

**MATHEMATICAL MODELLING OF COMPLEX
MECHANICAL SYSTEMS
Volume 1: Discrete Models**



MATHEMATICS AND ITS APPLICATIONS

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MATHEMATICAL MODELLING OF COMPLEX MECHANICAL SYSTEMS Volume 1: Discrete Models

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ELLIS HORWOOD

NEW YORK LONDON TORONTO SYDNEY TOKYO SINGAPORE

First published in 1993 by
ELLIS HORWOOD LIMITED
Market Cross House, Cooper Street,
Chichester, West Sussex, PO19 1EB, England



A division of
Simon & Schuster International Group
A Paramount Communications Company

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Printed and bound in Great Britain
by Bookcraft, Midsomer Norton

British Library Cataloguing in Publication Data

A catalogue record for this book is available from the British Library

ISBN 0-13-563750-3

Library of Congress Cataloging-in-Publication Data

Available from the publisher

To our teachers, from whose wisdom we drew, and to our students, to whom we want to transmit our experience

Wisdom begins when you sincerely want to learn. To desire Wisdom is to love her
Wisdom of Solomon 6:17

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