alfvén's velocity. Eqs. (1.10) are supplemented by the generalized equation of conduction of heat

$$\nabla^2\theta - \frac{1}{\varkappa}\dot{\theta} - \eta\dot{e} = -\theta/\varkappa. \tag{1.11}$$

The coupling of the electromagnetic field with the fields of deformation and temperature is described by the factor $a_0^2\varrho$. For $H_3 \to 0$ Eqs. (1.10), (1.11) assume the form of the thermoelasticity equations.

Eqs. (1.10) exhibit a symmetry with respect to axis x_3. The structure of these equations is analogous to that of the equations of motion, in terms of displacements, for the elastic body with transverse isotropy.

The set of equations is, finally, to be supplemented by the Duhamel—Neumann equations which connect the stress and strain tensors with the temperature

$$\sigma_{ij} = 2\mu\,\varepsilon_{ij} + (\lambda\varepsilon_{kk} - \gamma\,\theta)\,\delta_{ij}, \qquad i, j = 1, 2, 3, \tag{1.12}$$

where

$$\varepsilon_{ij} = \tfrac{1}{2}(\partial_i u_j + \partial_j u_i), \qquad \varepsilon_{kk} = e. \tag{1.13}$$

2. Two-dimensional problem

Passing to two-dimensional problems we can discuss three particular cases of Eqs. (1.10) by assuming that the displacements, body forces and heat sources are independent either of x_1 or x_2 or x_3. In the present paper we shall consider the simplest case only and we shall assume that all functions occurring in Eqs. (1.7)÷(1.13) are independent of variable x_3.

In this particular case Eqs. (1.7)÷(1.9) undergo considerable simplification and become:

$$E_1 = -\frac{\mu_0 H_3}{c}\dot{u}_2, \qquad E_2 = \frac{\mu_0 H_3}{c}\dot{u}_1, \qquad E_3 = 0, \tag{2.1}$$

$$\dot{h}_1 = 0, \qquad \dot{h}_2 = 0, \qquad \dot{h}_3 = -\frac{c}{\mu_0}(\partial_1 E_2 - \partial_2 E_1), \tag{2.2}$$

$$j_1 = \frac{c}{4\pi}\partial_2 h_3, \qquad j_2 = -\frac{c}{4\pi}\partial_1 h_3, \qquad j_3 = 0. \tag{2.3}$$

Eqs. (1.10) and (1.11) can be written in the form:

$$\mu\nabla_1^2\vec{u} + (\lambda + \mu + a_0^2\,\varrho)\,\mathrm{grad}\,\mathrm{div}\,\vec{u} + \vec{F} - \gamma\,\mathrm{grad}\,\theta = \varrho\ddot{\vec{u}}, \tag{2.4}$$

$$\nabla_1^2\theta - \frac{1}{\varkappa}\dot{\theta} - \eta\,\mathrm{div}\,\dot{\vec{u}} = -Q/\varkappa, \tag{2.5}$$

where

$$\vec{u} = (u_1, u_2, 0), \qquad \vec{F} = (F_1, F_2, 0), \qquad \nabla_1^2 = \partial_1^2 + \partial_2^2.$$

Here we deal with the state of plane strain, and the equations of motion, in terms of displacements, have a form analogous to that of the equations of motion, in terms of displacements, for the isotropic body; for $a_0^2\varrho \to 0$ they assume the form of the equations of plane thermoelastic strain.

Finally, Eqs. (1.12) and (1.13) assume the form

$$(2.6)\qquad \begin{aligned} &\sigma_{ij} = 2\mu\varepsilon_{ij} + (\lambda\varepsilon_{kk} - \gamma\theta)\,\delta_{ij}, \quad \varepsilon_{kk} = \partial_1 u_1 + \partial_2 u_2, \quad i,j = 1,2, \\ &\sigma_{33} = \lambda\varepsilon_{kk} - \gamma\theta, \end{aligned}$$

$$(2.7)\qquad \varepsilon_{ij} = \tfrac{1}{2}(\partial_i u_j + \partial_j u_i),$$

Let us perform on Eq. (2.4) the divergence operation. In this way we obtain relation:

$$(2.8)\qquad \Box_1^2 e + \frac{1}{a^2\varrho}(\partial_1 F_1 + \partial_2 F_2) - \frac{\gamma}{a^2\varrho}\nabla_1^2\theta = 0,$$

where

$$\Box_1^2 = \nabla_1^2 - \frac{1}{a^2}\partial_t^2, \quad \partial_t^2 = \frac{\partial^2}{\partial t^2}, \quad a^2 = c_1^2 + a_0^2, \quad c_1^2 = \frac{\lambda + 2\mu}{\varrho}.$$

After performing on Eq. (2.4) the curl operation we arrive at relation

$$(2.9)\qquad \Box_2^2\Omega + \frac{1}{\mu}(\partial_1 F_2 - \partial_2 F_1) = 0,$$

where

$$\Omega = \partial_1 u_2 - \partial_2 u_1, \quad \Box_2^2 = \nabla_1^2 - \frac{1}{c_2^2}\partial_t^2, \quad c_2^2 = \mu/\varrho.$$

It should be observed that the temperature θ does not occur in Eq. (2.9).

Let us decompose vectors $\vec{u}$ and $\vec{F}$ into the potential and rotational parts

$$(2.10)\qquad u_1 = \partial_1\Phi - \partial_2\psi, \qquad u_2 = \partial_2\Phi + \partial_1\psi,$$

$$(2.11)\qquad F_1 = \varrho(\partial_1\vartheta - \partial_2\chi), \quad F_2 = \varrho(\partial_2\vartheta + \partial_1\chi).$$

Inserting (2.10), (2.11) into (2.4) and (2.5), we obtain a system of three equations, where the first and third equations are coupled with each other

$$(2.12)\qquad \Box_1^2\Phi - m\theta + \frac{1}{a^2}\vartheta = 0,$$

$$(2.13)\qquad \Box_2^2\psi + \frac{1}{c_2^2}\chi = 0,$$

$$(2.14)\qquad \Box_3^2\theta - \eta\nabla_1^2\dot{\Phi} = -Q/\varkappa, \quad m = \gamma/a^2\varrho.$$

Here we have introduced a new operator

$$\Box_3^2 = \nabla_1^2 - \frac{1}{\varkappa}\partial_t.$$

Let us now express the components of vectors $\vec{E}$, $\vec{h}$, $\vec{j}$ in terms of functions Φ, ψ. In this way we arrive at equations

$$(2.15)\qquad E_1 = -\frac{\mu_0 H_3}{c}(\partial_2\dot{\Phi} + \partial_1\dot{\psi}), \quad E_2 = \frac{\mu_0 H_3}{c}(\partial_1\dot{\Phi} - \partial_2\dot{\psi}), \quad E_3 = 0,$$

$$h_1 = 0, \quad h_2 = 0, \quad h_3 = H_3 \nabla_1^2 \Phi, \tag{2.16}$$

$$j_1 = \frac{cH_3}{4\pi} \partial_2 \nabla_1^2 \dot{\Phi}, \quad j_2 = -\frac{cH_3}{4\pi} \partial_1 \nabla_1^2 \dot{\Phi}, \quad j_3 = 0. \tag{2.17}$$

By eliminating function θ from Eqs. (2.12)—(2.14), we obtain a system of two equations which are independent of each other

$$(\Box_1^2 \Box_3^2 - m\eta\partial_t \nabla_1^2)\Phi = -\frac{m}{\varkappa} Q - \frac{1}{a^2} \Box_3^2 \vartheta, \tag{2.18}$$

$$\Box_2^2 \psi + \frac{1}{c_2^2} \chi = 0. \tag{2.19}$$

It follows from the above equations that the heat sources and the potential part of the body forces produce in the infinite space longitudinal waves only, which are subject to damping and dispersion, as it is seen from the nature of Eq. (2.18), while the rotational part of the body forces gives rise to transverse waves only.

Let us consider in more detail the two above cases.

a) For $\vec{\chi} = 0$ we have $\psi \equiv 0$ at any point of the space. Function Φ is determined by the particular solution of Eq (2.18). The components of vectors $\vec{E}$, $\vec{h}$, $\vec{j}$ are given by formulae (2.15)÷(2.17), where $\psi = 0$ is to be inserted. Let us observe that in this case we also have $\Omega = 0$, $c = \nabla_1^2 \Phi$, $\operatorname{div} \vec{E} = 0$.

Expressing the stresses by function Φ and taking into account (2.12), we obtain

$$\begin{aligned} \sigma_{ij} &= 2\mu(\Phi_{ij} - \delta_{ij}\nabla_1^2\Phi) - a_0^2 \varrho \nabla_1^2 \Phi \delta_{ij} + \varrho\delta_{ij}(\ddot{\Phi} - \vartheta), \qquad i,j = 1,2, \\ \sigma_{33} &= -(2\mu + a_0^2\varrho)\nabla_1^2\Phi + \varrho(\ddot{\Phi} - \vartheta). \end{aligned} \tag{2.20}$$

The temperature can be determined from Eq. (2.12).

b) If $\vartheta = 0$ and $Q = 0$, then we have $\Phi \equiv 0$ at any point of the space, and also the temperature θ vanishes. For $\vec{\chi} \neq 0$ we determine function ψ as the particular solution of Eq. (2.19). In the infinite space there propagate longitudinal waves only, their length being equal to $c_2 = \mu/\varrho$. These waves undergo neither damping nor dispersion. Among the electromagnetic quantities only two components of vector $\vec{E}$ are different from zero

$$E_1 = -\frac{\mu_0 H_3}{c} \partial_1 \dot{\psi}, \quad E_2 = \frac{\mu_0 H_3}{c} \partial_2 \dot{\psi}, \quad E_3 = 0, \tag{2.21}$$

where

$$\operatorname{div} \vec{F} = -\frac{\mu_0 H_3}{c} \nabla_1^2 \psi, \quad \partial_1 E_2 - \partial_2 E_1 = 0. \tag{2.22}$$

Furthermore, the following relations hold

$$e = 0, \quad \Omega = \nabla_1^2 \psi, \tag{2.23}$$

$$\sigma_{11} = -\sigma_{22} = -2\mu\partial_1\partial_2\psi, \quad \sigma_{12} = \mu(\partial_1^2 - \partial_2^2)\psi, \quad \sigma_{33} = 0. \tag{2.24}$$

3. Examples

a) Let in the infinite thermoelastic space with the steady, primary and constant magnetic field $\bar{H}=(0,0,H_3)$ act a linear heat source $Q(r,t)=Q_0 e^{i\omega t}\dfrac{\delta(r)}{2\pi r}$. uniformly distributed along axis x_3. In this case we deal with the cylindric longitudinal wave moving in the radial direction *ad infinitum*. Eq. (2.18) assumes here the form

$$\left[\left(\nabla_r^2-\frac{1}{a^2}\,\partial_t^2\right)\left(\nabla_r^2-\frac{1}{\varkappa}\,\partial_t\right)-m\eta\partial_t\,\nabla_r^2\right]\Phi=-\frac{mQ_0\,e^{i\omega t}}{\varkappa}\,\frac{\delta\,(r)}{2\pi r},\tag{3.1}$$

where

$$\nabla_r^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\,\frac{\partial}{\partial r}.$$

The solution of this equation is given by function

$$\Phi(r,t)=\frac{mQ_0}{2\pi\varkappa(k_1^2-k_2^2)}\,[K_0(k_1 r)-K_0(k_2 r)],\tag{3.2}$$

where $K_0(k_j r)$, $j=1,2$ are the modified Bessel functions of the third kind and k_1, k_2 are the complex roots of equation

$$k^4+k^2[\sigma^2-q(1+\varepsilon)]-\sigma^2 q=0,\qquad q=i\omega/\varkappa,\qquad \sigma^2=\omega^2/a^2,\tag{3.3}$$

where

$$k_j=a_j+ib_j,\qquad a_j>0,\qquad j=1,2.$$

Function Φ being known, we can determine from formulae (2.12) and (2.15)÷(2.17) the radial displacement $u_r=\partial\Phi/\partial r$, the temperature θ and the quantities connected with the electromagnetic field. Thus, for the temperature θ and for the quantity h_3 we obtain the following formulae

$$\theta=\frac{Q_0\,e^{i\omega t}}{2\varkappa(k_1^2-k_2^2)}\,[(\sigma^2+k_1^2)\,K_0(k_1\,r)-(\sigma^2+k_2^2)\,K_0(k_2\,r)],\tag{3.4}$$

$$h_3=\frac{H_3\,mQ_0\,e^{i\omega t}}{2\varkappa(k_1^2-k_2^2)}\,[k_1^2\,K_0(k_1\,r)-k_2^2\,K_0(k_2\,r)].\tag{3.5}$$

b) Let in the infinite space act a plane heat source varying harmonically in time $Q(x_1,t)=Q_0\,e^{i\omega t}\,\delta(x_1)$. In this case Eq. (2.18) reduces to the form

$$\left[\left(\partial_1^2-\frac{1}{a^2}\,\partial_t^2\right)\left(\partial_1^2-\frac{1}{\varkappa}\,\partial_t\right)-m\eta\partial_t\,\partial_1^2\right]\Phi=-\frac{mQ_0}{\varkappa}\,e^{i\omega t}\,\delta(x_1),\tag{3.6}$$

its solution being given by function

$$\Phi=\frac{Q_0 m\,e^{i\omega t}}{2\varkappa(k_1^2-k_2^2)}\left(\frac{e^{-k_1 x_1}}{k_1}-\frac{e^{-k_2 x_1}}{k_2}\right),\qquad x_1>0,\tag{3.7}$$

where k_1, k_2 are the roots of Eq. (3.3) and

$$k_j = a_j + ib_j, \quad a_j > 0, \quad j = 1, 2.$$

Bearing in mind that

$$u_1 = \partial_1 \Phi, \quad \theta = \frac{1}{m}\left(\partial_1^2 - \frac{1}{a^2}\partial_t^2\right), \tag{3.8}$$

we obtain

$$u_1 = \frac{Q_0\, m}{2\varkappa(k_1^2 - k_2^2)}\left[e^{i\omega\left(t - \frac{x_1}{v_2}\right) - a_2 x_1} - e^{i\omega\left(t - \frac{x_1}{v_1}\right)}\right], \tag{3.9}$$

$$\theta = \frac{Q_0}{2\varkappa(k_1^2 - k_2^2)}\left[\frac{\sigma^2 + k_1^2}{k_1} e^{i\omega\left(t - \frac{x_1}{v_1}\right) - a_1 x_1} - \right.$$
$$\left. - \frac{\sigma^2 + k_2^2}{k_2} e^{i\omega\left(t - \frac{x_1}{v_2}\right) - a_2 x_1}\right], \quad x_1 > 0, \tag{3.10}$$

where

$$v_j = \frac{\omega}{\mathrm{Im}(k_j)}, \quad a_j = \mathrm{Re}(k_j), \quad j = 1, 2. \tag{3.11}$$

Here v_j is the phase velocity, and a_j — the attenuation factor. These quantities are functions of frequency ω and of parameters ε and a.

Knowing function Φ, we can determine the components of the electromagnetic field from formulae (2.1) and (2.2). Let us observe that for the one-dimensional problem we have $E_1 = E_3 = 0$, $h_1 = h_2 = 0$ and $j_1 = j_3 = 0$.

Introducing the notations

$$k = \frac{a}{\varkappa}\zeta, \quad \beta = \omega/\omega^*, \quad \omega^* = a^2/\varkappa, \tag{3.12}$$

into Eq. (3.3), we can write it in the form

$$\zeta^4 + \zeta^2\beta\left[\beta - i\left(1 + \frac{\varepsilon_0}{1+\alpha}\right)\right] - i\beta^3 = 0, \tag{3.13}$$

where

$$\alpha = a_0^2/c_1^2, \quad \varepsilon_0 = \frac{\gamma}{\varrho c_1^2}\eta\varkappa, \quad \varepsilon = \frac{\varepsilon_0}{1+\alpha}.$$

For $H_3 \to 0$ Eq. (3.13) takes the form of an analogous equation for the thermoelastic medium. The latter equation, with $a \to c_1$, $\alpha \to 0$, $k \to \frac{c_1}{\varkappa}$, $\omega^* \to c_1^2/\varkappa$, has been discussed in detail by P. Chadwick and I. N. Sneddon [6]. Following the results of this discussion, we may assume that for the actually occurring mechanical vibrations with the frequency of the order of ultrasonic vibrations the inequality $\omega \ll \omega^*$ holds, thus $\beta \ll 1$. Expanding the solution of Eq. (3.13) into a power series of β and using formulae (3.13), we obtain

$$
(3.14) \qquad \begin{aligned} v_1 &= \left(\frac{2\varkappa\omega}{1+\varepsilon}\right)^{1/2} \frac{1}{1+\dfrac{\varepsilon\beta}{2(1+e)^2}}, \qquad \alpha_1 = \left[\frac{\omega}{2\varkappa}(1+\varepsilon)\right]^{1/2}\left[1+\frac{\beta\varepsilon}{2(1+\varepsilon)^2}\right], \\ v_2 &= a(1+\varepsilon)^{1/2}, \qquad \alpha_2 = \frac{\varepsilon}{2}(1+\varepsilon)^{-5/2}\frac{a}{\varkappa}\left(\frac{\omega}{\omega^*}\right)^2. \end{aligned}
$$

It can readily be shown that the quantities v_1, α_1 correspond to the modified thermic wave, while v_2, α_2 correspond to the modified elastic wave. It follows from formulae (3.9), (3.10) that the modified waves of both types occur both in the expression for u_1 and in that for θ. The modified thermic wave undergoes damping and dispersion, while the modified elastic wave undegoes damping but is not subject to dispersion, since $v_2 \approx \text{const}$.

Since we have $v_2 = a(1+\varepsilon)^{1/2} = c_1(1+\alpha+\varepsilon_0)^{1/2}$, thus in the case considered the velocity of modified wave exceed that for the thermoelastic medium where the phase velocity is equal to $v_2 = c_1(1+\varepsilon_0)^{1/2}$.

The phase velocity in the magnetothermoelastic medium increases with the increase of component H_3 of the primary and constant magnetic field.

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В. НОВАЦКИЙ, ПЛОСКАЯ ЗАДАЧА МАГНИТОТЕРМОУПРУГОСТИ

В настоящей работе рассматривается термоупругая среда, в которой имеется начальное и постоянное магнитное поле $\vec{H}$. Рассуждения ограничиваются рассмотрением однородной и изотропной среды с идеальной электропроводностью при пренебрежении токами смещения. Действие массовых сил и источников тепла вызывает в рассматриваемом неограниченном пространстве образование температурного поля, поля деформации, а также возбуждение электромагнитного поля.

В первой части работы дается замкнутая система уравнений, описывающих эти возмущения, во второй же части — обсуждается динамическая двухмерная задача, в которой, принимая что $\vec{H} = (0,0, H_3)$, всякие функции зависят единственно от переменных x_1, x_2 и от времени t.

Путем разложения векторов смещения и массовых сил на части потенциальную и ротационную получаем уравнение продольных волн (затухающих и подвергающихся рассеянию), а также уравнение продольных волн (не затухающих и распространяющихся с постоянной скоростью c_2).

В третьей части работы приводится пример продольной цилиндрической волны, вызванной действием линейного источника тепла, а также пример плоской волны, вызванной действием плоского источника тепла.