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The Stability of Rectangular Plates with Ribs

by

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Let us consider a rectangular plate with a system of longitudinal and transversal ribs, simply supported at the edges. Denote by EJ, A_1 the flexural rigidity and the cross-sectional area of longitudinal ribs respectively and by EJ, A_2 — the corresponding quantities for transversal ribs. Let us assume that the ribs are symmetrical with respect to the mean surface of the plate and there is no friction between the ribs and the plate.

Let the plate be uniformly loaded at the edges, the loads being denoted by q_1, q_2. We assume that the ribs are loaded by part of that load, proportional to the cross-sectional area of the ribs, the concentrated forces P_1, P_2 (Fig. 1) constituting an additional load. Let us denote by r the number of longitudinal and by p that of transversal ribs. If the forces exceed their critical values, the plate and the ribs will undergo a deflection. The forces acting between the plate and the ribs are continuous functions of x or y, denoted $t^{(i)}(x)$ $(i=1, 2, ..., r)$ for longitudinal and $r^{(i)}(y)$ $(i=1, ..., p)$ for transversal ribs. The differential equation of the deflection surface of the plate will take the form

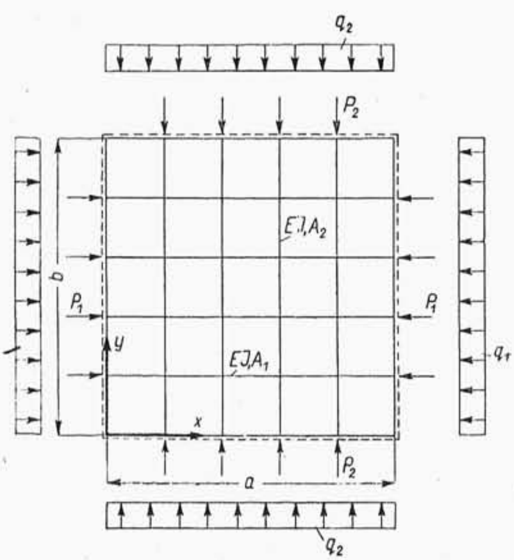

Fig. 1

$$N \nabla^2 \nabla^2 w + q_1 \frac{\partial^2 w}{\partial x^2} + q_2 \frac{\partial^2 w}{\partial y^2} = p(x, y), \tag{1}$$

where N denotes the flexural rigidity of the plate, w — the deflection, $p(x, y)$ — the load. The solution of Eq. (1) is found by means of a double trigonometric series

$$w(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} a_{n,m} \sin \alpha_n x \sin \beta_m y, \tag{2}$$

$$\alpha_n = \frac{n\pi}{a}, \quad \beta_m = \frac{m\pi}{b}.$$

The load $p(x, y)$ is also expressed in the form of a series

$$p(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} p_{n,m} \sin \alpha_n x \sin \beta_m y, \tag{3}$$

where

$$p_{n,m} = \frac{2}{b} \sum_{i=1}^{i=r} t_n^{(i)} \sin \alpha_n \xi_i + \frac{2}{a} \sum_{i=1}^{i=p} r_m^{(i)} \sin \beta_m \eta_i,$$

Substituting (2) and (3) in Eq. (1) we obtain

$$w(x,y) = \frac{2}{b} \sum_{i=1}^{i=r} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{t_n^{(i)} \sin \alpha_n \xi_i \sin \alpha_n x \sin \beta_m y}{D_{n,m}} + \tag{4}$$

$$+ \frac{2}{a} \sum_{i=1}^{i=p} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{r_m^{(i)} \sin \beta_m \eta_i \sin \alpha_n x \sin \beta_m y}{D_{n,m}},$$

where

$$D_{n,m} = N(\alpha_n^2 + \beta_m^2)^2 - (q_1 \alpha_n^2 + q_2 \beta_m^2).$$

The deflection of the i-th transversal rib will be expressed by the following differential equation

$$EI_i \frac{d^4 \overline{w}^{(i)}}{dy^4} + (P_2^{(i)} + q_2 A_2^{(i)}) \frac{d^2 \overline{w}^{(i)}}{dy^2} = - \sum_{m=1}^{\infty} r_m^{(i)} \sin \beta_m y, \tag{5}$$

and that of the i-th longitudinal rib by

$$EJ_i \frac{d^4 \overline{\overline{w}}^{(i)}}{dx^4} + (P_1^{(i)} + q_1 A_1^{(i)}) \frac{d^2 \overline{\overline{w}}^{(i)}}{dx^2} = - \sum_{n=1}^{\infty} t_n^{(i)} \sin \alpha_n x. \tag{6}$$

The solutions of Eqs. (5) and (6) are as follows:

$$\overline{w}^{(i)} = - \sum_{m=1}^{\infty} \frac{r_m^{(i)} \sin \beta_m y}{\Delta_m^{(i)}}, \qquad \overline{\overline{w}}^{(i)} = - \sum_{n=1}^{\infty} \frac{t_n^{(i)} \sin \alpha_n x}{\Delta_n^{(i)}}, \tag{7}$$

where

$$\Delta_m^{(i)} = EI_i \beta_m^4 - (P_2^{(i)} + q_2 A_2^{(i)}) \beta_m^2, \qquad \Delta_n^{(i)} = EJ_i \alpha_n^4 - (P_1^{(i)} + q_1 A_1^{(i)}) \alpha_n^2.$$

From the condition that the deflections of the rib and of the plate along the lines $x = \xi_k$ $(k = 1, 2, ..., p)$ and $y = \eta_k$ $(k = 1, 2, ..., r)$ must be identical, the following system of equations can be found:

$$(8) \quad \frac{r_m^{(k)}}{\Delta_m^{(k)}} + \frac{2}{a} \sum_{i=1}^{i=p} r_m^{(i)} \sum_{n=1}^{\infty} \frac{\sin \alpha_n \xi_i \sin \alpha_n \xi_k}{D_{n,m}} + \frac{2}{b} \sum_{i=1}^{i=r} \sin \beta_m \eta_i \sum_{n=1}^{\infty} \frac{t_n^{(i)} \sin \alpha_n \xi_k}{D_{n,m}} = 0,$$

$$\frac{t_n^{(k)}}{\Delta_n^{(k)}} + \frac{2}{b} \sum_{i=1}^{i=r} t_n^{(i)} \sum_{m=1}^{\infty} \frac{\sin \beta_m \eta_i \sin \beta_m \eta_k}{D_{n,m}} + \frac{2}{a} \sum_{i=1}^{i=p} \sin \alpha_n \xi_i \sum_{m=1}^{\infty} \frac{r_m^{(i)} \sin \beta_m \eta_k}{D_{n,m}} = 0.$$

By equating to zero the determinant of this system we obtain the condition of buckling.

The problem comprises a series of particular cases. Let us analyse some of them.

a. A plate having transversal ribs only. Let $q_2 = 0$ $P_2 = 0$ $P_1 = 0$. The system of equations (8) reduces to

$$(9) \quad \frac{r_m^{(k)}}{EI_k \beta_m^4} + \frac{2}{a} \sum_{i=1}^{i=p} r_m^{(i)} \sum_{n} \frac{\sin \alpha_n \xi_i \sin \alpha_n \xi_k}{N(\alpha_n^2 + \beta_m^2)^2 - q_1 \alpha_n^2} = 0.$$

Let us consider in particular the case of one rib in the middle. We have

$$(10) \quad \frac{1}{\gamma \varrho^3} + 2 \sum_{n=1,3,\dots}^{\infty} \frac{1}{(n^2 + \varrho^2)^2 - \varrho^2 n^2 s} = 0,$$

where

$$\varrho = a/b, \quad \gamma = \frac{EI}{Nb}, \quad s = \frac{q_1}{q_E}, \quad q_E = \frac{N\pi^2}{b^2}.$$

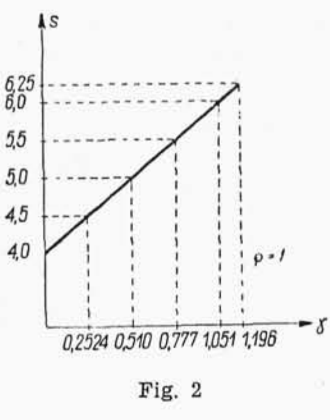

Fig. 2

In Eq. (10) we have put $m = 1$, because the form of buckling will be that of one half wave in the y-direction.

From Eq. (10) we determine, for fixed values of ϱ, the function $s = s(\gamma)$. This function is shown in Fig. 2, for $\varrho = 1$.

For two transversal ribs, symmetrical with respect to the line $x = a/2$, we have, for a symmetrical buckling, $r^{(1)} = r^{(2)}$, and for an antisymmetrical buckling, $r^{(1)} = -r^{(2)}$.

The condition of buckling is expressed in this case $\xi_1 = a/3$, $\xi_2 = 2a/3$ as follows:

(11)
$$\frac{1}{\gamma\varrho^3} + 4 \sum_{n=1,3,\dots}^{\infty} \frac{(-1)^{\frac{n-1}{2}} \sin\frac{n\pi}{3} \cos\frac{n\pi}{6}}{(n^2+\varrho^2)^2 - n^2\varrho^2 s} = 0,$$
$$\frac{1}{\gamma\varrho^3} + 4 \sum_{n=2,4,\dots}^{\infty} \frac{(-1)^{\frac{n-2}{2}} \sin\frac{n\pi}{3} \sin\frac{n\pi}{6}}{(n^2+\varrho^2)^2 - n^2\varrho^2 s} = 0.$$

b. A plate with longitudinal ribs only. Let $P_2 = q_2 = 0$.

The system (8) reduces to

(12)
$$\frac{t^{(k)}}{n^4\chi_k - sn^2\varrho^2\delta_k - p_k n^2\varrho^2} + 2\sum_{i=1}^{i=r} t^{(i)} \sum_{m=1,2,\dots}^{\infty} \frac{\sin\beta_m\eta_i \sin\beta_m\eta_k}{(n^2+\varrho^2 m^2)^2 - n^2\varrho^2 s} = 0,$$
$$(k = 1, 2, \dots r)$$

where

$$\chi_k = \frac{EJ_k}{bN}, \quad \delta_k = \frac{A_1^{(k)}}{b}, \quad p_k = \frac{P_1^{(k)} b}{N\pi^2}.$$

In the case of one longitudinal rib along the line $y = b/2$, we obtain

(13)
$$\frac{1}{n^4\chi - sn^2\varrho^2\delta - pn^2\varrho^2} + 2\sum_{n=1,3,\dots}^{\infty} \frac{1}{(n^2+\varrho^2 m^2)^2 - n^2\varrho^2 s} = 0.$$

Substituting in Eq. (13) $n = 1, 2, \dots$, we obtain the critical forces for one, two, etc. half-waves.

Les us assume in Eq. (13) $\delta = 0$, $s = 0$. This is the case of buckling of a rib reinforced by a plate. After some simple transformations we obtain from Eq. (13)

(14)
$$P_{1,kr} = \frac{EJ\pi^2 n^2}{a^2} + \frac{N\pi^2}{2n^2 a} \cdot \frac{1}{\varrho \sum\limits_{m=1,3,\dots}^{\infty} \frac{1}{(n^2+\varrho^2 m^2)^2}}.$$

Let us substitute now $p = 0$, $\varrho = 1$ in Eq. (13). We have

(15)
$$\chi = \frac{s\varrho^2\delta}{n^2} - \frac{1}{2n^4 \sum\limits_{m=1,3,\dots}^{\infty} \frac{1}{(n^2+\varrho^2 m^2)^2 - n^2\varrho^2 s}}.$$

The diagram of $s = s(\chi)$ for constant δ and for $n = 1$ $\varrho = 1$ is shown in Fig. 3.

c. A square plate with two identical ribs ($EI = EJ, A_1 = A_2$), which are compressed by the forces P. Let $q_1 = q_2 = 0$.

In this particular case the actions along the lines $x = a/2y = a/2$ are identical. From the first of Eqs. (8) we obtain

$$(16) \quad r_m \left[\frac{1}{EI\beta_m^4 - P\beta_m^2} + \frac{2}{a} \sum_{n=1,3,\ldots}^{\infty} \frac{1}{N(\alpha_n^2+\beta_m^2)^2} \right] + \\ + \frac{2}{a} \sin\frac{n\pi}{2} \sum_{n}^{\infty} \frac{r_n \sin\frac{n\pi}{2}}{N(\alpha_n^2+\beta_m^2)^2} = 0.$$

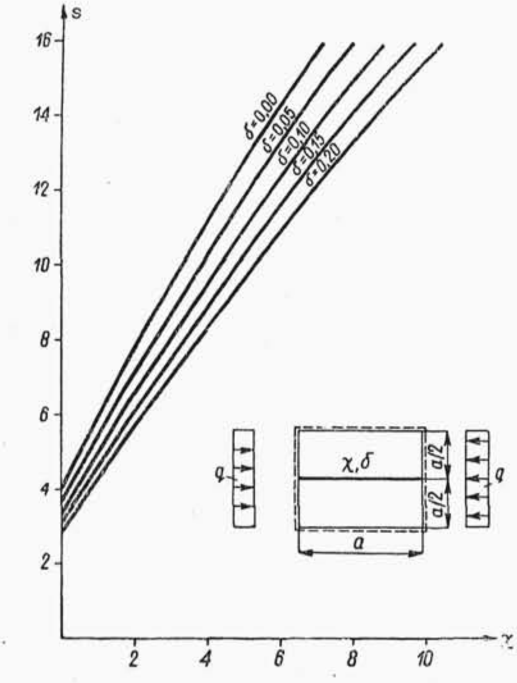

Fig. 3

Let us assume $m = 1$ and take from the second series the first term only. We have

$$P = \frac{EJ\pi^2}{a^2} + \frac{N\pi^2}{a} \cdot 0.9766.$$

Assuming $m = 1, 2, 3$ and taking three terms of the second series of Eq. (16), we obtain a more exact result:

$$P = \frac{EJ\pi^2}{a^2} + \frac{N\pi^2}{a} \cdot 0.9530.$$

For two longitudinal and two transversal ribs of the same flexural rigidity and the same cross-section, dividing the area of the plate into nine squares, we obtain

$$P = \frac{EJ\pi^2}{a^2} + \frac{N\pi^2}{a} \cdot 0.655$$

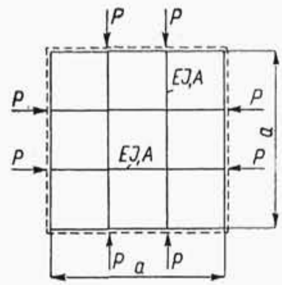

Fig. 4

The method described gives an exact solution of the problem of buckling of a plate reinforced by ribs. Its applicability is limited, however, by the difficulties of calculation for a greater number of ribs. It can therefore be used for symmetrical systems only, with not more than two longitudinal and two transversal ribs. For systems having longitudinal or transversal ribs only, it can be successfully applied for three and four ribs symmetrically arranged.

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