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Ewa Orłowska, Zdzisław Pawlak

Logical foundations of knowledge representation

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INSTYTUT PODSTAW INFORMATYKI POLSKIEJ AKADEMII NAUK
INSTITUTE OF COMPUTER SCIENCE POLISH ACADEMY OF SCIENCES
00-901 WARSAW, P.O. Box 22, POLAND

Ewa Orłowska, Zdzisław Pawlak

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Part I

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Pracę zgłosił Andrzej Blikle

Mailing address: Ewa Orłowska
Zdzisław Pawlak
Institute of Computer Science
Polish Academy of Sciences
P.O. Box 22
00-901 Warszawa PKiN

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Abstract . **Содержание** . Streszczenie

In the paper we attempt to precize basic notations of the field of knowledge representation and to discuss properties of knowledge representation systems. The special attention is drown to indiscernibility of objects in knowledge representation systems and to definability and approximate definability of concepts. In the existing literature there seems to be no reference to these properties of knowledge representation.

In part two of this paper we are going to discuss machine learning and induction along the same lines of reasonig, as used in part one.

Логические основы представления знаний

В данной работе производится попытка уточнения основных понятий в области представления знаний, а также дискутируются свойства систем представления знаний. Особое внимание уделено неразличаемости объектов в системах представления знаний, определяемости, а также приближенной определяемости понятий. В существующей литературе данные аспекты систем представления знаний освещены недостаточно.

Во второй части данной работы мы намерены проанализировать проблемы машинного учения, а также индукции, принимая в качестве отправной точки теже предпосылки, что в первой части.

Logiczne podstawy reprezentacji wiedzy

W pracy tej próbujemy sprecyzować podstawowe pojęcia z dziedziny reprezentacji wiedzy oraz przedyskutować własności systemów reprezentacji wiedzy. Specjalną uwagę poświęcono nierozerwalności obiektów w systemach reprezentacji wiedzy, definiowalności oraz przybliżonej definiowalności pojęć. W istniejącej literaturze nie zajmowano się tymi aspektami systemów reprezentacji wiedzy.

W części drugiej tej pracy mamy zamiar przeanalizować problemy komputerowego uczenia się oraz indukcji, przyjmując jako punkt wyjścia te same przesłanki co w części pierwszej.

Preface

This paper is intended primarily for those researchers in AI interested in symbolic reasoning processes and the symbolic representation of knowledge for use in machine inference.

Two major issues of knowledge engineering are representation and utilization of knowledge. Knowledge representation research is focused on developing methods for representing expert - level knowledge as symbolic data structures for computer use. Knowledge utilization research consists in designing flexible control structures and heuristics for plausible reasoning and decision making. We discuss foundational aspects of both representation and utilization of various kinds of information and we develop methods of dealing with knowledge at a conceptual level. We illustrate the importance and, in fact, the necessity of considering both semantic and syntactic levels of representation. We present the conceptual organization of knowledge on these two levels. We use description tools which are as neutral as possible from the implementation point of view and can be realized by various AI techniques. The idea of organizing representation of information according to the pattern: schemes - instances of schemes is not new. It was suggested by the view of representations as extended data types and by the need to represent knowledge about those data types. We follow this pattern at a conceptual level, considering two stages of representation. We introduce formal languages which provide a framework for defining schemes of information items. Semantics of these languages provides instances of the schemes. Moreover, we develop deduction methods which can provide mechanisms for using knowledge. The languages introduced in the book are expressive enough to represent a variety of types of knowledge explicitly. They can provide a direct, manipulatory access both to "facts" related to knowledge about a domain of applications, and to "heuristics" that guide decision making at a meta-level. The presented theory provide a medium for the formalization of knowledge in domains where it is as yet highly informal.

The work we present in this paper is to a great extent inspired by general discussions of knowledge engineering research, case studies, and experiments. We report a sample of these publications in references. The theory developed here, although deriving its motivation from knowledge engineering, can be viewed as a theory for reasoning in empirical theories in general. The publications which had some impact on that work are reported in references.

1. INTRODUCTION

1.1. Representation = Semantics + Syntax + Deduction Method

The problems of knowledge representation shown in this paper are in the sphere of Artificial Intelligence even though many of the problems analysed here have a longer genealogy going back to the methodology of sciences and logic. The term knowledge representation in the narrower sense means the way of presenting information about a fragment of reality and the way of using such information. But a broader interpretation of that terms has become common in recent years: it is used to denote the sphere of research concerned with the search for methods of presenting and handling information in artificial intelligence systems. Those methods can refer to any sphere of applications and any level of knowledge: one seeks ways of representing both ob- ject-level knowledge and meta-level knowledge, the latter being the knowledge about the former. In this book the methods of knowledge representation will be analysed in accordance with the following schema:

Representation = Semantics + Syntax + Deduction Method

Usually our primary concern about a given domain is semantic in character. Our views of the respective part of the real world are formed in abstraction from language. Hence the component Semantics is to provide a conceptual model of the domain to be represented. The term domain is understood here both generally and broadly. In object-level knowledge we are interested in a given sphere of applications, for instance, in medical system, in patients and certain data about them. In meta-level knowledge the domain consists of the knowledge at the former level. We can continue this sequence of domains through an arbitrary number of levels. The domain at each level determines the use of the knowledge at the next lower level. The conceptual primitives specific to the knowledge level under consideration should be defined in such a model. Thus the object-level primitives should characterize the domain of applications. The meta-level primitive concepts should accordingly describe the characteristics of the object-level knowledge in question.

The component Syntax provides the linguistic counterpart of the conceptual model adopted in the component Semantics. The point here is to define a formal language to be used in expressing information about those domains to which a given conceptual model pertains. There must be a strict correspondence between the model and the language connected with it. The primitive concepts included in the model should have their linguistic counterparts at the level of atomic formulas. Further, compound formulas should be constructed from atomic formulas with the use of logical operators, selected according to the type of the domain.

The component Deduction Method is to provide the methods of handling the knowledge presented by means of the formal language introduced within the component Syntax. The working out of such a deduction method should consist in formulating the logic to be used and the methods of inference that are in agreement with that logic. Such logic should obviously include classical propositional logic, but, as it turns out, classical logic alone does not suffice in many cases. For instance, making use of knowledge with the consideration of the temporal dimension of information requires the use of temporal logic.

The methods of knowledge representation suggested in this book cover all the three aspects of representation listed above. We are also concerned with the methodology of knowledge representation interpreted in terms of the said three components. The methodological problems related to the component Semantics refer generally to unambiguity and redundance of the knowledge of a given domain. Next, the methodological problems related to the component Syntax cover the expressibility of the concepts pertaining to the domain represented in a given case in the language linked to the model of that domain. Finally, the methodological problems related to the component Deduction Methods cover primarily the completeness of inference techniques in logics under consideration.

1.2. Conceptual primitives

The methods of knowledge representation shown in this book cover those domain which can be described by the listing of the following conceptual primitives:

Object

Attribute

Value of attribute

Each element considered here is treated as the designatum of a class of objects of a certain kind. The choice of the sphere of applications is linked to the indication of the objects which are elements of each of those classes.

It is assumed that anything that can be spoken about in the subject position of a natural language sentence, e.g., book, company, etc., is an object. Objects need not be atomic and indivisible. They can be compound and structured, but are treated as single wholes.

It is further assumed that properties of objects are fundamental elements of the knowledge of a given domain. A property is defined by a verb phrase in a natural language sentence, e.g., is red, is tall, etc. Those properties are given by means of attributes and values of attributes which are meaningful for given objects. For instance, in order to express the property of an object being of a certain colour we assume that we have at our disposal the attribute Colour which for a given object takes on a definite value from among a fixed set called the set of values of that attribute. For instance, the attribute Colour in a given domain may have the values red, green, white. Obviously, objects of different kinds have in principle different sets of characteristic attributes, although some attributes can be common to objects of different kinds. For instance, the attribute Weight is common to human beings and machines, but the attribute Education is proper to human beings only. It is assumed that objects of a given kind are characterized by attributes connected with certain practical considerations. Hence in different domains we can use different sets of attributes for objects of a given kind provided that these attributes are admissible for that kind of objects.

Every attribute can take on values from a definite set of values: for instance, the value of the attribute Weight can be a real number from a certain interval, the attribute Number of Children will have values which are natural numbers. Note that, according to our needs, one and the same attribute can draw its values from different sets; for instance, the attribute Colour may assume values from the set: red, blue, black, ... , but its value may also be defined by light wavelenght; the values of the attribute Height may be given in centimetres or in inches, but they may also be defined as small, medium, large. Thus the set of values of each attribute is defined, on the one hand, by our needs, and on the other, by our possibilities of measuring or observing that attribute. We shall hereafter assume that the sets of objects, attributes, and values of attributes are fixed for a given domain.

The methods of knowledge representation suggested in this book offer us the opportunity for presenting those three basic conceptual primitives for any domain at both the semantic and the syntactic level of representation.

Next to the properties of objects, which we treat as elementary components of knowledge, we consider more complex elements of knowledge, namely concepts. In its semantic interpretation a concept is represented - in accordance with the tradition current in set-theoretical considerations - by a set of objects. In its syntactic interpretation a concept is a formula of the appropriate formal language. The

relationship between the semantic and the syntactic interpretation of concepts is the fundamental problem of concept representation. The methods of knowledge representation given in this book make it possible rigorously to formulate those relationships and to show the resulting possibilities and limitations in the handling of concepts. Those problems are discussed in chapter five.

1.3. Deterministic information

As stated in the preceding section, the adoption of the domain the knowledge of which is to be represented consists, among other things in listing the objects connected with that domain, fixing the attributes which characterize those objects, and listing the sets of values of the respective attributes.

The description of an object by the listing of the values of all its attributes adopted in a given domain will be termed deterministic information about that object. When we speak about deterministic information we mean the fact that every object takes on exactly one value for each attribute and that the value of each attribute is defined for each object. Thus deterministic information about an object is exhaustive and exclusive. Deterministic information about an object is given by a set of pairs of the form: attribute - value of attribute. Self-evidently, that information depends on the set of attributes and the set of values of attributes we have at our disposal. It may occur that in a given domain certain two different objects are described by the same information. This is to say that those objects are indiscernible in terms of the given set of attributes and values of those attributes.

The knowledge representation (KR) system of deterministic information given in Sec. 2.1 is, for the domains with deterministic information, the model that corresponds to the semantic level of representation. The concept was first used in Pawlak (1981). Such a system consists of a set of objects, a set of attributes, a family of sets of values of attributes, and the deterministic information function, which to each object and each attribute assigns a certain value of that attribute.

At the syntactic level, each KR system of deterministic information has the language of that system assigned to it. The language described in this book is a modification of the language introduced in Marek and Pawlak (1976). Deterministic descriptors, i.e., formulas of the form (name of attribute, name of value of attribute) are atomic formulas in that language. Compound formulas are obtained from atomic formulas linked by classical propositional operations of negation, disjunction, conjunction, implication, and equivalence. Sets of objects considered in a given system naturally correspond to the formulas of the language of that system. The atomic formulas of the form (a v) have their counterpart in the set of those objects to which the information function assigns value v of the attribute a. Compound formulas have their counterparts in the sets of objects obtained from the sets that correspond to the components of a given formulas following the appropriate set theoretical operations. The usual correspondence between propositional and set theoretical operations is preserved: negation has its counterpart in the complement; disjunction, in the union; conjunction, in the intersection of sets. Implication and equivalence are definable in terms of negation and disjunction, and hence the operations of complement, union, and intersection suffice to define the set of objects for every formula of the language of a given system.

The component Deduction Method is, in the case of domains with deterministic information, defined in terms of the classical propositional calculus in which schemata of descriptors play the role of propositional variables. The inference method complies with the axioms and rules of the propositional calculus. The models of the language of that logic are determined by KR systems of deterministic information.

The representation of deterministic information at the three levels of representation is discussed in chapter 2.

1.4. Nondeterministic information

Deterministic information, discussed in the preceding section, is related to those domains in which we have complete knowledge of the objects as far as the attributes adopted in a given domain are concerned. But it often occurs that we are not in a position to state with certainty what is the value taken by a given attribute for a given object, and are merely able to indicate a set of the potential values of that attribute for that object. For instance, we may not know the colour of the eyes of a given person, but can only say that they were blue or green, and certainly neither brown nor black. In such a case the concept of deterministic information does not suffice. This is why the concept of nondeterministic information is used to cope with such situations.

We have to do with nondeterministic information about an object if a certain subset of the set of values of a given attribute is as-

signed to that object and to every attribute in the domain under consideration. That subset indicates the range of the values within which the value of a given attribute is to be found, even though that value is not explicitly assigned to the object in the domain in question. Thus nondeterministic information is, in a sense, incomplete.

At the semantic level, the domain of nondeterministic information has its model in the KR system of nondeterministic information. Such systems were first discussed by Pawlak (1983) and later investigated by Orłowska and Pawlak (1984). Those systems differ from the systems with deterministic information by the definition of the information function. The nondeterministic information function assigns to every object and every attribute a set of values of that attribute. Each subset of the set of values of a given attribute will be termed the generalized value of that attribute. As in the case of deterministic information, in this case, too, it may occur that certain objects in a given domain are not discenible in terms of nondeterministic information.

As in the previous case, a special language of the system is the syntactic counterpart of a KR system of nondeterministic information. Nondeterministic descriptors, that is formulas of the form (name of attribute, name of generalized value of attribute) are atomic formulas of that language. Compound formulas are obtained from atomic formulas with the use of propositional operations. This time, next to the classical propositional operations used in deterministic information languages we use certain other operators which are modal in character. They make it possible to compare objects relative to the generalized values of attributes they take. We are in particular interested in inclusions and intersections of generalized values of attributes. Two objects are treated as similar if the generalized values of attributes assigned to them have pairwise nonempty intersections. Further, an object is treated as informationally contained in another if the generalized values of the attributes assigned to the former are included in the generalized values of those attributes of the latter. Likewise, as in the case of deterministic information languages, each formula of the language of a KR system of nondeterministic information has its counterpart in a set of objects of that system.

In the case of domains with nondeterministic information the component Deduction Method requires application of other logical means than in the case of deterministic information. The logic on which inferences in the languages of KR systems of nondeterministic information can be based is developed on the basis of S4 and B modal logics

(Gabbay (1976)). In the language of that logic, schemata of nondeterministic descriptors are atomic formulas. Models of the language of logic are defined with the use of KR systems of nondeterministic information.

Problems of representation of nondeterministic information are discussed in chapter 3.

1.5. Temporal information

The domains considered so far were linked to static knowledge, in which the temporal dimension was not taken into account. This is certainly a limitation with respect to real life situations, in which properties of objects usually change with the lapse of time. We are interested in such attributes as Height, Temperature, Blood Pressure, usually at given moments of time. Further, their change in a given ti interval may be of essential importance, too. This is why we have to consider such information about an object which is explicitly relate to time. Such information may be deterministic or nondeterministic, but it must additionally include the parameter which represents the moment to which that information applies.

The KR system of temporal information, suggested and investigate in Orłowska (1982) and Orłowska (1983 (c)) is the conceptual model o domains with temporal information. That system, next to the elements discussed earlier in this Introduction, includes a set whose element are interpreted as moments of time, and a relation of linear order i that set, interpreted as the earlier-later relation.

The language of KR systems of temporal information, presented in this book, makes it possible to formulate the dependence of information upon time. The atomic formulas in those languages have the sche (moment of time, property of object). Compound formulas are formed o atomic formulas by linking the latter with classical propositional operations. Properties of objects are represented by formulas constr ed of formulas of the form (name of object, name of attribute, name value of attribute) with the use of the classical propositional oper tions and propositional operations related to time. The intuitive s of those temporal operations is: possibly in the past, possibly in t future, definitely in the past, definitely in the future. Thus the pr erties of objects, expressed in such a language, have a reference to time, and moreover time is explicitly indicated in the formulas of t language as one of the parameters. That makes it possible to use dir ly the temporal context of information.

The logic on the basis of which the component Deduction Method rests is defined in terms of tense logic with linearly ordered time (Burgess (1979)). A special form of the atomic formulas is adopted and, as in the case of deterministic and nondeterministic information, the models of the language of logic are determined by KR systems of temporal information.

Representation of temporal information is discussed in chapter 4.

1.6. Concepts

In accordance with the reservations made in the previous sections, the fixing of a representation of a domain reduces to formulating an appropriate KR system, the language of that system, and the method of inference in that language. The problem arises which concepts pertaining to a given domain can be represented with the use of those means. It has been said that both deterministic and nondeterministic information about an object need not necessarily describe it in an unambiguous way since there may be several objects information about which in a given domain is the same. The same applies to information about an object in a fixed moment of time. This fact essentially affects the possibilities of representing the concepts related to a given domain.

In accordance with what was mentioned in Section 1.1, each subset of the set of objects in a given KR system will be treated as a semantic representation of a concept. Further, the formula in the language of that KR system which corresponds to the subset will be a syntactic representation of that concept. The problem whether every concept which can be introduced semantically can also be defined syntactically is an essential aspect of the methodology of knowledge representation. It turns out that in the general case the answer to the question is in the negative, and that in view of the indiscernibility of objects, with which we have to do when information about objects does not define them unambiguously. Thus in every KR system the information function establishes the indiscernibility relation in the set of objects. Two objects are indiscernible if and only if the values of the information function for those objects are the same for every attribute. That relation is an equivalence relation. The equivalence classes of that relation will be termed elementary sets. Hence every elementary set contains those objects which are mutually indiscernible by the attributes adopted in a given KR system. Of course, it may occur that all elementary sets contain one element each so that each object is discernible from all the remaining ones, but that need not be so in the general case. If it is assumed that information about a subset of objects is the "sum" of information about its elements, then it may turn out that certain objects cannot be defined syntactically. For it may occur that some objects which are in the same elementary set may be in the set that corresponds to the concept in question while others are not. It follows there from that attributes can be used to define those sets only which are sums of the equivalence classes of the indiscernibility relation determined by the set of attributes fixed in a given KR system. The notion of approximate definability and the notions of lower and upper approximation are introduced for those concepts which are not definable. The lower and the upper approximation of a concept in a given KR system are defined in terms of the indiscernibility relation in that system. Those problems are discussed in Sections 5.1, 5.2, 5.3, and 5.4.

The indiscernibility relation turns out to be decisive for the expressive power of KR systems, that expressive power being interpreted as the possibility of an adequate representation of the concepts related to given domain. A logical formalism which makes it possible to prove facts connected with the expressive power of KR systems understood in this way is suggested in the book. A logic is analysed in the language of which there are constants which stand for indiscernibility relations. Those constants are used to define the operations that correspond to the operations of lower and upper approximation. That logic is discussed in Section 5.8. The problem of expressive power of KR systems discussed in the book was formulated in Orłowska and Pawlak (1984 (a)).

Another important methodological problem connected with the representation of concepts is such a choice of the attributes for a given KR system which would eliminate all attributes superfluous for the description of the concepts considered in that KR system. Those problems are analysed in Section 5.6 and 5.7.

Research on the representation of concepts outlined in this book is continued in Pawlak (1982), Pawlak (1984), Orłowska (1983 (a)).

2. REPRESENTATION OF DETERMINISTIC INFORMATION

2.1. Systems of deterministic information

In this section we present and discuss a mathematical model of knowledge representation (KR) system of deterministic information. A deterministic KR system is intended to represent knowledge about some objects in an application - specific domain. Hence the basic component of the system is a nonempty set OB of objects e.g., human beings, books. We assume that knowledge about these objects can be expressed through assignment of some characteristic features to the objects. For example, human beings can be characterized by means of sex and age, books by means of title and author's name etc. The characteristics of objects are represented by attributes and values of the attributes. Hence a nonempty set AT of attributes and for each $a \in AT$ a set VAL_a of values of attribute a are the components of the system. From the formal point of view the assignment of attribute values to objects can be considered to be a total function from the Cartesian product OBxAT into set $VAL = \bigcup_{a \in AT} VAL_a$.

Now, we give the formal definition of a deterministic KR system. A deterministic KR system is a quadruple

$S = (OB, AT, \{VAL_a\}_{a \in AT}, f)$

where OB is a nonempty set whose elements are called objects

AT is a nonempty set whose elements are called attributes

VAL_a is a nonempty set whose elements are called values of attribute a

f is a total function from set OBxAT into set $VAL = \bigcup_{a \in AT} VAL_a$

such that $f(o,a) \in VAL_a$ for every $o \in OB$ and every $a \in AT$

For any attribute a set VAL_a is referred to as the domain of attribute a and function f is called information function.

Example 2.1.1

Let us consider a very simple deterministic KR system defined as follows:

$OB = \{o_1, o_2, o_3, o_4, o_5, o_6\}$

$AT = \{Sex, Age\}$

$VAL_{Sex} = \{male, female\}$

$VAL_{Age} = \{young, medium, old\}$

and the information function is defined by the following table:

	Sex	Age
o_1	male	young
o_2	male	medium
o_3	female	old
o_4	male	medium
o_5	female	old
o_6	female	young

Observe, that according to the definition of deterministic KR system one attribute value only can be associated with each object, and for each object the value of each attribute is uniquely determined. That is why we consider information of this kind to be deterministic.

Given a deterministic KR system $S = (OB, AT, VAL, f)$, for every object $o \in OB$ we define the function f_o from set AT into set VAL such that

$$f_o(a) = f(o,a)$$

This function is called information (or data) about object o in system S. Consider, for example, information about object o_2 in the system given in example 2.1.1. It consists of the following pairs:

(Sex, male) (Age, medium)

Let us notice that information about an object is exhaustive and exclusive i.e., values of all the attributes are determined for the object and only one attribute value can be associated with the object.

2.2. Examples of systems of deterministic information

In this section we give some real life examples of deterministic KR systems.

Example 2.2.1

Table 1 is a part of a large criminal file containing records of crimes, criminals, arms, locations, methods and other information collected over a long period of time (see Ashany (1976)).

Criminals are the objects in this system and we can identify them with their identification number. The criminals are characterized by the following attributes: Name, Age, Height, Weight, Sex, Colour of Eyes, Race, Profession, Home Address, Crime and Arm. Domain of each attribute consists of the elements of the column labelled by the attribute. Information about each criminal can easily be reconstructe

ID	NAME	AGE	HGHT	WGHT	SEX	EYES	RACE	PROF.	H. ADDR.	CRIME	ARM
1	John	23	5.10	180	M	Blue	Oriental	Plumber	Detroit	Murder	A
2	Claude	35	5.11	170	M	Green	Caucasian	B. Keeper	Houston	Rape	B
3	Robert	27	6.02	210	M	Hazel	Oriental	Sailor	Los Ang.	Hijacking	D
4	Nancy	19	5.04	130	F	Blue	Caucasian	Secretary	Boston	Homicide	A
5	Ingmar	39	5.07	160	M	Blue	Caucasian	Student	Chicago	Rape	D
6	Roberto	31	6.03	220	M	Black	Negro	Barber	New York	Murder	A
7	Marcel	42	5.08	180	M	Green	Eurasian	Actor	New York	Hijacking	C
8	Johanna	21	5.05	140	F	Blue	Eurasian	Student	Los Ang.	Drug push.	E
9	Jurgen	18	5.11	190	M	Hazel	Caucasian	Actor	Houston	Kidnapping	B
10	Tom	44	6.00	200	M	Blue	Caucasian	Mechanic	San Fran.	Armed rob.	C
11	Albert	18	5.09	180	M	Black	Negro	Barber	Detroit	Rape	E
12	Mark	24	5.06	150	M	Brown	Eurasian	B. Keeper	Los Ang.	Murder	A
13	Valerie	25	5.08	170	F	Hazel	Caucasian	Actor	Chicago	Armed rob.	B
14	George	37	5.05	150	M	Green	Caucasian	Plumber	San Fran.	Hijacking	D
15	Gary	31	5.10	170	M	Blue	Eurasian	Sailor	Houston	Drug push.	E
16	Peter	22	5.07	160	M	Black	Oriental	Student	New York	Hijacking	A
17	Rudy	25	6.03	210	M	Green	Caucasian	B. Keeper	San Fran.	Homicide	C
18	Homer	32	5.09	180	M	Brown	Oriental	Student	Chicago	Murder	D
19	Jeff	36	5.11	190	M	Hazel	Caucasian	Plumber	Los Ang.	Rape	A
20	Julia	24	5.05	140	F	Hazel	Caucasian	Actor	Boston	Homicide	E
21	Karen	27	5.08	170	F	Blue	Eurasian	Secretary	Houston	Murder	C
22	Steve	29	6.02	190	M	Brown	Oriental	B. Keeper	Detroit	Kidnapping	E
23	Gerald	32	5.06	160	M	Blue	Caucasian	Sailor	Boston	Armed rob.	D
24	Margot	19	5.09	180	F	Black	Negro	Secretary	Detroit	Hijacking	A
25	Frank	48	5.10	190	M	Brown	Eurasian	Barber	New York	Rape	B
26	Laurie	21	5.11	190	F	Hazel	Caucasian	Secretary	Los Ang.	Murder	B
27	Donald	26	6.01	220	M	Green	Eurasian	Plumber	New York	Drug push.	C
28	Adrian	28	5.09	180	M	Hazel	Caucasian	B. Keeper	Chicago	Murder	A
29	Ronald	21	5.10	170	M	Brown	Oriental	Sailor	New York	Homicide	A
30	Joan	26	5.07	160	F	Black	Negro	Student	Detroit	Armed rob.	B

Tab. 1, Representation of data from a criminal file from the row of the table corresponding to the object.

Example 2.2.2

Shown below is the other instance of criminal data file (Ashany (1976))

ID	Name	Hair Cover-age	Hair Tex-ture	Eyebrow Weight	Eyebrow Separa-tion	Eyes Open-ing	Eyes Separa-tion	Eyes Colour
1	John	Full	Wavy	Thin	Sep.	Narr.	Medium	Blue
2	Claude	Rec.	Str.	Bushy	Meet.	Narr.	Wide	Brown
3	Robert	Bald	Str.	Medium	Meet.	Medium	Close	Green
4	Nancy	Full	Wavy	Thin	Sep.	Wide	Medium	Hazel
5	Ingmar	Rec.	Curly	Bushy	Meet.	Wide	Wide	Blue
6	Roberto	Full	Curly	Thin	Sep.	Narr.	Close	Blue
7	Marcel	Rec.	Str.	Bushy	Meet.	Narr.	Medium	Black
8	Johanna	Full	Curly	Thin	Sep.	Medium	Medium	Green
9	Jurgen	Bald	Wavy	Thin	Sep.	Wide	Close	Green
10	Tom	Full	Wavy	Medium	Meet.	Narr.	Medium	Hazel

Tab. 2

The table contains data needed for identification of human face. There are the following attributes in the system: Name, Hair Coverage, Hair Texture, Eyebrow Weight, Eyebrow Separation, Eyes Opening, Eyes Separation, Eyes Colour.

The domains of these attributes are given below:

VAL_{Name} = {John, Claude, Robert, Nancy, Ingmar, Roberto, Marcel, Johanna, Jurgen, Tom}

$VAL_{Hair\ Coverage}$ = {Full, Receding (Rec.), Bald}

$VAL_{Hair\ Texture}$ = {Straight (Str.), Wavy, Curly}

$VAL_{Eyebrow\ Weight}$ = {Thin, Medium, Bushy}

$VAL_{Eyebrow\ Separation}$ = {Separate (Sep.), Meeting (Meet.), Narrow (Nar.)}

$VAL_{Eyes\ Opening}$ = {Narrow, Medium, Wide}

$VAL_{Eyes\ Separation}$ = {Close, Medium, Wide}

$VAL_{Eyes\ Colour}$ = {Black, Blue, Brown, Green, Hazel}

In the pharantheses the abbreviations used in the table are given. Thus information about the face of Roberto is as follows:

(Hair Coverage, Full)
(Hair Texture, Curly)
(Eyebrow Weight, Thin)
(Eyebrow Separation, Separate)
(Eyes Opening, Narrow)

(Eyes Separation, Medium)
(Eyes Colour, Blue).

Example 2.2.3

Our next example concerns pathomorfological changes in cells and cell organelles.

In table 3 we give after Moore et al. (1977) pathologic state definitions in terms of the general pathology of the growth disorder.

State	Volume Density	Numerical Density	Surface Density
Normal	Normal	Normal	Normal
Proliferation	Normal	Increased	Increased
Hypertrophy	Increased	Normal	Normal
Hyperplasia	Increased	Increased	Increased
Hypoplasia	Decreased	Decreased	Decreased
Atrophy	Decreased	Normal	Decreased
Ageneration	Normal	Decreased	Decreased
Dysplasia	Increased	Decreased	Decreased
Dystrophy	Decreased	Increased	Increased

Tab. 3

Objects in the system are states of cell organelle systems. The organelle systems are characterized by attributes called Volume Density, Numerical Density and Surface Density. The domain of each attribute is {Normal, Increased, Decreased}. The biological meaning of these attributes and their values is immaterial for the purpose of this paper. Let us observe that each pathological state is described by a certain combination of attribute values. In other words each pathological state is determined by the information about this state contained in the table.

Example 2.2.4

In this example we give instances of microorganisms description in the "object, attribute, value" fashion after Michalski et al. (1981). The objects to be described are some microorganisms shown in Figure 1.

There are the following attributes chosen to characterize the microorganisms: Body Parts, Body Spots, Texture, Tail Type.

The domains of these attributes are as follows:

$VAL_{Body\ Parts}$ = {one part, two parts, many parts}

$VAL_{Body\ Spots}$ = {one spot, many spots}

$VAL_{Texture}$ = {blank, striped, crosshatched}

$VAL_{Tail\ Type}$ = {non, single, multiple}

Table 4 gives information about all the microorganisms shown in Figure 1.

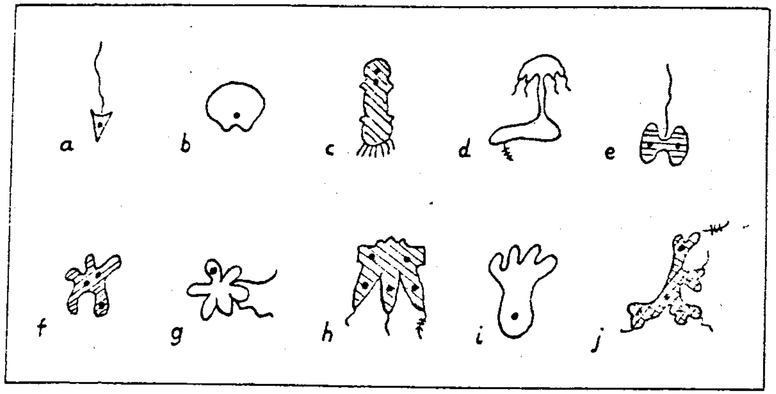

Fig. 1

	Body Parts	Body Spots	Texture	Tail Type
a	one	one	blank	single
b	one	one	blank	none
c	one	many	striped	multiple
d	two	one	blank	multiple
e	two	many	striped	single
f	many	many	striped	none
g	many	one	blank	multiple
h	many	many	striped	multiple
i	many	one	blank	none
j	many	many	crosshatched	multiple

Tab. 4

As in the previous examples information about a microorganism in the table consists of values of all the attributes for this microorganism.

Example 2.2.5

Knowledge in the form of deterministic KR systems often is used in medicine. The medical data used in this example are taken from Warmus (1983). Table 5 contains a sample from a data file of patients suffering from heart disease seen in one of the hospitals in Warsaw.

There are the following attributes in the table: Gasometry, Dyspnea, Cyanosis, Pulmonary Stasis, Heart Rate, Hepatomegaly, Edema, Degree of Disease Advance.

The set of objects in this system consists of patients $P_1, \ldots P_{10}$. The domains of the attributes consists of integers. For example, values of attribute "Heart Rate" are integers ranging from 50 to 250.

	Gasometry	Dyspnea	Cyanosis	Pulmonary Stasis	Heart Rate	Hepato-megaly	Edema	Degree of Disease Advance
P_1	37	1	1	1	62	0	0	1
P_2	43	2	3	4	76	8	3	3
P_3	42	1	2	1	71	1	0	1
P_4	43	0	3	2	80	5	1	1
P_5	48	1	3	3	92	6	3	3
P_6	38	1	3	2	87	5	1	2
P_7	54	0	0	0	95	1	0	2
P_8	40	3	0	0	128	1	0	0
P_9	40	1	0	0	111	1	0	1
P_{10}	50	0	1	0	68	2	1	1

Tab. 5

The last column contains data about a health status of patients. The degree of disease advance increases according to the natural ordering of the values of this attribute. In reality there are six values of this attribute but not all of them occur in the presented part of the table.

To make use of information about objects given by means of a KR system we need a language for expressing explicit knowledge of the system and rules for deriving implicit information. In the following section we present a formal language and a deduction system for the language. The language is intended to express deterministic information.

2.3. Logic DIL of deterministic information

In this section we define a formalized language which is expressive enough to represent a variety of types of deterministic information. Since information about an object in a KR system of deterministic information is a set of attribute - value pairs, expressions of the language include schemes of such pairs as atomic formulas. From atomic formulas we construct compound formulas by using the usual propositional operations.

We can formally define these formulas to be expressions built up from symbols taken from the following nonempty at most denumerable and pairwise disjoint sets:

a set CONAT of constants representing attributes a set CONVAL of constants representing values of attributes

set $\{\neg, \vee, \wedge, \rightarrow, \leftrightarrow\}$ of propositional operations of negation, disjunction, conjunction, implication and equivalence, respectively

set $\{(,)\}$ of brackets.

Set FORDIL of all the formulas is the least set satisfying the following conditions:

$(a\ v) \in$ FORDIL for any $a \in$ CONAT and $v \in$ CONVAL

if $A, B \in$ FORDIL then $\neg A$, $A \vee B$, $A \wedge B$, $A \rightarrow B$, $A \leftrightarrow B \in$ FORDIL.

Formulas are the schemes of sentences expressing information about objects. Each formula will be interpreted as a set of objects obeying the property described by the formula. A formula of the form $(a\ v)$ will be interpreted as the set of objects taking the value denoted by v for the attribute denoted by a. Formula of the form $\neg A$ will correspond to the complement of the set of objects represented by A. Formulas $A \vee B$ and $A \wedge B$ will represent the union and intersection of sets determined by A and B, respectively. By using formulas of the form $A \rightarrow B$ and $A \leftrightarrow B$ we shall express inclusion and equality of sets.

If the following we define semantics of the given language by means of the notions of model and satisfiability of the formulas in a model. By a model determined by a KR system S = (OB, AT, VAL, f) we mean a tuple

$M = (S, m)$

where m: CONAT $\cup$ CONVAL $\rightarrow$ AT $\cup$ VAL is a meaning function such that

m(CONAT) = AT and m(CONVAL) = VAL.

Thus we give an interpretation for formulas by assigning correspondence between elements of the language and entities in the domain of discourse represented by a given KR system. In this way constants representing attributes and values of attributes are considered to be an indication of a whole class of entities, with individual instances supplied by the domain od application. If we give to each constant its proper meaning in a model M = (S, m) then we obtain a sentence stating a property which is meaningful for objects from system S. Some objects may have this property and the other may not. To express this we introduce the notion of satisfiability of formulas by objects. We say that an object o satisfies a formula A in a model M(M, o sat A) iff the following conditions are satisfied:

M, o sat$(a\ v)$ iff $f(o, m(a)) = m(v)$

M, o sat $\neg A$ iff not M, o sat A

M, o sat $A \vee B$ iff M, o sat A or M, o sat B

M, o sat $A \wedge B$ iff M, o sat A and M, o sat B

M, o sat $A \rightarrow B$ iff M, o sat $\neg A \vee B$

M, o sat $A \leftrightarrow B$ iff M, o sat $A \rightarrow B$ and M, o sat $B \rightarrow A$.

According to this definition to each formula A of the language there is associated the set of those objects which satisfy the formula in a model M. We call this set extension of formula A in model M, ($ext_M A$):

$ext_M A = \{o \in OB : M, o \text{ sat } A\}$

The extensions of compound formulas depend on the extensions of their components in the following way.

<u>Fact 2.3.1</u>

(a) $ext_M(a\ v) = \{o \in OB : f(o, m(a)) = m(v)\}$

(b) $ext_M \neg A = -ext_M A$

(c) $ext_M A \vee B = ext_M A \cup ext_M A$

(d) $ext_M A \wedge B = ext_M A \cap ext_M B$

(e) $ext_M A \rightarrow B = -ext_M A \cup ext_M B$

(f) $ext_M A \leftrightarrow B = ext_M A \cap ext_M B \cup (-ext_M A) \cap (-ext_M B)$

Thus an object o satisfies a formula A whenever o has the property described by A, and extension of a formula A consists of the objects possessing the property expressed by A.

We say that a formula A is true in a model M($\models_M A$) iff $ext_M A$ = OB. A formula A is valid ($\models A$) iff it is true in every model. A set T of formulas is satisfied by an object o in a model M(M, o sat T) iff M, o sat A for every formula $A \in T$. A set T is satisfiable if there exists a model M and an object o such that M, o sat T. A formula A is a semantical consequence of a set T of formulas ($T \models A$) iff M, o sat A whenever M, o sat T for every model M and for every object o from the set of objects of M. Formulas A and B are said to be equivalent in a model M iff $ext_M A = ext_M A$. Formulas A and B are equivalent iff they are equivalent in all models.

We now have an easily established fact:

<u>Fact 2.3.2</u>

(a) $\models_M A$ iff $ext_M A$ = OB

(b) $\models_M \neg A$ iff $ext_M A = \emptyset$

(c) $\models_M A \rightarrow B$ iff $ext_M A \subseteq ext_M B$

(d) $\models_M A \leftrightarrow B$ iff $ext_M A = ext_M B$

The lemma shows that we can express inclusion and equality of sets of objects in our language.

We now give a deductive structure to the language. We specify a recursive set of axioms and inference rules. The axioms correspond very closely to the axioms for the classical propositional logic.

Axioms of DIL

A1. $A \to (B \to A)$

A2. $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$

A3. $A \to (\neg A \to B)$

A4. $(\neg A \to A) \to A$

Rule of inference

$\frac{A,\ A \to B}{B}$ modus ponens

The given axioms characterize the operation of negation and implication only, but in our language the remaining propositional operations are definable by means of $\neg$ and $\to$, namely we have the following lemma.

Fact 2.3.3

(a) $ext_M A \vee B = ext_M \neg A \to B$

(b) $ext_M A \wedge B = ext_M \neg(A \to \neg B)$

(c) $ext_M A \leftrightarrow B = ext_M (A \to B) \wedge (B \to A)$

We say that a formula A is derivable from a set T of formulas ($T \vdash A$) iff it is obtainable from the axioms and the formulas from T by repeated application of the rule. A formula A is said to be a theorem of logic DIL($\vdash A$) iff it is derivable from the axioms only. A set T of formulas is consistent iff a formula of the form $A \wedge \neg A$ is not derivable from T.

A logic is said to be sound if every formula A that can be derived from a set T is also a semantical consequence of T. We show that logic DIL has soundness property.

Fact 2.3.4 (Soundness theorem)

(a) $\vdash A$ implies $\models A$

(b) $T \vdash A$ implies $T \models A$

(c) T satisfiable implies T consistent

Proof: The axioms of DIL are easily seen to be valid, and the rule clearly preserves validity. This proves (a), from which (b) and (c) follow immediately.

In the following we list some important theorems and metatheorems of logic DIL. They represent facts that are true in all models, that is the facts expressing those properties of objects which do not depend on a choice of a domain of applications.

Fact 2.3.5

(a) $\vdash \neg\neg A \leftrightarrow A$

(b) $\vdash \neg(A \vee B) \leftrightarrow (\neg A \wedge \neg B)$

(c) $\vdash \neg(A \wedge B) \leftrightarrow (\neg A \vee \neg B)$

(d) $\vdash A \wedge (B \vee C) \leftrightarrow (A \wedge B) \vee (A \wedge C)$

(e) $\vdash A \vee (B \wedge C) \leftrightarrow (A \vee B) \wedge (A \vee C)$

(f) $\vdash (A \to B) \leftrightarrow (\neg B \to \neg A)$

(g) $\vdash (A \to B) \leftrightarrow (\neg A \vee B)$

The given theorems of logic DIL provide a basis for equivalent transformations of formulas.

In many artificial intelligence systems implicational formulas of the form $A_1 \wedge \ldots \wedge A_n \to B$, referred to as production rules, are used. The next lemma shows how we can transform production rules.

Fact 2.3.6

(a) $\vdash (A \to B \vee C) \leftrightarrow (A \to B) \vee (A \to C)$

(b) $\vdash (A \wedge B \to C) \leftrightarrow (A \to C) \vee (B \to C)$

(c) $\vdash (A \vee B \to C) \to ((A \to C) \wedge (B \to C))$

(d) $\vdash (A \to B \wedge C) \leftrightarrow (A \to B) \wedge (A \to C)$

(e) $\vdash (A \to B) \wedge (C \to D) \to (A \vee C \to B \vee D)$

(f) $\vdash A \wedge B \to A \vee B$

Fact 2.3.7

(a) $A \in T$ implies $T \vdash A$

(b) $T \vdash A$ and $T \subseteq Z$ imply $Z \vdash A$

(c) $\vdash A$ implies $T \vdash A$ for any T

(d) $T \vdash A$ and $T \vdash A \to B$ imply $T \vdash B$

(e) $T \cup \{A\} \vdash B$ iff $T \vdash A \to B$

(f) $T \vdash A$ iff $T \cup \{\neg A\}$ is not consistent

A logic is said to be complete if every formula A that is a semantical consequence of a set T can also be derived from T.

In the following we prove the completeness theorem for logic DIL. The proof follows closely the proof of completeness for classical propositional logic. The only difference is that a special canonical KR system should be constructed.

Let T be a consistent set of formulas and let relation $\approx$ be defined as follows:

$A \approx B$ iff $T \vdash A \leftrightarrow B$

Fact 2.3.8

(a) Relation $\approx$ is an equivalence on set FORDIL

(b) Relation $\approx$ is a congruence with respect to operations $\neg$, $\vee$, and $\wedge$.

Let $FORDIL/_{\approx}$ denote the set of all the equivalence classes of relation $\approx$, and let $[A]$ denote the equivalence class of formula A. We consider the algebra

$ADIL = (FORDIL/_{\approx}, -, \cup, \cap, 1, 0)$

where $-[A] = [\neg A]$

$[A] \cup [B] = [A \vee B]$

$[A] \cap [B] = [A \wedge B]$

$1 = [A \vee \neg A]$

$0 = [A \wedge \neg A]$

Fact 2.3.9

(a) Algebra ADIL is a non-degenerate Boolean algebra

(b) $[A] \leq [B]$ iff $T \vdash A \rightarrow B$

(c) $T \vdash A$ iff $[A] = 1$

(d) $[\neg A] \neq 0$ iff not $T \vdash A$

Let FT be the family of all the maximal filters in algebra ADIL. Set FT is non-empty since the algebra is non-degenerate. We define a canonical KR system S_o as follows:

$S_o = (OB_o, AT_o, VAL_o, f_o)$

where OB_o = FT

AT_o = CONAT

VAL_o = CONVAL

$f_o(F,a) = v$ iff $[(a\ v)] \in F$ for any $F \in OB_o$, $a \in AT_o$ and $v \in VAL_o$.

Canonical model M_o determined by the canonical system is defined as follows:

$M_o = (S_o, m_o)$

where $m_o(a) = a$ for $a \in$ CONAT

$m_o(v) = v$ for $v \in$ CONVAL

Fact 2.3.10

The following conditions are equivalent:

(a) M_o, F sat A

(b) $[A] \in F$

Proof: The proof is by induction with respect to the lenght of formula

Case 1. A is (a v)

By the definition of satisfiability M_o, F sat(a v) iff $f_o(F,a) =$ and hence by the definition of the canonical model the theorem hold

Case 2. A is $\neg B$

We have M_o, F sat $\neg B$ iff not M_o, F sat B. By the induction hyp

thesis not $[B] \in F$. Since F is a maximal filter, we have $[\neg B] \in F$.

Case 3. A is $B \rightarrow C$

We have M_o, F sat $B \rightarrow C$ iff not M_o, F sat B or M_o, F sat C. Hence $[\neg B] \in F$ or $[C] \in F$. Since F is a prime filter, we have $[\neg B \vee C] \in F$. By 2.3.5 (g) we have $[B \rightarrow C] \in F$.

We are now ready to prove the completeness theorem.

Fact 2.3.11 (Completeness theorem)

(a) $\models A$ implies $\vdash A$

(b) $T \models A$ implies $T \vdash A$

(c) T consistent implies T satisfiable

Proof: Suppose not $T \vdash A$. By 2.3.9 (d) we have $[\neg A] \neq 0$. Thus there is a maximal filter $F_o \in FT$ such that $[\neg A] \in F_o$. By 2.3.10 we have M_o, F_o sat A. For any formula $B \in T$ we have $T \vdash B$ by 2.3.7 (a) and $[B] = 1$ by 2.3.9 (c). Hence $[B] \in F_o$, but by 2.3.10 M_o, F_o sat B, a contradiction. Condition (a) follows from (b), and condition (c) follows from 2.3.10.

As a corollary we obtain the following

Fact 2.3.12 (Compactness theorem)

The following conditions are equivalent:

(a) T is satisfiable

(b) Every finite subset of T is satisfiable

The material presented in this section provides a tool for defining languages of KR systems. These languages can be considered as linguistic counterparts of the respective systems. They enable us to express explicit information about objects and to infer implicit information.

2.4. Languages of systems of deterministic information

The formalized language defined in section 2.3 provides a means for representing information determined by a KR system. The formulas of the language of logic DIL can be treated as schemes of sentences which express knowledge about objects. In this section we consider languages obtained from the language of DIL by assigning meaning to attribute constants and attribute value constants. In other words with any KR system we associate a language determined by this system.

Let a system S = (OB, AT, VAL, f) and a meaning function m be given such that m(CONAT) = AT and m(CONVAL) = VAL. Let M = (S,m) be the model determined by system S. We consider set FORDIL(S) of formulas of system S. It is the least set containing all the pairs of the form (m(a) m(v)) for a ∈ CONST and v ∈ CONVAL and closed with respect to the operations ¬, ∨, ∧, ⇒, and ⇔. We define satisfiability of formulas of system S by objects of the system and extensions of these formulas by means of the respective notions introduced in logic DIL. For an object o ∈ OB we have

o sat(m(a)m(v)) iff M, o sat(a v)

For compound formulas the inductive definition of satisfiability is the same as in logic DIL. Similarly

ext(m(a)m(v)) = ext_M(a v)

Extensions of compound formulas are defined in a way similar to that followed in 2.3.1 (b) - (f). A formula A ∈ FORDIL(M) is true iff extA = OB.

The formulas from set FORDIL(M) express properties of objects of the given system S. We can describe all the properties of objects which are expressible in terms of information about these objects provided by system S. Consider, for example, the system presented in Hunt, Marin and Stone (1966).

Example 2.4.1

We are given a characterization of various animals in terms of attributes: Size, Animality, and Colour. We have

OB = {A_1, A_2, A_3, A_4, A_5, A_6, A_7}

AT = {Size, Animality, Colour}

VAL_{Size} = {small, medium, large}

$VAL_{Animality}$ = {bear, dog, cat, horse}

VAL_{Colour} = {black, brown}

The information function is given by means of the following table

	Size	Animality	Colour
A_1	small	bear	black
A_2	medium	bear	black
A_3	large	dog	brown
A_4	small	cat	black
A_5	medium	horse	black
A_6	large	horse	black
A_7	large	horse	brown

We have, for example, the following true formulas in the language of the given system:

(Animality bear) ∧ (Size small) ∨
∨ (Animality bear) ∧ (Size medium) ⇔
(Animality bear)

ext(Animality bear) = {A_1, A_2}

(Animality dog) ∧ (Size large) ∨
∨ (Animality horse) ∧ (Size large) ⇔
⇔ (Size large)

ext(Size large) = {A_3, A_6, A_7}

(Size small) ⇒ (Colour black)

ext(Size small) = {A_1, A_4}

ext(Colour black) = {A_1, A_2, A_4, A_5, A_6}

ext(Size small) ⊆ ext(Colour black)

(Size small) ⇔ ¬(Size medium) ∧ ¬(Size large)

ext¬(Size medium) = {A_1, A_3, A_4, A_6, A_7}

ext¬(Size large) = {A_1, A_2, A_4, A_5}

ext¬(Size medium) ∩ ext¬(Size large) = {A_1, A_4}

(Colour black) ∨ (Colour brown)

ext(Colour black) = {A_1, A_2, A_4, A_5, A_6}

ext(Colour brown) = {A_3, A_7}

ext(Colour black) ∪ ext(Colour brown) = OB

Example 2.4.2

Suppose we are given a true formula F in the language of a system with the following sets of attributes and attribute values:

AT = {Profession, Address}

$VAL_{Profession}$ = {programmer, actor, mathematician}

$VAL_{Address}$ = {Warsaw, Paris}

F : (Profession programmer) ∧ (Address Warsaw) ∨
∨ (Profession actor) ∧ (Address Warsaw) ∨
∨ (Profession mathematician) ∧ (Address Paris)

We can see that this formula is true in the following systems S_1 and S_2 with the sets OB_1 and OB_2 of objects, where

OB_1 = {John, Mary, Bob}

OB_2 = OB_1 ∪ {Jill, Robert}

and with the following information functions:

		Profession	Address
f_1	John	programmer	Warsaw
	Mary	actor	Warsaw
	Bob	mathematician	Paris

		Profession	Address
f_2	John	programmer	Warsaw
	Mary	actor	Warsaw
	Bob	mathematician	Paris
	Jill	programmer	Warsaw
	Robert	actor	Warsaw

We conclude that information function characterize objects up to undistinguishable objects and as a consequence formulas do not uniquely determine sets of objects. The problem of indiscernibility in KR systems will be considered in chapter 6.

Example 2.4.3

Consider system S_2 from example 2.4.2 and the following true formulas in the language of this system:

F_1 (Address Warsaw) → (Profession programmer) ∨ (Profession actor)

F_2 (Address Paris) → (Profession mathematician)

F_3 (Address Warsaw) ∨ (Address Paris)

From these formulas we can derive

F_4 (Profession programmer) ∨ (Profession actor) ∨ (Profession mathematician)

by using 2.3.6 (a) and modus ponens rule.

In the next example we show that meta - level knowledge can also be conceptualized according to the schema: syntax-semantics-deduction method, where a semantic representation is provided by a KR system, a syntactic representation is given by means of the language of this system and a deduction method is determined by logic DIL.

Example 2.4.4

Let us assume that a certain KR system S_1 is given and let the following formulas of the language of system S_1 be true:

F_1 $A_1 \wedge A_2 \to B_1$

F_2 $A_3 \wedge A_4 \to B_2$

F_3 $A_5 \to B_3$

F_4 $A_2 \wedge B_2 \to B_4$

F_5 $B_1 \wedge B_3 \to B_5$

F_6 $B_2 \wedge A_5 \to B_6$

F_7 $B_4 \wedge B_6 \to B_5$

F_8 $B_2 \wedge A_2 \to B_7$

F_9 A_2

F_{10} A_3

F_{11} A_4

F_{12} A_5

Furthermore, let us assume that meta - level knowledge about system S_1 is given by means of system S_2 in which we indicate the utility of object - level knowledge. System S_2 is defined as follows:

$OB = \{F_1, \ldots, F_{12}\}$

$AT = \{$Premise$_1$, Premise$_2$, Conclusion, Utility Under Condition$_1$, Utility Under Condition$_2\}$

$VAL_{Premise_1}$, $VAL_{Premise_2}$, $VAL_{Conclusion} \subseteq FORDIL(S_1)$

$VAL_{Utility\ Under\ Condition_1} = VAL_{Utility\ Under\ Condition_2} = \{$definitely useful, probably useful, especially useful, useless, probably useless$\}$

It is easy to see that a formula of the form $A \to B$ is equivalent to $A \wedge (C \vee \neg C) \to B$ and any formula A is equivalent to $C \vee \neg C \to A$. Hence any implicational formula can have a valid formula $C \vee \neg C$ as a premise and any assertional formula can be interpreted as an implicational formula with valid premises. Hence the values of an information function of system S_2 can be defined for all the objects and all the attributes. Let us assume that this function is defined by means of the following table. For the sake of simplicity we use the abbreviation Utility$_1$ and Utility$_2$ for the respective attributes.

	Premise$_1$	Premise$_2$	Conclusion	Utility$_1$	Utility$_2$
F_1	A_1	A_2	B_1	useful	useless
F_2	A_3	A_4	B_2	def. useful	useless
F_3	$C \vee \neg C$	A_5	B_3	def. useful	prob. useless
F_4	A_2	B_2	B_1	useful	useless
F_5	B_1	B_3	B_5	esp. useful	def. useful
F_6	B_2	A_5	B_6	useless	useless
F_7	B_4	B_6	B_5	esp. useful	def. useful
F_8	B_2	A_2	B_7	useful	prob. useless
F_9	$C \vee \neg C$	$C \vee \neg C$	A_2	prob. useful	useless
F_{10}	$C \vee \neg C$	$C \vee \neg C$	A_3	prob. useful	useless

F_{11}	CvIC	CvIC	A_4	def. useful	prob. useless
F_{12}	CvIC	CvIC	A_5	def. useful	prob. useless

Shown below are examples of formulas from set FORDIL(S_2) and their intuitive meaning.

G_1 (Premise$_1$ A_2) v (Premise$_2$ A_2)

ext G_1 = $\{F_1, F_4, F_8\}$

G_2 (Premise$_1$ A_2) v (Premise$_2$ A_2) $\rightarrow$ (Utility$_1$ useful)

ext G_2 = OB

Under condition$_1$ production rules which mention A_2 in their premises are useful

G_3 (Conclusion B_5)

ext G_3 = OB - $\{F_5, F_7\}$

G_4 (Conclusion B_5) $\rightarrow$ (Utility$_2$ probably useless) v (Utility$_2$ useless)

ext G_4 = OB

Under Condition$_2$ production rules which do not mention B_5 in their conclusion are probably useless or useless

If we are interested in indicating a partial ordering of the object level production rules then we can introduce the attributes: First, Last, Before F, After F, etc. for some production rule F. Values of these attributes range over set $\{$yes, no$\}$.

2.5. Summary

In this chapter we presented the three major elements providing the conceptual framework for representation of deterministic information. First, we gave a conceptual counterpart of domain of application. We assumed that a domain consists of a set of objects which are characterized by means of some attributes and values of the attributes. We assumed that information about an object consists of a set of attribute - value pairs such that a single value of each attribute is specified for the object. Second, we defined formal languages which enable us to express information about the objects. Deduction method for these languages was developed, by using logic DIL of deterministic information. The logic was obtained from the classical propositional logic by assuming a special form of atomic propositions.

3. REPRESENTATION OF NONDETERMINISTIC INFORMATION

3.1. Systems of many valued information

In many real situations it is not sufficient to associate a single value of an attribute with an object. For example, if a person knows more than one language, say English, German and Polish, then information about the person should include the three pairs:

(language, English) (language, German) (language, Polish)

To cover such situations we introduce a notion of many - valued KR system. In many - valued systems information about objects is given by means of information relation. That is we assume that many values of an attribute may be associated with an object.

By a many - valued KR system we mean a quadruple

$S = (OB, AT, \{VAL_a\}_{a \in AT}, g)$

where OB, AT, and VAL = $\bigcup_{a \in AT} VAL_a$ are sets of objects, attributes, and values of attributes respectively, and $g \subseteq OB \times AT \times VAL$ is a relation such that if $(o, a, v) \in g$ then $v \in VAL_a$, and for each $o \in OB$ and each $a \in AT$ there is a value $v \in VAL_a$ such that $(o, a, v) \in g$.

By an information about object $o \in OB$ we mean relation $g_o \in AT \times VAL$ such that

$(a, v) \in g_o$ iff $(o, a, v) \in g$

Let us observe that according to the given definitions information about an object includes at least one pair for each attribute of the system.

<u>Example 3.1.1</u>

Consider a KR system which contains facts about languages (Lan) which persons $P_1, P_2, P_3, P_4, P_5, P_6$ speak, and about degrees (Deg) they have. The respective many - valued KR system is defined as follows:

OB = $\{P_1, P_2, P_3, P_4, P_5, P_6\}$

AT = $\{$Lan, Deg$\}$

VAL = $\{$French (F), Hungarian (H), German (D), Swedish (S), Romanian (R)$\}$

VAL_{Deg} = $\{$Bachelar of Science (BS), Master of Science (MS), Philosophy Doctor (PhD)$\}$

The information relation of the system is given by means of the following table.

In the table we use the following notation. In the place of the table labelled by an object o and an attribute a we put all the values v of attribute a such that $(o, a, v) \in g$. For example, information about person P_2 consists of the following pairs:

(Lan H) (Lan R) (Deg BS)

g		Lan	Deg
	P_1	F, D	BS, MS, PhD
	P_2	H, R	BS
	P_3	F, D, S	BS, MS
	P_4	F	BS, MS
	P_5	F, D	BS
	P_6	R	BS

Example 3.1.2

Assume that we are interested in representing knowledge about som patients in a hospital. We are interested among others in illnesses which the patients were through and in medicines they took. These ar examples of attributes which for a given object may assume more than one value.

Similarly, among attributes which characterize medicines we may have attribute Contraindications which usually assumes several value for a given medicine.

In the next section we discuss a generalization of many - valued systems. We consider domains of objects which connot be characterize neither by an information function nor by an information relation, but the only information we can get is a set of possible values of an attribute for an object.

3.2. Systems of nondeterministic information

By a system of nondeterministic information we mean a quadruple

$$S = (OB, AT, \{VAL_a\}_{a \in AT}, f)$$

where OB, AT and VAL_a, for each $a \in AT$, are non-empty sets of objects attributes, and attribute values, respectively,

$$f : OB \times AT \rightarrow P(VAL) \text{ where } VAL = \bigcup_{a \in AT} VAL_a$$

is a total function such that $f(o,a) \subseteq VAL_a$ for every $o \in OB$ and $a \in AT$

Function f is referred to as nondeterministic information functio It does not specify a single value of an attribute for an object. Fo each object there is associated a set of possible values of every at tribute. We do not specify how many values an attribute may take for a given object. Sets $f(o,a)$ are said to be generalized values of at- tribute a.

For any object $o \in OB$ we define function $f_o : AT \rightarrow P(VAL)$ which is referred to as nondeterministic information about object o:

$f_o(a) = V$ iff $f(o,a) = V$ where $V \subseteq VAL$

Example 3.2.1

Consider a criminal data file providing information about some criminals given by a wittness. This information is usually vague. The wittness specifies attributes characterizing the criminals say colour of eyes and age with some tolerance. He suggests that the proper value of the respective attribute belongs to a certain set of values, but he is not able to point out it definitely.

Example 3.2.2

Given below is a part of the table which provides information about some comets.

Comet	A_1	1/a
1899 I	$+2.9 \pm 0.4$	-46 ± 91
1946 I	$+3.0$	-5
1948 I	$+0.8 \pm 0.2$	$+47 \pm 18$
1955 V	$+1.5 \pm 0.8$	-284 ± 289
1975 XI	$+0.8 \pm 0.5$	-154 ± 889

The above table can be treated as a nondeterministic KR system in which the given comets are objects, and they are characterized by attribute A_1, corresponding to nongravitational effects in cometary motion, and reciprocal semimajor axis 1/a. The values of these attributes are obtained by measurement and therefore they cannot be specified exactly. We can only know some intervals of their possible values.

Example 3.2.3

Consider a table in which the accuracy of the comet orbit determination is characterized by a quantity $\frac{1}{2}(L+M+N)$ where the integers L, M, and N depend, among others, on the determination of the osculating 1/a and the span of time covered by the observations

L, M, N	Mean error of 1/a	Time span of observations
6	1 - 4 units	12 - 24 months
5	5 - 20	6 - 12
4	21 - 100	3 - 6
3	101 - 500	1.5 - 3
2	501 - 2500	0.75 - 1.5

In the nondeterministic KR system determined by the given table we have, for example, the following information about the degree 5 of accuracy:

(Mean error 1/a, 5-20) (Time span of observations, 6-12)

This means that to achieve the accuracy 5 one has to measure 1/a with the error not exceeding 20 and possibly greater than 5, and to observe a comet for at least 6 months and possibly not more than 12.

Example 3.2.4

Consider, for example, a system of medical information. Let set OB of objects be a set of diseases, set AT of attributes be the set of some parameters of patient's body e.g. temperature, blood pressure, state of throat etc. Set VAL_a of values of parameter a is a set of possible values of that parameter. For example, $VAL_{temperature}$ is the set of elements of the interval $35^o - 42^c$. For a disease o and a parameter a the set f(o,a) is the set of values of a which may occur during disease o. A nondeterministic information f_o about disease o indicates what are the generalized values of all the attributes for object o.

Given a system S of nondeterministic information, we define binary relations of informational inclusion (in(S)) and informational similarity (sim(S)) in the set OB as follows:

$(o,o') \in in(S)$ iff $f(o,a) \subseteq f(o',a)$ for all $a \in AT$

$(o,o') \in sim(S)$ iff $f(o,a) \cap f(o',a) \neq \emptyset$ for all $a \in AT$.

Hence an object o is informationally included in object o' whenever for every attribute $a \in AT$ the possible values of a for o are among the possible values of a for o'. For example, a disease o is informationally included in a disease o' if the symptoms of o occur during o', or loosely speaking, if disease o' is accompanied by disease o, or if o' may be caused by o. Objects o and o' are informationally similar if for every attribute $a \in AT$ the generalized values of a for o and o' have an element in common.

The following properties of relations in(S) and sim(S) follow immediately from the definition.

Fact 3.2.1

(a) Relation in(S) is reflexive and transitive

(b) Relation sim(S) is reflexive and symmetric.

Fact 3.2.2

(a) $(o_1,o_2) \in in(S)$ implies $(o_1,o_2) \in sim(S)$

(b) $(o_1,o_2) \in sim(S)$, $(o_1,o_3) \in in(S)$, and $(o_2,o_4) \in in(S)$ imply $(o_3,o_4) \in sim(S)$.

Example 3.2.5

Let us consider a KR system S which provides information about some medicines. Each medicine is characterized by means of indications and contraindications of their usage and their possible side effects. We assume that

$OB = \{M_1, M_2, M_3, M_4, M_5, M_6\}$

$AT = \{$Indication, Contraindication, Side Effect$\}$

$VAL_{Indication} = \{i_1, i_2, i_3, i_4\}$

$VAL_{Contraindication} = \{c_1, c_2, c_3\}$

$VAL_{Side\ Effect} = \{e_1, e_2, e_3, e_4\}$

Attribute Indication gives the names of all diseases which can be treated with the corresponding medicine. Attribute Contraindication gives all disease which exclude the use of the medicine. Attribute Side Effects gives information about all possible unwanted effects which might be caused by the medicine.

The nondeterministic information function of the system is defined below:

	Indication	Contraindication	Side Effect
M_1	i_1, i_2, i_3	c_1, c_2	c_1, c_2, c_3
M_2	i_2, i_4	c_2, c_3	c_1, c_2, c_4
M_3	i_4	c_2	c_2, c_3
M_4	i_2, i_3	c_2	c_1, c_3
M_5	i_3	c_1, c_3	c_1, c_2
M_6	i_4	c_2	c_2, c_4

Relation of informational inclusion and informational similarity of the system consist of the following pairs, respectively:

in(S): (M_4, M_1) (M_6, M_2) (M_i, M_i) for i = 1,...,6

sim(S): (M_1, M_2) (M_1, M_4) (M_2, M_3) (M_2, M_4) (M_2, M_6) (M_3, M_4)

(M_i, M_i) for all i = 1,...,6

(M_j, M_i) for (M_i, M_j) listed above

Let us observe that any many - valued KR system can be treated as a particular nondeterministic KR system. Given an information relation $g \subseteq OB \times AT \times VAL$, we can define a nondeterministic information function $f: OB \times AT \rightarrow P(VAL)$ as follows:

$f(o,a) = \{v \in VAL: (o,a,v) \in g\}$

In the next section we present a formal language whose formulas are schemes of sentences expressing properties of objects in system of nondeterministic information. We develop a deductive system for the language based on axiomatization of propositional modal logics (Gabbay (1976)).

3.3. Logic NIL of nondeterministic information

To define formulas of the language of logic NIL we admit the following nonempty, at least denumerable, and pairwise disjoint sets of symbols:

a set CONAT of constants representing attributes

a set CONGVAL of constants representing generalized values of attributes

set $\{\neg, \vee, \wedge, \rightarrow, \leftrightarrow\}$ of classical propositional operations

set $\{\Diamond_1, \Diamond_g, \Diamond, \Box_1, \Box_g, \Box\}$ of unary modal propositional operations

set $\{(,)\}$ of brackets

The modal operations are related to relations in(S) and sim(S). Diamond ($\Diamond$) operators are referred to as possibility operators, and box ($\Box$) operators are treated as necessity operators. Their informal meaning is as follows:

$\Diamond_g$ possibly greater (with respect to informational inclusion)

$\Diamond_1$ possibly less

$\Diamond$ possibly similar (with respect to informational similarity)

$\Box_g$ definitely greater

$\Box_1$ definitely less

$\Box$ definitely similar

Set FORNIL of all formulas is the least set satisfying the following conditions:

(a V)$\in$ FORNIL for any a$\in$ CONAT and V$\in$ CONGVAL

if A,B$\in$ FORNIL then $\neg$A, A$\vee$B, A$\wedge$B, A$\rightarrow$B, A$\leftrightarrow$B$\in$ FORNIL

if A$\in$ FORNIL then $\Diamond_g$A, $\Diamond_1$A, $\Diamond$A, $\Box_g$A, $\Box_1$A, $\Box$A$\in$ FORNIL

Formulas of the form (aV) are called nondeterministic descriptors. Let DESNIL denote the set of all the nondeterministic descriptors.

Formulas are intended to be schemes of sentences providing definitions of sets of objects. For example, a formula of the form (aV) represents the set of those objects for which the set of possible values of attribute denoted by a coincides with the set corresponding to V. Modal operations enable us to express facts connected with informational inclusion and informational similarity of objects. They provide a means for considering Boolean structure of families of generalized

values of attributes. Formula $\Diamond_g$(a V) represents the set of those object which informationally include at least one object assuming V as a value of a. In particular if we consider a system with the single attribute a then this set coincides with the set of those objects o for which V is included in f(o,a). Similarly, formula $\Diamond_1$(a V) corresponds to the set of objects which are informationally included in objects assuming V as a value of a. If a is the only attribute of a system then this set coincides with the set of those objects o for which f(o,a) is included in V. Formula $\Diamond$(a V) represents the set of objects which are informationally similar to some objects assuming V for a.

Semantics of the given language is defined by means of notions of model and satisfiability of the formulas in a model. By a model we mean a system

M = (OB, R, Q, m)

where OB is a non-empty set of objects

R is a reflexive and transitive relation in set OB

Q is a reflexive and symmetric relation in set OB

and moreover R and Q satisfy the condition:

$(o_1, o_2)\in Q$, $(o_1, o_3)\in R$, and $(o_2, o_4)\in R$ imply $(o_3, o_4)\in R$

m: DESNIL$\rightarrow$ P(OB) is a meaning function assigning sets of objects to nondeterministic descriptors.

We say that an object o$\in$OB satisfies a formula A in a model M (M, o sat A) iff the following conditions are satisfied:

M, o sat (a V) iff o$\in$ m(a V)

M, o sat $\neg$A iff not M, o sat A

M, o sat A$\vee$B iff M, o sat A or M, o sat B

M, o sat A$\wedge$B iff M, o sat A and M, o sat B

M, o sat A$\rightarrow$B iff M, o sat $\neg$A$\vee$B

M, o sat A$\leftrightarrow$B iff M, o sat (A$\rightarrow$B)$\wedge$(B$\rightarrow$A)

M, o sat $\Diamond_g$A iff there is an o'$\in$ OB such that (o', o)$\in$R and M, o' sat A

M, o sat $\Diamond_1$A iff there is an o'$\in$ OB such that (o, o')$\in$R and M, o' sat A

M, o sat $\Diamond$A iff there is an o'$\in$ OB such that (o, o')$\in$Q and M, o' sat A

M, o sat $\Box_g$A iff for all o'$\in$ OB if (o', o)$\in$R then M, o' sat A

M, o sat $\Box_1$A iff for all o'$\in$ OB if (o, o')$\in$R then M, o' sat A

M, o sat $\Box$A iff for all o'$\in$ OB if (o, o')$\in$Q then M, o' sat A

To each formula A of the language we assign the set $ext_M A$ (extension of A in M) of those objects which satisfy the formula in a model:

$ext_M A = \{o \in OB: M, o \text{ sat } A\}$

Fact 3.3.1

(a) $ext_M(a\ V) = m(a\ V)$

(b) $ext_M \neg A = -ext_M A$

(c) $ext_M A \vee B = ext_M A \cup ext_M B$

(d) $ext_M A \wedge B = ext_M A \cap ext_M B$

(e) $ext_M A \rightarrow B = -ext_M A \cup ext_M B$

(f) $ext_M A \leftrightarrow B = ext_M A \cap ext_M B \cup (-ext_M A) \cap (-ext_M B)$

(g) $ext_M \Diamond_g A = \{o \in OB$: there is an $o' \in OB$ such that $(o', o) \in R$ and $o' \in ext_M A\}$

(h) $ext_M \Diamond_1 A = \{o \in OB$: there is an $o' \in OB$ such that $(o, o') \in R$ and $o' \in ext_M A\}$

(i) $ext_M \Diamond A = \{o \in OB$: there is an $o' \in OB$ such that $(o, o') \in Q$ and $o' \in ext_M A\}$

(j) $ext_M \Box_g A = ext_M \neg \Diamond_g \neg A$

(k) $ext_M \Box_1 A = ext_M \neg \Diamond_1 \neg A$

(l) $ext_M \Box A = ext_M \neg \Diamond \neg A$.

We say that a formula A is true in a model M ($\models_M A$) iff $ext_M A$ = OB. A formula A is valid ($\models A$) iff it is true in every model. A set T of formulas is satisfied by on object o in a model M (M, o sat T) iff M, o sat A for every formula $A \in T$. A set T is satisfiable iff there exist a model M and an object o such that M, o sat T. A formula A is a semantical consequence of a set T of formulas ($T \models A$) iff M, o sat A whenever M, o sat T for every model M and for every object o from the set of objects of M.

We admit the following axioms and inference rules for logic NIL.

Axioms of NIL

A1. All formulas having the form of tautologies of the classical propositional calculus.

A2. $\Box_g(A \rightarrow B) \rightarrow (\Box_g A \rightarrow \Box_g B)$

A3. $\Box_1(A \rightarrow B) \rightarrow (\Box_1 A \rightarrow \Box_1 B)$

A4. $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$

A5. $A \rightarrow \Box_1 \Diamond_g A$

A6. $A \rightarrow \Box_g \Diamond_1 A$

A7. $\Box_g A \rightarrow A$

A8. $\Box_1 A \rightarrow A$

A9. $\Box A \rightarrow A$

A10. $\Box_g A \rightarrow \Box_g \Box_g A$

A11. $\Box_1 A \rightarrow \Box_1 \Box_1 A$

A12. $A \rightarrow \Box \Diamond A$

A13. $\Box A \rightarrow \Box_g \Box \Box_1 A$

Due to axioms A2, A3, and A4 logic NIL is a normal modal logic. Axioms A5 and A6 show that operation $\Diamond_g$ is inverse with respect to operation $\Diamond_1$. Axioms A7 and A8 provide reflexivity of relations R and Q, respectively. Axioms A9 and A10 provide transitivity of relation R and symmetry of relation Q, respectively.

Rules of inference

R1	$\dfrac{A,\ A \rightarrow B}{B}$	R3	$\dfrac{A}{\Box_1 A}$
R2	$\dfrac{A}{\Box_g A}$	R4	$\dfrac{A}{\Box A}$

Rules R2, R3, and R4 are counterparts of the necessity rule in modal logics.

The given axioms and rules characterize the operations $\neg$, $\rightarrow$, $\Box_g$, $\Box_1$, and $\Box$ only, but it is sufficient due to 3.3.1 (f), (g), (k), (l) and the following

Fact 3.3.2

(a) $ext_M A \vee B = ext_M \neg A \rightarrow B$

(b) $ext_M A \wedge B = ext_M \neg(A \rightarrow \neg B)$

We say that a formula A is derivable from a set T of formulas ($T \vdash A$) iff it is obtainable from the axioms and the formulas from T by repeated application of inference rules. A formula A is said to be a theorem of logic NIL ($\vdash A$) iff it is derivable merely from the axioms. A set T of formulas is consistent if a formula of the form $A \wedge \neg A$ is not derivable from T.

Fact 3.3.3 (Soundness theorem)

(a) $\vdash A$ implies $\models A$

(b) $T \vdash A$ implies $T \models A$

(c) T satisfiable implies T consistent.

Proof: The axioms of NIL are easily seen to be valid, and rules clearly preserve validity. This proves (a) from which (b) and (c) follow immediately.

Examples of theorems of logic NIL are presented below.

Fact 3.3.4

(a) $\vdash A \to \Diamond_g A$

(b) $\vdash \Diamond_g(A \vee B) \leftrightarrow (\Diamond_g A \vee \Diamond_g B)$

(c) $\vdash \Diamond_g(A \wedge B) \to (\Diamond_g A \wedge \Diamond_g B)$

(d) $\vdash \Box_g(A \wedge B) \leftrightarrow (\Box_g A \wedge \Box_g B)$

(e) $\vdash \Box_g A \to \Diamond_g \Box_g A$

(f) $\vdash \Diamond_g \Box_g A \to A$

(g) $\vdash \neg \Box_g A \leftrightarrow \Diamond_g \neg A$

(h) $\vdash \Diamond \Box A \to A$

(i) $\vdash \neg \Box A \leftrightarrow \Diamond \neg A$

(j) $\vdash \Box_1 A \wedge \Diamond_g B \to \Diamond_g(A \wedge B)$

(k) $\vdash \Box_1 A \wedge \Diamond_1 B \to \Diamond_1(A \wedge B)$

Theorems related to operations $\Diamond_1$ and $\Box_1$ are similar to that presented in (a),...,(g). They can be obtained through replacement of $\Diamond_g$ by $\Diamond_1$ and $\Box_g$ by $\Box_1$.

In the following completeness theorem for logic NIL will be presented. Let T be a consistent set of formulas and let relation $\approx$ in set FORNIL be defined as follows:

$A \approx B$ iff $T \vdash A \leftrightarrow B$

Fact 3.3.5

(a) Relation $\approx$ is an equivalence on set FORNIL

(b) Relation $\approx$ is a congruence with respect to $\neg$, $\vee$, and $\wedge$.

(c) If $A \approx B$ then $\Box_g A \approx \Box_g B$, $\Box_1 A \approx \Box_1 B$, and $\Box A \approx \Box B$.

The proof of conditions (a) and (b) is the same as for the classical propositional logic (Rasiowa and Sikorski (1970)). Condition (c) follows from A2, A3, A4, and necessity rules.

We construct the quotient algebra

ANIL = (FORNIL/$\approx$, $-$, $\cup$, $\cap$, 1, 0)

where FORNIL/$\approx$ is the set of the equivalence classes $[A]$ of relation $\approx$ for all formulas A

$-[A] = [\neg A]$

$[A] \cup [B] = [A \vee B]$

$[A] \cap [B] = [A \wedge B]$

$1 = [A \vee \neg A]$

$0 = [A \wedge \neg A]$

Fact 3.3.6

(a) Algebra ANIL is a nondegenerate Boolean algebra

(b) $[A] \leq [B]$ iff $T \vdash [A \to B]$

(c) $T \vdash A$ iff $[A] = 1$

(d) $[\neg A] \neq 0$ iff not $T \vdash A$

Let FT be the family of all the maximal filters in algebra ANIL. Set FT is nonempty since the algebra is nondegenerate. We define relation $R_o \subseteq FT \times FT$ as follows:

$(F,G) \in R_o$ iff for any formula A if $[\Box_1 A] \in F$ then $[A] \in G$.

Fact 3.3.7

The following conditions are equivalent:

(a) $(F,G) \in R_o$

(b) If $[\Box_g A] \in G$ then $[A] \in F$

(c) If $[A] \in F$ then $[\Diamond_g A] \in G$

(d) If $[A] \in G$ then $[\Diamond_1 A] \in F$

Proof: Assume condition (a), and suppose $[\Box_1 A] \in G$ and $[A] \notin F$. It follows that $[\neg A] \in F$ and by A5 $[\Box_1 \Diamond_g \neg A] \in F$. By (a) we obtain $[\Diamond_g \neg A] \in G$. By 3.3.4 (j) we have $[\Diamond_g(A \wedge \neg A)] \in G$, but G is a proper filter, a constradiction. Hence condition (b) holds.

Let us now assume that condition (b) holds and suppose $[A] \in F$ and $[\Diamond_g A] \notin G$. Hence $[\Box_g \neg A] \in G$ and by (b) we have $[\neg A] \in F$, a contradiction. Hence condition (c) holds.

Assume condition (c) and suppose $[A] \in G$ and $[\Diamond_1 A] \notin F$. Then $[\neg \Diamond_1 A] \in F$ and by (c) we have $[\Diamond_g \neg \Diamond_1 A] \in G$. By A6 and 3.3.4 (g) we have $[\neg A] \in G$, a contradiction. Hence condition (d) holds.

We also have (d) implies (a). For suppose not, then $[\neg A] \in G$, and by (d) $[\Diamond_1 \neg A] \in F$. By 3.3.4 (k) we have $[\Diamond_1(A \wedge \neg A)] \in F$, a contradiction.

We define relation $Q_o \subseteq FT \times FT$ as follows

$(F,G) \in Q_o$ iff for any formula A if $[\Box A] \in F$ then $[A] \in G$

Fact 3.3.8

(a) Relation R_o is reflexive and transitive

(b) Relation Q_o is reflexive and symmetric

(c) If $(F_1,F_2) \in Q_o$, $(F_1,F_3) \in R_o$, and $(F_2,F_4) \in R_o$ then $(F_3,F_4) \in Q_o$

Proof: Condition (a) follows from A7, A8, A10, A11. Condition (b) follows from A9 and A12. Condition (c) follows from A13.

Fact 3.3.9

(a) If $[\Diamond_g A] \in F$ then there exists a $G \in FT$ such that $(F,G) \in R_o$ and $[A] \in G$

(b) If $[\Diamond_1 A] \in F$ then there exists a $G \in FT$ such that $(G,F) \in R_o$ and $[A] \in G$

(c) If $[\Diamond A] \in F$ then there exists a $G \in FT$ such that $(F,G) \in Q_o$ and $[A] \in$ $\in G$.

Proof: Let $[\Diamond_1 A] \in F$ and consider set $X_F = \{[B] : [\Box_1 B] \in F\}$. Set X_F is nonempty, since $1 \in X_F$. Consider filter F' generated by set $X_F \cup \{[A]\}$. We have $F' = \{[B]$: there exist $[A_1], \ldots, [A_n] \in X_F$, $n \geq 1$, such that $[A_1] \wedge \ldots \wedge [A_n] \wedge [A] \leq [B]\}$. We shall show that for any $[A_1], \ldots, [A_n] \in X_F$ we have $[A_1] \wedge \ldots \wedge [A_n] \wedge [A] \neq 0$. Suppose conversely, then $T \vdash A_1 \wedge \ldots \wedge A_n \rightarrow A$. By A3 and R3 we have $T \vdash \Box_1(A_1 \wedge \ldots \wedge A_n) \rightarrow \Box_1 \neg A$. Since $[\Box_1 A_1], \ldots, [\Box_1 A_n] \in F$, we have $[\Box_1 A_1 \wedge \ldots \wedge \Box_1 A_n] \in F$. Since $\vdash \Box_1 A \wedge \Box_1 B \leftrightarrow \Box_1 (A \wedge B)$, we have $[\Box_1(A_1 \wedge \ldots \wedge A_n)] \in F$. Hence $[\Box_1 \neg A] \in F$, so $[\neg \Diamond_1 A] \in F$, what contradicts the assumption. Thus filter F' is proper. Let G be the maximal filter containing F'. We clearly have $[A] \in G$ and $(F,G) \in R_o$. Hence condition (a) is satisfied. The proof of conditions (b) and (c) is similar.

We define canonical model M_o as follows:

$M_o = (OB_o, R_o, Q_o, m_o)$

where $OB_o = FT$

R_o and Q_o are relations defined above

$F \in m_o(aV)$ iff $[(aV)] \in F$.

Fact 3.3.10

The following conditions are equivalent:

(a) M_o, F sat A

(b) $[A] \in F$

Proof: If A is of the form (aV) then the theorem holds by the definition of meaning function m_o in the canonical model. If A is of the form $\neg B$ or $B \rightarrow C$ we use the definition of satisfiability and the fact that filter F is maximal and prime. If A is of the form $\Diamond_g B$ or $\Diamond_1 B$ then the theorem follows from 3.3.7 and 3.3.9 (a) and (b). If A is of the form $\Diamond B$ then we use 3.3.9 (c). Now consider a formula of the form $\Box_1 A$ and suppose that M_o, F sat $\Box_1 A$ and $[\Box_1 A] \notin F$. Hence $[\Diamond_1 \neg A] \in F$ and M_o, F sat $\Diamond_1 \neg A$. Thus M_o, F sat $\neg \Box_1 A$, a contradiction. Suppose now $[\Box_1 A] \in F$ and consider set $X_F = \{[B] : [\Box_1 B] \in F\}$. We have $[A] \in X_F$. Moreover, set X_F is a filter, since we have $[B]$ and $[C] \in X_F$ iff $[B] \wedge [C] = [B \wedge C] \in X_F$ for any formulas B and C. Set X_F is a proper

filter, since $0 \notin X_F$. By Kuratowski - Zorn lemma there is a maximal filter G such that $(F,G) \in R_o$ and $[A] \in G$. But X_F is included in every filter G such that $(F,G) \in R_o$, thus $[A]$ belongs to every such filter. By the induction hypothesis we have M_o, G sat A for all G satisfying $(F,G) \in R_o$. Hence M_o, F sat $\Box_1 A$. For formulas of the form $\Box_g A$ and $\Box A$ the proof is similar.

Lemma 3.3.10 enables us to prove completeness and compactness of logic NIL.

Fact 3.3.11 (Completeness theorem)

(a) $\models A$ implies $\vdash A$

(b) $T \models A$ implies $T \vdash A$

(c) T consistent implies T satisfiable.

Proof: We now prove condition (b). Suppose not $T \vdash A$. By 3.3.6 (d) we have $[\neg A] \neq 0$. Thus there is a maximal filter $F_o \in FT$ such that $[\neg A] \in F_o$. By 3.3.10 we have M_o, F_o sat $\neg A$. For any formula $B \in T$ we have $T \vdash B$ by 3.3.6 (c). Hence $[B] \in F_o$ and by 3.3.10 M_o, F_o sat B, a contradiction. Condition (a) follows from (b), and condition (c) follows from 3.3.10.

As a corollary we obtain

Fact 3.3.12 (Compactness theorem)

The following conditions are equivalent:

(a) T is satisfiable

(b) Every finite subset of T is satisfiable.

Deductive methods based on logic NIL enable us to determine when a formula expressing a property of objects is implied by some other formulas. In NIL all the tautologies of classical logic are valid and hence its deductive power is not less than that of the classical logic. The modal operations enable us to reason in the presence of nondeterminism understood as indefiniteness of information about objects. These operations enable us to penetrate in a sense a Boolean structure of families of generalized values of attributes. In the next section we discuss languages of systems of nondeterministic information based on logic NIL.

3.4. Languages of systems of nondeterministic information

Let S = (OB, AT, VAL, f) be given, and let in(S) and con(S) be relations of informational inclusion and informational similarity

lations of informational inclusion and informational similarity deter-
mined by system S. Let n be the function

$n : CONAT \cup CONGVAL \rightarrow AT \cup P(VAL)$

such that $n(CONAT) = AT$

the range of function f is included in $n(CONGVAL)$

Function n is referred to as naming function. It assigns attribute to attribute constants and generalized values of attributes to the con stants of generalized values.

We consider model

$M = (OB,\ in(S),\ con(S),\ m)$

where $m(a\ V) = \{o : f(o,n(a)) = n(V)\}$

Thus the meaning function m assigns sets of objects to nondeterministic descriptors according to the information about these objects

We can now define the set FORNIL(S) of formulas of system S to be the least set containing all pairs of the form $(n(a)\ n(V))$ for any a $\in$ CONAT and $V \in$ CONGVAL and closed with respect to operations $\neg,\ \vee,\ \wedge,$ $\rightarrow,\ \leftrightarrow,\ \Diamond_g,\ \Diamond_1,\ \Diamond,\ \Box_g,\ \Box_1,\ \Box$.

In a natural way we define satisfiability of formulas of system S by objects of the system, and extensions of formulas, namely

o sat $(n(a)\ n(V))$ iff M, o sat $(a\ V)$

$ext(n(a)\ n(V)) = ext_M(a\ V)$

For compound formulas the respective inductive definitions are sim ilar to that presented in section 3.3.

A formula $A \in FORNIL(S)$ is true iff $extA = OB$.

By using formulas from set FORNIL(S) we can express many properti of objects which are characterized by nondeterministic information.

Fact 3.4.1

(a) If $\Diamond_g(n(a)\ n(V))$ is true then $n(V) \subseteq f(o,n(a))$ for all $o \in OB$

(b) If $\Diamond_1(n(a)\ n(V))$ is true then $f(o,n(a)) \subseteq n(V)$ for all $o \in OB$

(c) If $\Diamond\ (n(a)\ n(V))$ is true then $f(o,n(a)) \subseteq n(V) \neq \emptyset$ for all $o \in OB$

Proof: Formula $\Diamond_g(n(a)\ n(V))$ is true iff each object o in a given system has associated with it a certain object o′ which is informationally included in o and assumes generalized value n(V) of attribu n(a). It follows that n(V) is a subset of a generalized value of attribute n(a) for object o. In a similar way conditions (b) and (c) c be easily seen.

Fact 3.4.2

For any system such that $AT = \{a\}$ the following conditions are sa

isfied:

(a) If ext $\Diamond_g(n(a)n(V)) \neq \emptyset$ then there is an object assuming generalized value n(V) for attribute n(a) and it is possible that there are objects assuming supersets of n(V) for n(a)

(b) If ext $\Diamond_1(n(a)n(V)) \neq \emptyset$ then there is an object assuming n(V) for n(a) and it is possible that there are objects assuming subsets of n(V) for n(a)

(c) If ext $\Box_g(n(a)n(V)) \neq \emptyset$ then there is an object assuming n(V) for n(a) and there are no objects assuming supersets of n(V) for n(a)

(d) If ext $\Box_1(n(a)n(V)) \neq \emptyset$ then there is an object assuming n(V) for n(a) and there are no objects assuming subsets of n(V) for n(a).

Example 3.4.1

Let us consider the following system of medical information:

$OB = \{D_1, \ldots, D_6\}$ is a set of diseases

$AT = \{a,b\}$ is a set of symptoms occurring during diseases from OB

$VAL_a = \{v_1,\ v_2,\ v_3,\ v_4,\ v_5\}$

$VAL_b = \{u_1,\ u_2,\ u_3\}$

$VAL = VAL_a \cup VAL_b$

$f : OB \times AT \rightarrow P(VAL)$ is given by the following table

	a	b
D_1	$\{v_1,\ v_3\}$	$\{u_1,\ u_2,\ u_3\}$
D_2	$\{v_2,\ v_5\}$	$\{u_1\}$
D_3	$\{v_1,\ v_3,\ v_4\}$	$\{u_1,\ u_2\}$
D_4	$\{v_1\}$	$\{u_1,\ u_2\}$
D_5	$\{v_1,\ v_3\}$	$\{u_1\}$
D_6	$\{v_5\}$	$\{u_1\}$

The relation of informational inclusion of the given system consists of the following pairs of diseases:

all the pairs $(D_i,\ D_i)$ for $i = 1,\ldots,6$

$(D_4,\ D_1)$ $(D_5,\ D_1)$ $(D_6,\ D_2)$ $(D_4,\ D_3)$ $(D_5,\ D_3)$

In the following we list extensions of some formulas of the language of the system and we give their intuitive interpretation.

ext $\Diamond_g(a\{v_1\}) = \{D_1,\ D_3,\ D_4\}$

Diseases D_1, D_3, and D_4 can be caused by a disease in which symptom a assumes value v_1: in other words if a patient suffers from one of diseases D_1, D_3 or D_4 then he possibly suffered from a disease satisfying $(a\{v_1\})$.

$$ext \Diamond_1(a\{v_1, v_3, v_4\}) = \{D_3, D_4, D_5\}$$

Diseases D_3, D_4, and D_5 are possibly followed by a disease in which possible values of a are among v_1, v_3, and v_4; or if a patient suffers from D_3, D_4 or D_5 then he will possibly suffer from a disease satisfying $(a\{v_1, v_3, v_4\})$.

$$ext \Box_5(b\{u_1\}) = \{D_2, D_6\}$$

Each disease causing D_2 or D_6 assumes value u_1 of symptom a_2.

$$ext \Box_1(b\{u_1, u_2\}) = \{D_3\}$$

Each disease caused by D_3 assumes u_1 or u_2 for symptom b.

Let us observe that

$$ext \Diamond_g(a\{v_3\}) = \emptyset$$

since in our system there is no object which assumes generalized value $\{v_3\}$ of attribute a. This means that although in our system $\{v_3\}$ is a subset of generalized values of a for diseases D_1, D_3, and D_5, knowledge provided by the system does not enable us to point out a disease which satisfies $(a\{v_3\})$ and possibly causes diseases D_1, D_3 or D_5.

The relation of informational similarity of the given system consists of the following pairs:

all the pairs (D_i, D_i) for $i = 1,\dots,6$

(D_1, D_3) (D_1, D_4) (D_1, D_5) (D_2, D_6) (D_3, D_4) (D_3, D_5) (D_4, D_5)

all the pairs (D_i, D_j) for (D_j, D_i) given above.

Consider, for example, the following extensions:

$$ext \Diamond(a\{v_1, v_3\}) = \{D_1, D_3, D_4, D_5\}$$

For diseases D_1, D_3, D_4, and D_5 there are diseases informationally similar to them which may take v_1 or v_3 as the values of symptom a.

$$ext \Box(b\{u_1\}) = \{D_2, D_6\}$$

All diseases similar to D_2 or D_6 in the sense of informational similarity may assume u_1 for attribute b.

In the next example we show that in some cases meta-level knowledge can be specified by using nondeterministic KR systems and their languages

Example 3.4.2

Assume that we are given a KR system S_1 such that the following formulas of the language of S_1 are true:

F_1 $A_1 \wedge A_2 \rightarrow B_1 \wedge B_2 \wedge B_3$

F_2 $A_3 \wedge A_4 \rightarrow B_2$

F_3 $A_1 \wedge A_2 \wedge A_3 \rightarrow B_1 \wedge B_2 \wedge B_4$

F_4 $B_1 \rightarrow B_5$

F_5 $A_1 \wedge A_3 \wedge B_1 \rightarrow B_4 \wedge B_5$

F_6 $A_3 \wedge A_4 \rightarrow B_2 \wedge B_3$

F_7 $A_3 \wedge A_4 \wedge B_1 \rightarrow B_2 \wedge B_3 \wedge B_5$

We assume that for each of the above formulas an importance measure is determined with respect to a certain condition. The importance measure takes values from the real interval $\langle 0; 1 \rangle$. We define the respective nondeterministic KR system S_2 as follows:

$OB = \{F_1, \dots, F_7\}$

$AT = \{$Premises, Conclusions, Importance Measure$\}$

$VAL_{Premises}, VAL_{Conclusions} \subseteq FORDIL(S_1)$

$VAL_{Importance\ Measure} = \langle 0; 1 \rangle$

Nondeterministic information function in our system is defined as follows:

	Premises	Conclusions	Importance Measure
F_1	A_1, A_2	B_1, B_2, B_3	0.7 - 1
F_2	A_3, A_4	B_2	0.5 - 0.6
F_3	A_1, A_2, A_3	B_1, B_2, B_4	0.6 - 0.9
F_4	B_1	B_5	0.5 - 0.7
F_5	A_1, A_3, B_1	B_4, B_5	0.5 - 0.8
F_6	A_3, A_4	B_2, B_3	0.4 - 0.8
F_7	A_3, A_4, B_1	B_1, B_3, B_5	0.5 - 0.9

Relations of informational inclusion and informational similarity of S_2 consist of the following pairs of formulas, respectively:

$in(S_2)$: (F_2, F_6) (F_2, F_7) (F_4, F_5) (F_4, F_7) (F_6, F_7) (F_i, F_i) for $i = 1,\dots,7$

$sim(S_2)$: (F_1, F_3) (F_2, F_3) (F_2, F_6) (F_2, F_7) (F_3, F_5) (F_3, F_6)
(F_5, F_7) (F_4, F_5) (F_4, F_7) (F_5, F_7) (F_6, F_7) (F_i, F_j)
for (F_j, F_i) given above

Listed below are extensions of some formulas from set FORNIL(S_2) and their intuitive interpretation.

ext $\Diamond_1$(Conclusions $\{B_2, B_3, B_5\}$) = $\{F_2, F_4, F_6, F_7\}$

Each production rules among F_2, F_4, F_6, and F_7 is informationally less than a certain rule having conclusions B_2, B_3 and B_5: this means that both their premises, conclusions, and importance measures are included in the generalized values of the respective attributes of this rule.

ext $\Diamond_g$(Importance Measure 0.5 - 0.6) = $\{F_2, F_6, F_7\}$

Importance measure intervals of rules F_2, F_6, and F_7 include interval 0.5 - 0.6, and moreover the remaining attributes have their generalized values not less than the respective attributes of a certain rule with importance measure between 0.5 and 0.6; it follows that F_2, F_6, and F_7 are possibly more important than 0.5 - 0.6

ext $\Diamond$(Conclusions $\{B_2\}$) = $\{F_1, F_2, F_3, F_6, F_7\}$

Each rule F_1, F_2, F_3, F_6 and F_7 are similar to certain rule having B_2 as a Conclusion: in other words these rules have B_2 among their conclusions and moreover both their premises and importance measure intervals have an element in common with generalized values of the respective attributes of this rule.

We now conclude that by using modal operations of the language we can express those relationships between objects of a system which are determined by the algebraic structure of the family of generalized values of attributes. These relationships are not stated explicitly in a KR system, that is on the level of semantics we do not have a direct access to them. However, we can have a manipulatory access to these relationships by using the language based on logic NIL.

3.5 Summary

In this chapter we focused on determining the appropriate concepts to represent knowledge about domains in which objects are characterized nondeterministically. We introduced a notion of nondeterministic KR system which corresponds to semantic view of knowledge. Next, we introduced a special modal logic providing a means for defining languages

of nondeterministic KR systems and we developed deduction methods for the logic. The crucial difference between deterministic and nondeterministic systems, is that the first refer to domains in which characterization of objects is given by means of some attributes and their values while the second correspond to domains in which knowledge about objects is incomplete in a sense. Incompleteness is understood as a lack of definite information about values of attributes for the objects in the domain. It follows that we are not able to characterize the objects precisely.

The logic of nondeterministic information considered in this chapter is an extension of the logic of deterministic information. The language is augmented by modal operations and deduction method enables us to reason in the presence of these modalities. The modal operators provide a means for comparing objects with respect to informational inclusion and informational similarity.

4. REPRESENTATION OF TEMPORAL INFORMATION

4.1. Systems of temporal information

In the previous chapters two major forms of knowledge were considered. However, in these approaches knowledge was regarded as representing a static information about some parts of reality. But static knowledge is insufficient to model reality correctly. A KR system cannot be seen as a collection of information items which represent only the current state of knowledge.

In this section we introduce a semantic component of representation of temporal information. The task is then to determine a set of conceptual primitives which are expressive enough to model temporal information. As previously we assume that objects, attributes and values of attributes are atomic pieces of information. Moreover we admit a set TM of moments of time and a linear order R in this set as components of each KR system of temporal information. We shall consider the time dimension in its full generality, that is we consider set TM to be an arbitrary linearly ordered set. If decisions must be made resticting a number of time versions of information items then it will be possible to add the respective assumptions for set TM and relation R and then to supply the corresponding axioms for the logic of temporal information.

We can formally define a KR system of temporal information to be a system

$$S = (OB, AT, \{VAL_a\}_{a \in AT}, TM, R, f)$$

where OB is a nonempty set of objects

AT is a nonempty set of attributes

VAL_a is a nonempty set of values of attribute a

TM is a nonempty set whose elements are called moments of time

$R \subseteq TM \times TM$ is a linear order

$f : OB \times TM \times AT \to VAL = \bigcup_{a \in AT} VAL_a$ is an information function such

that $f(o,t,a) \in VAL_a$ for each $o \in OB$, $t \in TM$, and $a \in AT$.

By means of the information function each object has associated with it a characteristic feature at a moment t. This characteristic feature is determined by a set of attribute - value pairs which may change over time. We define an information about object o at a moment t to be a function $f_{ot} : AT \to VAL$ such that

$$f_{ot}(a) = v \text{ iff } f(o,t,a) = v$$

Example 4.1.1

Consider the part of the table containing the results of photoelectric observations of stars, presented in the Astrophysical Journal.

	J.D	V	B-V
S C Mi	1688.788	11.12	1.97
R Cne	1798.538	9.28	1.76
R Cne	1719.750	3.38	1.47
	1800.558	9.51	2.02
R Ieo	1688.821	6.27	1.62
	1833.481	9.91	2.87
T Cen	1687.826	6.05	1.44
	1717.816	6.12	1.73

The above table can be treated as a temporal KR system such that:

the set OB of objects consists of the stars: S Canis Minoris, R. Canori, R Leonis and T Centauri

the set AT of attributes consists of the two wavelenght regions of the spectrum: visual and blue-visual

the set VAL of values of attributes consists of the magnitude of a star in the given wavelenght regions

the set TM of moments of time consists of nonnegative real numbers, representing Julian Days, given in the second column of the table

the information function is determined by the given table.

Example 4.1.2

Presented below is a part of the table from The Biochemical Journal determining a time course of $[^{35}S]$ cysteine incorporation into metallothionein and high - molecular weight proteins after exposure to Cd or Hg

Time(h)		High-molecular--weight Proteins	Metallothionein
1	Control	6854±739	1005±115
	Cd	5923±682	901±38
	Hg	6940±787	972±84
12	Control	8709±581	1307±49
	Cd	10786±798	4111±170
	Hg	9685±348	1841±51

We can represent this table as the following KR system:

OS = {Control, Cd, Hg}

AT = {High-molecular-weight Proteins, Metallothionein}

VAL is a family of subsets of the set of integers including the subsets given in the third and fourth columns of the table

TM = {1, 12}

R is the natural order in the set of integers

Information function can easily be reconstructed from the table

Example 4.1.3

Consider a system which consists of the examination results of some patients in a hospital.

$OB = \{P_1, P_2, P_3\}$

$AT = \{$Temperature, Ache, Nausea$\}$

$VAL_{Temperature} = \{$below normal, normal, slightly above normal, high above normal$\}$

$VAL_{Ache} = \{$none, present$\}$

$VAL_{Nausea} = \{$no, yes$\}$

$TM = \{$83/05/01, 83/05/04, 83/05/06$\}$

R is the earlier - later relation in the set TM

Information function f is given by the following table:

	Date of examination	Temperature	Ache	Nausea
P_1	83/05/01	high above normal	present	yes
	83/05/04	slightly above normal	present	no
	83/05/06	normal	none	no
P_2	83/05/04	high above normal	present	no
	83/05/06	below normal	present	no
P_3	83/05/01	normal	none	yes
	83/05/04	normal	present	yes

In the given system information about patient P_1 at the 4th May 83 consists of the following pairs:

(Temperature, slightly above normal)

(Ache, present)

(Nausea, no)

In the following section a logic of temporal information is introduced providing a complete notation for dealing with the definition of time varying information and for temporal reasoning. We introduce the tense operators which enable us to recall any past state or any future state of an object with respect to a given state.

4.2. Logic of temporal information

In this section we introduce a logic TIL called temporal informa-

tion logic. The language of logic TIL is a propositional language with the modal tense operations. Formulas of the language are built up from symbols taken from the following nonempty at least denumerable and pairwise disjoint sets:

a set CONOB of constants representing objects

a set CONAT of constants representing attributes

a set CONVAL of constants representing values of attributes

set $\{\neg, \vee, \wedge, \rightarrow, \leftrightarrow\}$ of classical propositional operations

set $\{\Diamond_p, \Diamond_f, \Box_p, \Box_f\}$ of modal propositional operations

set $\{(,)\}$ of brackets

The informal meaning of the modal operations is as follows:

$\Diamond_p$ possibly in the past

$\Diamond_f$ possibly in the future

$\Box_p$ definitely in the past

$\Box_f$ definitely in the future

Set FORTIL of formulas of the language is the least set satisfying the following conditions:

$(o\ a\ v) \in$ FORTIL for any $o \in$ CONOB, $a \in$ CONAT, $v \in$ CONV

if $A, B \in$ FORTIL then $\neg A$, $A \vee B$, $A \wedge B$, $A \rightarrow B$, $A \leftrightarrow B \in$ FORTIL

if $A \in$ FORTIL then $\Diamond_p A$, $\Diamond_f A$, $\Box_p A$, $\Box_f A \in$ FORTIL

The formulas of the language are intended to represent information about objects provided by a temporal KR system. In particular, if o, a, and v represent a certain object, attribute and attribute value in a given KR system, then formula (o a v) express the following proposition: object o assumes value v of attribute a. However, in a temporal KR system the truth or the falsity of such a statement depends on the moment of time. Hence semantics of the language should enable us to express time dependencies of statements.

We define semantics of logic TIL by means of a notion of model determined by a temporal KR system. By a model we mean a tuple

$M = (S, m)$

where $S = (OB, AT, VAL, TM, R, f)$ is a temporal KR system

m : CONOB $\cup$ CONAT $\cup$ CONVAL $\rightarrow$ OB $\cup$ AT $\cup$ VAL is a meaning function such that

$m(CONOB) = OB$, $m(CONAT) = AT$, and $m(CONVAL) = VAL$.

We now define satisfiability of the formulas in a model. We say that a formula A is satisfied in a model M at a moment $t \in TM$ (M, t, sat A) iff the following conditions are satisfied:

M, t sat (o a v) iff $f(m(o), t, m(a)) = m(v)$

M, t sat $\neg A$ iff not M, t sat A
M, t sat $A \vee B$ iff M, t sat A or M, t sat B
M, t sat $A \wedge B$ iff M, t sat A and M, t sat B
M, t sat $A \rightarrow B$ iff M, t sat $\neg A \vee B$
M, t sat $A \leftrightarrow B$ iff M, t sat $(A \rightarrow B) \wedge (B \rightarrow A)$
M, t sat $\Diamond_p A$ iff there is an $s \in TM$ such that $(s,t) \in R$ and M, s sat A
M, t sat $\Diamond_f A$ iff there is an $s \in TM$ such that $(t,s) \in R$ and M, s sat A
M, t sat $\Box_p A$ iff for all $s \in TM$ if $(s,t) \in R$ then M, s sat A
M, t sat $\Box_f A$ iff for all $s \in TM$ if $(t,s) \in R$ then M, s sat A

According to the given semantics a formula of the form $(o\ a\ v)$ is satisfied by t whenever an information about the object denoted by o at moment t includes the pair $(m(a),m(v))$. A formula of the form $\Box_p(o\ a\ v)$ is satisfied by t if information about object $m(o)$ at all the moments earlier than t includes $(m(a),m(v))$. Similarly, a formula of the form $\Diamond_f(o\ a\ v)$ is satisfied by t if information about object $m(o)$ at a certain moment later than t includes $(m(a),m(v))$.

A formula A is true in a model M iff for all $t \in TM$ we have M, t sat A. A formula A is valid ($\models A$) iff it is true in all models. A set T of formulas is said to be satisfied in a model at a moment t(M, t sat T) iff M, t sat A for every formula $A \in T$. A set T is satisfiable iff M, t sat T for some model M and moment t. A formula A is a semantical consequence of set T of formulas ($T \models A$) if M, t sat A whenever M, t sat T for every M and t.

We give a deductive structure to the language of logic TIL in the usual way, specifying a recursive set of axioms and inference rules.

Axioms of TIL

A1. All formulas having the form of tautologies of the classical propositional calculus,

A2. $\Box_f(A \rightarrow B) \rightarrow (\Box_f A \rightarrow \Box_f B)$

A3. $\Box_p(A \rightarrow B) \rightarrow (\Box_p A \rightarrow \Box_p B)$

A4. $\Box_f A \rightarrow A$

A5. $\Box_p A \rightarrow A$

A6. $A \rightarrow \Box_f \Diamond_p A$

A7. $A \rightarrow \Box_p \Diamond_f A$

A8. $\Box_f A \rightarrow \Box_f \Box_f A$

A9. $\Box_p A \rightarrow \Box_p \Box_p A$

A10. $\Diamond_f A \wedge \Diamond_f B \rightarrow (\Diamond_f(A \wedge B) \vee \Diamond_f(A \wedge \Diamond_f B) \vee \Diamond_f(\Diamond_f A \wedge B))$

A11. $\Diamond_p A \wedge \Diamond_p B \rightarrow (\Diamond_p(A \wedge B) \vee \Diamond_p(A \wedge \Diamond_p B) \vee \Diamond_p(\Diamond_p A \wedge B))$

R1 $\dfrac{A, A \rightarrow B}{B}$

R2 $\dfrac{A}{\Box_f A}$

R3 $\dfrac{A}{\Box_p A}$

The notion of a proof of a formula is defined as usual. We say that a formula A is derivable from a set T of formulas ($T \vdash A$) whenever there is a proof of A from T. A formula A is a theorem of logic TIL ($\vdash A$) if there is a proof of A from the empty set of formulas. A set T of formulas is consistent if a formula of the form $A \wedge \neg A$ is not derivable from T.

The following soundness theorem holds for logic TIL.

<u>Fact 4.2.1</u>

(a) $\vdash A$ implies $\models A$,
(b) $T \vdash A$ implies $T \models A$,
(c) T satisfiable implies T consistent

The given axiomatization follows closely the axiomatization of the tense logic with linearly ordered time presented in Burgess (1979). Axioms A4 and A5 provide reflexivity of time ordering. Axioms A8 and A9 correspond to transitivity of this relation, and axioms A10 and A11 guarantee that a time scale is linearly ordered. Axioms A6 and A7 show that past operations are inverse with respect to future operations.

In the following we list some theorems of logic TIL

<u>Fact 4.2.2</u>

(a) $\vdash A \rightarrow \Diamond_p A$
(b) $\vdash \Diamond_p(A \vee B) \leftrightarrow (\Diamond_p A \vee \Diamond_p B)$
(c) $\vdash \Diamond_p(A \wedge B) \rightarrow (\Diamond_p A \wedge \Diamond_p B)$
(d) $\vdash (\Box_p A \vee \Box_p B) \rightarrow \Box_p(A \vee B)$
(e) $\vdash \Box_p(A \wedge B) \leftrightarrow (\Box_p A \wedge \Box_p B)$
(f) $\vdash \neg \Diamond_p A \leftrightarrow \Box_p \neg A$
(g) $\vdash \neg \Box_p A \leftrightarrow \Diamond_p \neg A$

The respective theorems for future operations are mirror images of the above formulas, that is they can be obtained by switching past operations and future operations.

We now prove completeness theorem for logic TIL. We use the standard algebraic method and we adopt it for our special semantics, which is closely related to temporal KR systems. We show that a cononical KR system can be defined by using syntactical constructs.

Let T be a consistent set of formulas, we define relation $\approx$ in set FORTIL as follows

$A \approx B$ iff $T \vdash A \Leftrightarrow B$

<u>Fact 4.2.3</u>

(a) Relation $\approx$ is an equivalence on set FORTIL

(b) Relation $\approx$ is a congruence with respect to operations $\neg$, $\vee$, and $\wedge$.

(c) If $A \approx B$ then $\Box_p A \approx \Box_p B$ and $\Box_f A \approx \Box_f B$

The proof follows closely the earlier proofs of similar theorems for logics DIL and NIL. Condition (c) follows from axioms A2, A3 and the rules R2 and R3 of necessitation.

As previously, we consider set FORTIL$/_\approx$ of all the equivalence classes $[A]$ of relation $\approx$ for $A \in$ FORTIL, and we form algebra

ATIL = (FORTIL$/_\approx$, $\cup$, $\cap$, $-$, 1, 0)

which satisfies the same conditions as those presented in section 3.3 for algebra ANIL. Let FT be the set of all the maximal filters in algebra ATIL. We define a canonical KR system as follows:

$S_o = (OB_o, AT_o, VAL_o, TM_o, R_o, f_o)$

where OB_o = CONCB

AT_o = CONAT

VAL_o = CONVAL

TM_o = FT

$R_o = \{(F,G) \in FT \times FT$: for all $A \in$ FORTIL if $[\Box_f A] \in F$ then $[A] \in G\}$

$f_o(o,F,a) = v$ iff $[(o\ a\ v)] \in F$

<u>Fact 4.2.4</u>

The following conditions are equivalent:

(a) $(F,G) \in R_o$

(b) $[\Box_p A] \in G$ implies $[A] \in F$

(c) $[A] \in F$ implies $[\Diamond_p A] \in G$

(d) $[A] \in G$ implies $[\Diamond_f A] \in F$

Proof: Assume that condition (a) is satisfied and suppose $[\Box_p A] \in G$ and $[A] \notin F$. Hence $[\neg A] \in F$ and by A6 $[\Box_f \Diamond_p \neg A] \in F$. Using (a) we obtain $[\Diamond_p \neg A] \in G$. Since $\vdash \Box_p A \wedge \Diamond_p B \to \Diamond_p (A \wedge B)$, we have $[\Diamond_p (A \wedge \neg A)] \in G$, a contradiction since G is a proper filter. Hence condition (b) holds.

Let us now assume that condition (b) holds and suppose $[A] \in F$ and $[\Diamond_p A] \notin G$. Hence $[\Box_p \neg A] \in G$ and by (b) we have $[\neg A] \in F$, a contradiction.

Hence condition (c) holds.

Assume condition (c) and suppose $[A] \in G$ and $[\Diamond_f A] \notin F$. Then $[\neg \Diamond_f A] \in F$ and by (c) we have $[\Diamond_p \neg \Diamond_f A] \in G$. By A7 $[\neg A] \in G$, a contradiction.

Hence condition (d) holds.

We can also see that (d) implies (a). Otherwise we would have $[\neg A] \in G$ and by (d) $[\Diamond_f \neg A] \in F$. Since $\vdash \Box_f A \wedge \Diamond_f B \to \Diamond_f (A \wedge B)$, we would have $[\Diamond_f (A \wedge \neg A)] \in F$, a contradiction.

<u>Fact 4.2.5</u>

Relation R_o is a reflexive and transitive linear order in set FT.

Proof: Reflexivity and transitivity of R_o follows from A4 and A8, respectively. By using A10 and A11 we have $(F,G) \in R_o$ or $(G,F) \in R_o$ or $F = G$ for any $F,G \in FT$ and hence R_o is a linear order.

The following lemmas are needed for the completeness theorem for logic TIL.

<u>Fäct 4.2.6</u>

(a) If $[\Diamond_f A] \in F$ then there exists an $G \in FT$ such that $(F,G) \in R_o$ and $[A] \in G$

(b) If $[\Diamond_p A] \in G$ then there exists an $F \in FT$ such that $(F,G) \in R_o$ and $[A] \in F$

(c) If $[\Box_f A] \in F$ then for every $G \in FT$ if $(F,G) \in R_o$ then $[A] \in G$

(d) If $[\Box_p A] \in G$ then for every $F \in FT$ if $(F,G) \in R_o$ then $[A] \in F$

<u>Fact 4.2.7</u>

The following conditions are equivalent:

(a) M_o, F sat A

(b) $[A] \in F$.

The proofs of these facts follow closely the earlier proofs of theorems 3.3.9 and 3.3.10. Theorem 4.2.7 enables us to establish in a usual way completeness and compactness of logic TIL.

If in some applications it is desirable to consider irreflexive time scales than we can drop axioms A4 and A5 and consider models determined by temporal KR systems with transitive linear orders.

4.3. Languages of systems of temporal information

In this section we define two languages $L_1(S)$ and $L_2(S)$ based on the temporal information logic TIL and a model of the logic. Expressions of the language $L_1(S)$ are intended to represent assertions which may be true or false at a given moment of time. These assertions concern properties of objects which can be expressed in terms of attributes and values of attributes. Expressions of the language $L_2(S)$ are intended to represent properties of objects in their semantical sense, that is subsets of a set of objects.

Assume that we are given a model $M = (S, m)$ determined by a temporal KR system $S = (OB, AT, VAL, TM, R, f)$ where the meaning function m assigns objects, attributes and values of attributes to the respective constants. We consider set FORTIL(S) of formulas of system S which is obtained from set FORTIL in the usual way by substituting names of objects, attributes and attribute values for the respective constants.

The language $L_1(S)$

Atomic expressions of language $L_1(S)$ are all the pairs of the form (t,A) for $t \in TM$ and $A \in FORTIL(S)$. Other expressions are built up from atomic expressions by means of the classical propositional operators. We can formally define set $FOR_1(S)$ of all formulas of language $L_1(S)$ to be a least set satisfying the following conditions:

$(t,A) \in FOR_1(S)$ for all $t \in TM$ and $A \in FORTIL(S)$

$A, B \in FOR_1(S)$ implies $\neg A,\ A \vee B,\ A \wedge B,\ A \rightarrow B,\ A \leftrightarrow B \in FOR_1(S)$

An atomic formula of the form (t,A) can be considered as the assertion : condition A holds at the moment t. In particular, if A is of the form (o a v) then expression $(t,(o\ a\ v))$ is a representation of the fact that object o assumes value v of attribute a at moment t. Similarly, $(t, \Diamond_p A)$ and $(t, \Diamond_f A)$ represent the fact that there is a moment s earlier (later) than t such that condition A holds at s.

<u>Example 4.3.1</u>

Let us consider the results of the spirography test for a group of children suffering from fibrosis pneumonic. The following table is a part of the document presented in Milanowski (1972).

	Date of examination	Sex	Age	Height	VC%	FEV_1%
M.S.	VII 68	m	8	143	46-50	81-85
	X 68				40-45	81-85
	XII 68				40-45	81-85
	VI 69				40-45	74
	XII 69		9	145	40	85
	IX 70		10	147	40-45	85
	XI 70				46-50	81-85
E.S.	III 68	f	12	129	50	74
W.M.	XI 68	m	8	116	50	85
	I 69				46-50	74-80
	III 69				50	81-85
	V 69				46-50	81-85
	II 70		10	118	50	74-80
	V 71		11	121	50	81-85

The above table determines the information function of the following KR system of temporal information:

OB = {M.S., E.S., W.M.}

AT = {Sex, Age, Height, Vital Capacity, Maximal Expiratory, Capacity}

VAL_{Sex} = {m(male), f (female)}

VAL_{Age} and VAL_{Height} are subsets of the set of integers

$VAL_{Vital\ Capacity}$ consists of subsets of the set of integers and includes the subsets denoted informally by < 40 (integers less than 40), 40-45 (integers between 40 and 45), > 50 (integers greater than 50)

$VAL_{Maximal\ Expiratory\ Capacity}$ includes the following subsets of the set of integers: < 74, 74-80, 81-85, > 85.

Consider the following expression of the language determined by this KR system:

$(I.69,\ \Diamond_p(M.S.\ VC\% < 40))$

This is the representation of the fact that there is a moment later than the January 69 when patient M.S. had the percentage value of VC less than 40.

The language $L_2(S)$

Expressions of language $L_2(S)$ are intended to represent subsets of the set of objects of system S. First, we define an auxiliary set E(S) of expressions obtained from formulas from set FORTIL(S) by removing the names of objects.

Set E(S) is the least set satisfying the following conditions:

$(a\ v) \in E(S)$ for all $a \in AT$ and $v \in VAL$

$A, B \in E(S)$ implies $\neg A,\ A \vee B,\ A \wedge B,\ A \rightarrow B,\ A \leftrightarrow B,\ \Diamond_p A,\ \Diamond_f A,\ \Box_p A,$

$\Box_f A \in E(S)$.

Set $FOR_2(S)$ of all the expressions of language $L_2(S)$ is the least set defined as follows:

$(t,A) \in FOR_2(S)$ for all $t \in TM$ and $A \in E(S)$

A,B ∈ $FOR_2(S)$ implies ¬A, A∨B, A∧B, A→B, A↔B ∈ $FOR_2(S)$

An expression of the form (t,A) is intended to represent a set of those objects which at the moment t have the property A. To each formula A of language $L_2(S)$ we assign a set of objects called extension of A:

$ext(t,(a\ v)) = \{o \in OB : f_{ot}(a) = v\}$

$ext(t,\neg A) = -\ ext(t,A)$

$ext(t, A\vee B) = ext(t,A) \cup ext(t,B)$

$ext(t, A\wedge B) = ext(t,A) \cap ext(t,B)$

$ext(t, A\rightarrow B) = ext(t,\neg A \vee B)$

$ext(t, A\leftrightarrow B) = ext(t, (A\rightarrow B)\wedge(B\rightarrow A))$

$ext(t, \Diamond_p A) = \{o \in OB$: there is an $s \in TM$ such that $(s,t) \in R$ and $s \in extA\}$

$ext(t, \Diamond_f A) = \{o \in OB$: there is an $s \in TM$ such that $(t,s) \in R$ and $s \in extA\}$

$ext(t, \Box_p A) = \{o \in OB$: for all $s \in TM$ if $(s,t) \in R$ then $s \in extA\}$

$ext(t, \Box_f A) = \{o \in OB$: for all $s \in TM$ if $(t,s) \in R$ then $s \in extA\}$

Extensions of composed formulas of language $FOR_2(S)$ are defined in a usual way as the set - theoretical combinations of the extensions of their components.

Observe that language $L_2(S)$ enables us to express information about objects which in the time period between moment t_1 and moment t_2 possibly have a certain property. Such assertions have the form:

$(t_1, \Diamond_f A) \wedge (t_2, \Diamond_p A)$

The first conjunct represents the set of those objects which obey property A in a certain moment later than t_1. The second conjunct corresponds to the set of those objects which obey A at a certain moment earlier than t_2.

We can also express the fact that for some objects a moment t is the earliest (latest) moment in which those object's have a certain property. These formulas have the form

$(t,A) \wedge (t, \Box_p \neg A)$

$(t,A) \wedge (t, \Box_f \neg A)$

Formula $(t, \Box_p \neg A)$ represents the set of objects which do not obey property A at all the moments earlier than t. Formula (t,A) represents the set of objects which obey A at moment t. The intersection of these

two sets consists of those objects which for the first time obey A at moment t. In the similar way the interpretation can be given for the second formula.

Example 4.3.2

Consider a library catalogue which can be considered as a temporal KR system in the following way. The set OB of objects consists of the catalogue numbers of books, the set AT of attributes consists of title, author, publisher, and subject, and the set TM of moments of time indicates a year of edition of books. Some examples of assertions formulated in the language determined by this KR system are given below.

(1965, $\Diamond_f$((author Robinson A.)∧(subject Logic and Foundations)))∧ (1980, $\Diamond_p$((author, Robinson A.)∧(subject, Logic and Foundations))).

The formula given above represents the set consisting of the catalogue numbers of books by A. Robinson concerning logic and foundations of mathematics and edited in the period 1965 - 1980. It includes, for example, "Non-standard analysis" and "Complete theories".

(1975, $\Diamond_f$(subject Artificial Intelligence))

This formula corresponds to the set of catalogue numbers of the books on artificial intelligence edited later than 1975. It may include "Logic for problem solving" by R. Kowalski and "Understanding spoken language" by D.E. Walker.

4.4. Summary

In this chapter we have discussed how the temporal dimension can be incorporated in conceptual models of knowledge representation. We proceeded according to the scheme: semantics + syntax + deduction method and we introduced some methods of dealing with a time scale explicitly on each of these three levels of representation. We used the formalism of the temporal logic of linearly ordered time which occurred to be suitable for defining languages of systems of temporal information. The formal languages had been introduced providing a direct manipulatory access to time dependent information. The definition of these languages is devided into two sublevels:

- one in which we consider the time dimension semantically, in the sense of a modal approach to language semantics, and we define the temporal propositional logic whose interpretation structure is determined by a temporal KR system

- one in which we introduce time explicitly, and we define a propositional calculus which, with respect to the deductive power, is based on the classical propositional logic, and moreover the formulas of the calculus contain time constants and formulas of temporal logic.

5. METHODOLOGY OF KNOWLEDGE REPRESENTATION

5.1. Indiscernibility

In general, information about objects provided by a KR system is not sufficient to characterize objects uniquely that is we are not able to distinguish all the objects by means of the admitted attributes and their values. Consider, for example, the set of animals which are characterized by attributes Animality and Colour according to the following information function (Hunt et al (1966)):

	Animality	Colour
A_1	bear	black
A_2	bear	black
A_3	dog	brown
A_4	cat	black
A_5	horse	black
A_6	horse	black
A_7	horse	brown

In this KR system information about A_1 is the same as information about A_2 and consists of the following pairs:

(Animality, bear) (Colour, black)

Similarly, information about A_5 equals information about A_6 and hence these animals cannot be distinguished by means of attributes Animality and Colour. We can observe the other sets of undistinguishable objects in the given system. For example animals A_3 and A_7 cannot be distinguished by attribute Colour; animals A_5, A_6 and A_7 cannot be distinguished by attribute Animality.

To deal with such cases we define a family of indiscernibility relations determined by the attributes of a KR system. Given a KR system with a set OB of objects, a set AT of attributes and an information function f, we consider a subset A of set AT and a relation $\widetilde{A}$ in set OB defined as follows:

$(o_1,o_2) \in \widetilde{A}$ iff $f(o_1,a) = f(o_2,a)$ for all $a \in A$

$\widetilde{\emptyset} = OB \times OB$

Relation $\widetilde{A}$ is referred to as indiscernibility with respect to attributes from set A. A pair of objects belongs to relation $\widetilde{A}$ whenever they cannot be distinguished by means of attributes from A. An indiscernibility relation determined by the empty set does not enable us to tell an object from none of the others. Indiscernibility relation $\widetilde{AT}$ determined by all the attributes of a system S is called indiscernibility determined by S, and it is denoted by ind(S). Let us observe that the definition of indiscernibility relations does not depend on the kind of information function, that is the notion of indiscernibility is meaningful both for deterministic and nondeterministic KR systems.

<u>Fact 5.1.1</u>

(a) $\widetilde{A}$ is an equivalence relation for all $A \subseteq AT$

(b) $\widetilde{A \cup B} = \widetilde{A} \cap \widetilde{B}$

(c) If $A \subseteq B$ then $\widetilde{B} \subseteq \widetilde{A}$

Equivalence classes of a relation $\widetilde{A}$ are called indiscernibility classes of set A. In particular the indiscernibility classes of ind(S) are called elementary sets in system S.

<u>Example 5.1.1</u>

Let us consider a KR system of deterministic information determined by the table given below:

	a	b	c
o_1	0	0	1
o_2	0	0	1
o_3	0	1	0
o_4	1	2	0
o_5	1	0	1
o_6	1	2	0
o_7	0	1	0
o_8	0	2	1
o_9	1	0	1
o_{10}	0	1	0
o_{11}	0	2	1

The indiscernibility classes of the attributes are as follows:

$\widetilde{a}$: $\{o_1, o_2, o_3, o_7, o_8, o_{10}, o_{11}\}$ $\{o_4, o_5, o_6, o_9\}$

$\widetilde{b}$: $\{o_1, o_2, o_5, o_9\}$ $\{o_3, o_7, o_{10}\}$ $\{o_4, o_6, o_8, o_{11}\}$

$\widetilde{c}$: $\{o_3, o_4, o_6, o_7, o_{10}\}$ $\{o_1, o_2, o_5, o_8, o_9, o_{11}\}$

Elementary sets of the system are as follows:

$E_1 = \{o_1, o_2\}$

$E_2 = \{o_3, o_7, o_{10}\}$

$E_3 = \{o_4, o_6\}$

$E_4 = \{o_5, o_9\}$

$E_5 = \{o_8, o_{11}\}$

Example 5.1.2

Let us consider a many - valued KR system providing information about languages presons $P_1, \dots, P_{10}$ speak. We admit the following values of attribute Language: English (GB), French (F), German (D), Polish (PL)

	Language
P_1	GB, F
P_2	PL, GB, D
P_3	F, D
P_4	GB, D
P_5	GB, F
P_6	GB, D
P_7	PL, GB, D
P_8	F, D
P_9	F, D, PL
P_{10}	F, D, PL

We consider nondeterministic information function determined by this system and the indiscernibility relation determined by attribute Language. The elementary sets in the system are as follows:

$\{P_1, P_5\}$ $\{P_2, P_7\}$, $\{P_3, P_8\}$ $\{P_4, P_6\}$ $\{P_9, P_{10}\}$

Example 5.1.3

Consider the following KR system of nondeterministic information

	Colour of Eyes	Age
P_1	blue, green	20-30
P_2	green, hazel	30-35
P_3	blue, green	20-30
P_4	green, hazel	20-30
P_5	brown, black	35-40
P_6	brown, hazel	35-40
P_7	blue, green	30-35
P_8	brown, black	35-40

Indiscernibility classes of attribute Colour of Eyes:

$\{P_1, P_3, P_7\}$ $\{P_2, P_4\}$ $\{P_5, P_8\}$ $\{P_6\}$

Indiscernibility classes of attribute Age:

$\{P_1, P_3, P_4\}$ $\{P_2, P_7\}$ $\{P_5, P_6, P_8\}$

Elementary sets:

$\{P_1, P_3\}$ $\{P_2\}$ $\{P_4\}$ $\{P_5, P_8\}$ $\{P_6\}$

5.2. Definable sets of objects

In this section we discuss how indiscernibility of individual objects influences knowledge about sets of objects. Clearly, since objects are not necessarily distinguishable in a KR system, knowledge characterizing a set of objects may be ambiguous to some extent. Consider, for example, information about animals given in section 5.1., and assume that we are interested in set $X = \{A_1, A_3, A_6, A_7\}$. Information provided by the given table does not enable us to characterize set X precisely. We cannot say that an animal belongs to X iff it is a black bear, a brown dog, a black horse or a brown horse, because A_2 and A_5 satisfy this condition too. Some other sets can be defined precisely, for example, set $Y = \{A_5, A_6, A_7\}$ can be characterized by the following condition: an animal belongs to Y iff it is a black or brown horse.

We conclude, that we should distinguish sets of objects which can be completely characterized in a given KR system.

Assume that we are given a KR system S with set OB of objects and set AT of attributes. We say that

set $X \subseteq OB$ is definable by set $A \subseteq AT$ iff
X is either the empty set or the union of some
indiscernibility classes of A

In particular if X is definable by set AT then we say that X is definable in system S. Observe, that any finite set OB and the empty set are definable by a set A of attributes. Moreover, for a finite OB the family of sets definable by A is closed under union, intersection and complement, and hence it is a Boolean algebra.

Example 5.2.1

Let us consider the system given in example 5.1.1 and sets

$X = \{o_1, o_2, o_5, o_9\}$

$Y = \{o_4, o_6, o_8, o_{11}\}$

$Z = \{o_3, o_4, o_6, o_7, o_{10}\}$

These sets are definable in the given system:

$X = E_1 \cup E_4$

$Y = E_3 \cup E_5$

$Z = E_2 \cup E_3$

On the other hand sets

$T = \{o_1, o_4, o_5, o_6, o_9, o_{11}\}$

$U = \{o_2, o_8, o_9, o_{11}\}$

$W = \{o_3, o_4, o_5, o_9\}$

are not definable in the system.

Example 5.2.2

Consider the KR system from example 2.1.1. The elementary sets of the system are given below.

$\{o_1\}$ $\{o_2, o_4\}$ $\{o_3, o_5\}$ $\{o_6\}$

This means that knowledge provided by the system enables us to distinguish the following sets of individuals:

young males
males of medium age
old females
young females
all sets which can be obtained from the above sets by using set-theoretical operations

Set $X = \{o_1, o_2, o_4, o_6\}$ is definable in the system. It consists of young males, males of medium age and young females. Set $Y = \{o_1, o_3\}$ and set $Z = \{o_2, o_5\}$ are not definable in the system. We cannot say that Y coincides with a set of young males and old females, since o_5 does not belong to Y. Similarly, it is not true that set Z coincides with a set of males of medium age and old females, since o_4 and o_3 do not belong to Z.

It is now easy to see that any set of objects which is definable in a system can be described by a certain formula of the language of this system.

Fact 5.2.1

The following conditions are equivalent:

(a) A set X is definable in a system S

(b) There is a formula F in the language of S such that X = ext F.

Example 5.2.3

Let set OB consists of the following ten trains, presented in Michalski (1980).

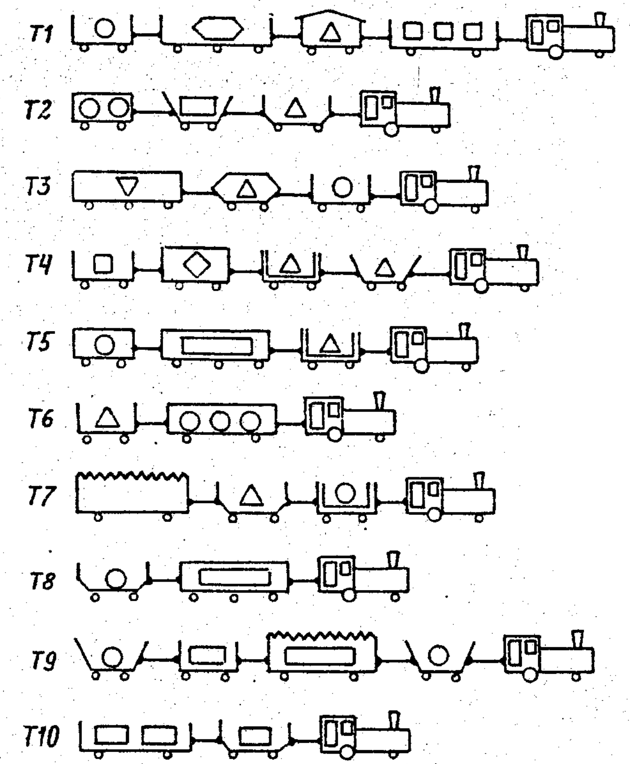

Fig. 2

Let set $AT = \{a_1, a_2, a_3\}$ be defined as follows:

a_1 number of cars

a_2 maximal number of wheels in cars

a_3 occurence of a zigzag line in cars

We have

$VAL_{a_1} = \{3, 4, 5\}$

$VAL_{a_2} = \{2, 3\}$

$VAL_{a_3} = \{yes, no\}$

The information function can easily be reconstructed from the given pictures.

The indiscernibility classes of set $\{a_1, a_2\}$ and the respective formulas of the language of the given system are as follows:

$\{T_1\}$	$(a_1\ 5)\wedge(a_2\ 3)$
$\{T_2, T_7\}$	$(a_1\ 4)\wedge(a_2\ 2)$
$\{T_3, T_5\}$	$(a_1\ 4)\wedge(a_2\ 3)$
$\{T_4, T_9\}$	$(a_1\ 5)\wedge(a_2\ 2)$
$\{T_6, T_{10}\}$	$(a_1\ 3)\wedge(a_2\ 2)$
$\{T_8\}$	$(a_1\ 3)\wedge(a_2\ 3)$

The indiscernibility classes of set $\{a_1, a_3\}$ and the respective formulas are as follows:

$\{T_1, T_4\}$	$(a_1\ 5)\wedge(a_3\ \text{no})$
$\{T_2, T_3, T_5\}$	$(a_1\ 4)\wedge(a_3\ \text{no})$
$\{T_6, T_8, T_{10}\}$	$(a_1\ 3)\wedge(a_3\ \text{no})$
$\{T_7\}$	$(a_1\ 4)\wedge(a_3\ \text{yes})$
$\{T_9\}$	$(a_1\ 5)\wedge(a_3\ \text{yes})$

Set $Z = \{T_6, T_7, T_8, T_9, T_{10}\}$ is not $\{a_1, a_2\}$ - definable, but it is $\{a_1, a_3\}$ - definable. We have

$ext((a_1\ 4)\wedge(a_2\ 2)\vee(a_1\ 5)\wedge(a_2\ 2)\vee(a_1\ 3)\wedge(a_2\ 2)\vee(a_1\ 3)\wedge(a_2\ 3))$

$= \{T_2, T_4, T_6, T_7, T_8, T_9, T_{10}\} = Z\cup\{T_2, T_4\}$

$ext((a_1\ 3)\wedge(a_3\ \text{no})\vee(a_1\ 4)\wedge(a_3\ \text{yes})\vee(a_1\ 5)\wedge(a_3\ \text{yes})) = Z$

Listed below are some formulas of the language of the given system and the corresponding sets of objects which are definable in the system.

F_1	$(a_1\ 4)\wedge(a_2\ 2)$	$extF_1 = \{T_2, T_7\}$
F_2	$\neg(a_1\ 4)$	$extF_2 = \{T_1, T_4, T_6, T_8, T_9, T_{10}\}$
F_3	$(a_1\ 5)\vee(a_2\ 3)$	$extF_3 = \{T_1, T_5, T_8, T_9\}$
F_4	$(a_1\ 3)$	$extF_4 = \{T_6, T_8, T_{10}\}$
F_5	$(a_1\ 3)\rightarrow(a_2\ 2)$	$extF_5 = \emptyset$
F_6	$(a_1\ 3)\rightarrow\neg(a_1\ 4)$	$extF_6 = OB$
F_7	$(a_1\ 3)\leftrightarrow\neg(a_1\ 4)\wedge\neg(a_1\ 5)$	$extF_7 = OB$

We have

$ext(a_1\ 3)\subseteq ext(\neg(a_1\ 4))$

$ext(a_1\ 3) = ext(\neg(a_1\ 4)\wedge\neg(a_1\ 5))$

A KR system S is said to be selective iff each elementary set con-

sists of exactly one object.

Fact 5.2.1

The following conditions are equivalent:

(a) A system S with a set OB of objects is selective

(b) Any set $X\subseteq OB$ is definable in S

Given a definable set $X\subseteq OB$, knowledge provided by the system enables us to decide when an object $o\in OB$ belongs to set X. However, if set X is not definable in system S we are not able to answer a membership question precisely. For example, knowledge provided by the system given in example 2.1.1 is not sufficient for establishing whether an object belongs to set $\{o_1, o_3\}$ since we are not able to define this set in terms of attributes Sex and Age. To deal with such cases we introduce notions of approximations of sets of objects.

5.3. Approximations of sets of objects

Let a system S = (OB, AT, VAL, f) be given, we define a pair of operations in set OB of objects, namely the operation of lower approximation and upper approximation of a set. These operations enable us to assign a pair of definable sets to any subset X of set OB. For a set X which is definable in the system its approximations coincide with X, and for a nondefinable set X its approximations are, roughly speaking, close enough to X. They determine limits of tolerance for deciding whether objects belong to X or not.

An upper approximation $\overline{S}X$ of set X in system S is the least set which is definable in S and includes set X.

A lower approximation $\underline{S}X$ of set X in system S is the greatest set which is definable in S and is included in X.

The following facts follow immediately from the given definitions.

Fact 5.3.1

(a) $\overline{S}X = \{o\in OB\text{: there is an } o'\in OB \text{ such that } (o,o')\in ind(S) \text{ and } o'\in X\}$

(b) $\underline{S}X = \{o\in OB\text{: for all } o'\in OB \text{ if } (o,o')\in ind(S) \text{ then } o'\in X\}$

Fact 5.3.2

The following conditions are equivalent:

(a) A set $X\subseteq OB$ is definable in system S

(b) $\underline{S}X = X = \overline{S}X$

Example 5.3.1

Let us consider system S determined by the following table

	a	b
o_1	1	3

o_2	2	4
o_3	2	3
o_4	1	4
o_5	2	3
o_6	3	4
o_7	2	4
o_8	3	3
o_9	1	4
o_{10}	3	4

Elementary sets of the system are as follows

$\{o_1\}$ $\{o_2, o_7\}$ $\{o_3, o_5\}$ $\{o_4, o_9\}$ $\{o_6, o_{10}\}$ $\{o_8\}$

We consider set $Z = \{o_6, o_7, o_8, o_9, o_{10}\}$. It is not definable in the given system and its approximations are as follows

$\overline{S}Z = \{o_2, o_4, o_6, o_7, o_8, o_9, o_{10}\} = Z \cup \{o_2, o_4\}$

$\underline{S}Z = \{o_6, o_8, o_{10}\} = Z - \{o_7, o_9\}$

Example 5.3.2

Let us consider the system from example 2.1.1 with the following elementary sets:

$\{o_1\}$ $\{o_2, o_4\}$ $\{o_3, o_5\}$ $\{o_6\}$

Sets $X = \{o_1, o_2\}$ $Y = \{o_1, o_3, o_6\}$ $Z = \{o_2, o_4, o_6\}$

have the following approximations:

$\overline{S}X = \{o_1, o_3, o_4\}$ $\underline{S}X = \{o_1\}$

$\overline{S}Y = \{o_1, o_3, o_5, o_6\}$ $\underline{S}Y = \{o_2, o_6\}$

$\overline{S}Z = \underline{S}Z = \{o_2, o_4, o_6\} = Z$

In the following we list some properties of the operations of lower and upper approximation.

Fact 5.3.3

(a) $\underline{S}(X \cap Y) = \underline{S}X \cap \underline{S}Y$

(b) $\underline{S}X \subseteq X$

(c) $\underline{S}\,\underline{S}\,X = \underline{S}X$

(d) $\underline{S}$ OB = OB

Fact 5.3.4

(a) $\overline{S}(X \cup Y) = \overline{S}X \cup \overline{S}Y$

(b) $X \subseteq \overline{S}X$

(c) $\overline{S}\,\overline{S}\,X = \overline{S}X$

(d) $\overline{S}\,\emptyset = \emptyset$

It follows that algebra P(OB) of all the subsets of set OB with additional operations $\overline{S}$ and $\underline{S}$ is a topological field of sets, where $\overline{S}$ is a closure operation and $\underline{S}$ is an interior operation.

Fact 5.3.5

(a) $\overline{S}X = -\underline{S}(-X)$

(b) $\underline{S}X = -\overline{S}(-X)$

(c) if $X \subseteq Y$ then $\overline{S}X \subseteq \overline{S}Y$ and $\underline{S}X \subseteq \underline{S}Y$.

Thus operations $\underline{S}$ and $\overline{S}$ are dual and monotonic with respect to inclusion.

5.4. Rough definability

Given a system S = (OB, AT, VAL, f) and a set $X \subseteq$ OB, for any object $o \in$ OB we say that

o is an S-positive instance of X iff $o \in \underline{S}X$

o is an S-negative instance of X iff $o \in OB - \overline{S}X$

o is an S-borderline instance of X iff $o \in \overline{S}X - \underline{S}X$.

It follows that if o is a positive instance of X then knowledge provided by system S enables us to state that o definitely belongs to X. For negative instances of X we know that they definitely do not belong to X. Borderline instances of X represent a doubtful region, they possibly belong to X but we cannot decide if for certain in virtue of knowledge given in the system. We say that

a set X is roughly definable in a system S iff $\underline{S}X \neq \emptyset$ and $\overline{S}X \neq$ OB.

Thus for roughly definable sets a membership question can be decided approximately. However, if lower approximation $\underline{S}X$ is empty then there are no S-positive instances of X and hence none of the objects can be recognized to be surely an element of X. Similarly, if upper approximation $\overline{S}X$ equals set OB then there are no negative instances of X and hence none of the objects can be definitely excluded from X. We say that

a set X is internally nondefinable in a system S iff $\underline{S}X = \emptyset$

a set X is externally nondefinable in a system S iff $\overline{S}X$ = OB

a set X is totally nondefinable in a system S iff X is internally nondefinable and externally nondefinable in S

Fact 5.4.1.

(a) A set X is internally nondefinable in a system S iff none of the objects is an S-positive instance of X

(b) A set X is externally nondefinable in S iff none of the objects is an S-negative instance of X

Example 5.4.1

Consider system S from example 2.1.1 with the following elementary sets:

$\{o_1\}$ $\{o_2, o_4\}$ $\{o_3, o_5\}$ $\{o_6\}$

Sets $X = \{o_2, o_3, o_4\}$ $Y = \{o_2, o_3\}$ $Z = \{o_1, o_2, o_3, o_6\}$ have the following approximations:

$\underline{S}X = \{o_2, o_4\}$ $\overline{S}X = \{o_2, o_3, o_4, o_5\}$

$\underline{S}Y = \emptyset$ $\overline{S}Y = \{o_2, o_3, o_4, o_5\}$

$\underline{S}Z = \{o_1, o_6\}$ $\overline{S}Z = OB$

It follows that objects o_2 and o_4 are the positive instances of X, objects o_1 and o_6 are the negative instances of X and objects o_3 and o_5 are bordeline instances of X. Set Y is internally nondefinable and set Z is externally nondefinable in system S.

Observe, that if an indiscernibility ind(S) generates a one-element elementary set then there are no totally nondefinable objects in system S.

5.5. Comparing knowledge representation systems

It can be seen from the previous considerations that expressive power of knowledge representation systems is closely related to their ability for defining sets of objects. In this section we consider a family $ST = \{S_i\}_{i \in I}$ of knowledge representation systems of the form

$S_i = (OB, AT_i, VAL_i, f_i)$

where set OB is the same for all the systems and I is a nonempty set of indices. We say that

a system $S_1 \in ST$ is more expressive than a system $S_2 \in ST$

$(S_1 \leq S_2)$ iff $ind(S_1) \subseteq ind(S_2)$.

This means that if $S_1 \leq S_2$ then the indiscernibility relation of system S_1 provides a finer partition of set OB into elementary sets than the indiscernibility relation of system S_2. It follows that approximations of sets of objects in system S_1 are closer to these sets than their approximations in system S_2, namely the following theorems hold.

Fact 5.5.1

The following conditions are equivalent:

(a) $S_1 \leq S_2$

(b) $\overline{S}_1X \subseteq \overline{S}_2X$ for any $X \subseteq OB$

Proof: Let $[o]_i$, for i=1,2 denote the equivalence class with respect to relation $ind(S_i)$ determined by object $o \in OB$. If $ind(S_1) \subseteq ind(S_2)$ then for any $o \in OB$ we have $[o]_1 \subseteq [o]_2$, and hence condition (b) holds. Let us now suppose that for any set $X \subseteq OB$ we have $\overline{S}_1X \subseteq \overline{S}_2X$ and not $S_1 \leq S_2$. Hence there is a pair (o,o') of objects such that $(o,o') \in ind(S_1)$ and $(o,o') \notin ind(S_2)$. Consider set $\{o\}$. We have $o' \in \overline{S}_1\{o\}$ and $o' \notin \overline{S}_2\{o\}$, which contradicts condition (b).

Fact 5.5.2

The following conditions are equivalent

(a) $S_1 \leq S_2$

(b) $\underline{S}_2X \subseteq \underline{S}_1X$ for any $X \subseteq OB$

A proof follows from 5.3.5 and 5.5.1.

In the following we list some properties of relation $\leq$.

Fact 5.5.3

(a) If $AT_1 \subseteq AT_2$ then $S_2 \leq S_1$

(b) Relation $\leq$ is a partial order in any family ST of systems

(c) Selective systems are minimal elements in any family ST ordered by relation $\leq$.

Example 5.5.1

Consider systems S_1 and S_2 such that

$OB_1 = OB_2 = \{o_1, o_2, o_3, o_4, o_5\}$

$AT_1 = \{a, b\}$ $AT_2 = \{a, c, d\}$

$VAL_a = \{p_1, p_2\}$

$VAL_b = \{q_1, q_2\}$

$VAL_c = \{r_1, r_2\}$

$VAL_d = \{s_1, s_2\}$

f_1	a	b
o_1	p_1	q_1
o_2	p_2	q_2
o_3	p_1	q_1

f_2	a	c	d
o_1	p_1	r_1	s_1
o_2	p_2	r_2	s_2
o_3	p_1	r_2	s_1

o_4 p_2 q_1 o_4 p_2 r_2 s_1

o_5 p_2 q_1 o_5 p_2 r_2 s_1

The indiscernibility relations of these systems generate the following elementary sets

$ind(S_1)$: $\{o_1, o_3\}$ $\{o_2\}$ $\{o_4, o_5\}$

$ind(S_2)$: $\{o_1\}$ $\{o_2\}$ $\{o_3\}$ $\{o_4, o_5\}$

We clearly have $S_2 \leq S_1$. Consider set $X = \{o_1, o_4\}$ and its approximations in the given systems:

$\overline{S}_1X = \{o_1, o_3, o_4, o_5\}$ $\underline{S}_1X = \emptyset$

$\overline{S}_2X = \{o_1, o_4, o_5\}$ $\underline{S}_2X = \{o_1\}$

5.6. Dependencies of attributes

Given a KR system with a set AT of attributes, we define a dependency relation $\rightarrow$ on set P(AT) of all the subsets of set AT as follows:

$Z \rightarrow T$ iff $\tilde{Z} \subseteq \tilde{T}$

Thus fulfilling condition $Z \rightarrow T$ means that if a pair of objects cannot be distinguished by means of attributes belonging to set Z, then it cannot be distinguished by attributes from set T. We say that

a set $T \subseteq AT$ is dependent on a set $Z \subseteq AT$ iff $Z \rightarrow T$ holds

Fact 5.6.1

The following conditions are equivalent:

(a) $Z \rightarrow T$ holds

(b) $\widetilde{Z \cup T} = \tilde{Z}$

This means that if set T is dependent on set Z then sets $Z \cup T$ and Z provide the same characterization of objects of the system. It follows that set T of attributes is superflous. The problem of reduction of sets of attributes will be discussed in the next section.

For any subsets Z, T, U and W of a set AT of attributes the following conditions are satisfied.

Fact 5.6.2

(a) $T \subseteq Z$ implies $Z \rightarrow T$

(b) $U \subseteq W$ and $Z \rightarrow T$ imply $Z \cup W \rightarrow T \cup U$

(c) $Z \rightarrow T$ and $T \rightarrow U$ imply $Z \rightarrow U$

(d) $Z \rightarrow T$ and $T \cup U \rightarrow W$ imply $Z \cup U \rightarrow W$

(e) $Z \rightarrow T$ and $Z \rightarrow U$ imply $Z \rightarrow T \cup U$

(f) $Z \rightarrow T \cup U$ implies $Z \rightarrow T$ and $Z \rightarrow U$

Let us observe that if $a \rightarrow b$ holds for a pair a, b of attributes in a system with an information function f then there is a functional re-

lationship between values of a and values of b, namely there exists a unique dependency function

$h: VAL_a \rightarrow VAL_b$

such that $f(o,b) = h(f(o,a))$

Let $E_{(a,v)}$ denote an equivalence class of indiscernibility $\tilde{a}$ consisting of those objects o for which $f(o,a) = V$. Then we have the following lemma.

Fact 5.6.3

The following conditions are equivalent:

(a) $a \rightarrow b$ holds

(b) $E_{(a,v)} \subseteq E_{(b,h(v))}$ for all $v \in VAL_a$

Let us observe that the definition of the dependency relation does not depend on a kind of information function. Hence all the facts presented in this section concern both deterministic and nondeterministic KR systems.

Example 5.6.1

In the system given by means of the table

	a_1	a_2	a_3	a_4
o_1	0	0	0	0
o_2	0	1	0	2
o_3	1	1	0	1
o_4	1	1	0	1
o_5	0	1	1	2

we have the following indiscernibility classes:

$\tilde{a}_1$: $\{o_1, o_2, o_5\}$ $\{o_3, o_4\}$

$\tilde{a}_2$: $\{o_1\}$ $\{o_2, o_3, o_4, o_5\}$

$\tilde{a}_3$: $\{o_1, o_2, o_3, o_4\}$ $\{o_5\}$

a_4 : $\{o_1\}$ $\{o_2, o_5\}$ $\{o_3, o_4\}$

Hence the following dependencies hold:

$a_4 \rightarrow a_2$

$a_4 \rightarrow a_1$

The corresponding dependency functions are as follows

h_1 h_2

$a_4 \rightarrow a_1$ $a_4 \rightarrow a_2$

0	0		0	0
2	0		2	1
1	1		1	1
2	0			

Example 5.6.2

In the system given in example 2.2.2 we have the dependency

Eyebrow Weight → Eyebrow Separation

The corresponding dependency function is as follows

Eyebrow Weight →	Eyebrow Separation
Thin	Sep
Bushy	Meet
Medium	Meet

Example 5.6.3

In the system given in example 2.2.3 we have the following dependencies:

{Volume Density, Numerical Density} → Surface Density

{Volume Density, Surface Density} → Numerical Density

Example 5.6.4

In the system given in example 2.2.4 we have the following dependency and the dependency function

Texture →	Body Spots
blank	one
shiped	many
crosshatched	many

We can generalize a concept of dependency function to sets of attributes. Let $Z = \{a_1,\dots,a_n\}$ and $T = \{b_1,\dots,b_k\}$. If $Z \to T$ holds then there exist functions h_b for $b \in T$ such that

$f(o,b_i) = h_{b_i}(f(o,a_1),\dots,f(o,a_n))$ for $i = 1,\dots,k$

Fact 5.6.4

The following conditions are equivalent:

(a) $Z \to T$ holds

(b) $\bigcap_{a \in Z} E(a,v) \subseteq E(b,h_b(v))$ for all $v \in VAL_a$ and for each $b \in T$

5.7. Reduction of sets of attributes

As we have seen in the previous section some attributes in a KR system may be removed from the system without a loss of information about objects. We discuss this problem in some details here. Let us

consider a system S with a set AT of attributes. We say that

a set $Z \subseteq AT$ is a reduct of AT in system S iff

Z is a minimal set such that $\widetilde{Z} = ind(S)$

Fact 5.7.1

The following conditions are equivalent

(a) A set Z is a reduct of AT in a system S

(b) $Z \to a$ holds for all attributes $a \in AT - Z$

The proof follows immediately from 5.6.2 (a), (f).

Example 5.7.1

Let us consider a system given by the following table

	a	b	c
o_1	0	0	1
o_2	0	1	0
o_3	1	1	1
o_4	1	1	1

We have the following dependencies in the system

{a, b} → c

{a, c} → b

{c, b} → a

and hence there are the three reducts of set {a, b, c} in the system: {a, b}, {a, c}, {c, b}.

Example 5.7.2

In the system given in example 2.2.3 due to the dependencies shown in example 5.6.3 we have the following reducts

{Volume Density, Numerical Density}

{Volume Density, Surface Density}

This means that in order to define pathological states of a cell it is not necessary to use all the three attributes, but it is sufficient to admit two of them as shown above.

Example 5.7.3

In the system presented in example 2.2.4 the only reduct of the set of attributes is as follows:

{Body Parts, Texture, Body Type}

These three attributes are sufficient to characterize uniquely the microorganisms considered in that example.

Given a system S with set AT of attributes and a reduct Z of set AT, by reduct of S we mean a system obtained from S by removing the attributes from set AT - Z and by considering the respective restriction of the information function of S.

Fact 5.7.2

The following conditions are equivalent:

(a) S' is a reduct of S

(b) $ind(S') = ind(S)$

(c) $S' \leq S$ and $S \leq S'$

Hence for any KR system S their reducts have the same expressive power as system S.

5.8. Logic INDL of indiscernibility relations

The logic considered in the following section is intended to provide a formal method for comparing an expressive power of knowledge representation systems. The expressive power of a system is represented by the indiscernibility relation of the system. A system S_1 is considered to be more expressive than a system S_2 iff indiscernibility relation $ind(S_1)$ is included in indiscernibility relation $ind(S_2)$. We define a formalized language which enables us to express facts concerning sets of objects in knowledge representation systems. We also give a deductive structure to the language and hence we are able to recognize valid facts or to infer facts from given facts. In particular we can axiomatize a class of selective systems.

Expressions of the logic are intended to represent sets of objects. They are built up from atomic expressions, i.e. variables by means of operations corresponding to set-theoretical operations and operations of upper and lower approximation. To define formulas of the logic we use symbols from the following non-empty at most denumerable and pairwise disjoint sets:

set VAROB of variables representing sets of objects

set CONREL of constants representing indiscernibility relations

set $\{\neg, \vee, \wedge, \rightarrow, \leftrightarrow\}$ of propositional operations of negation disjunction, implication and equivalence

set $\{\_, \bar{\ }\}$ of operations of lower approximation and upper approximation.

Set FORINDL of all formulas of the logic is the least set satisfying the following conditions:

VAROB $\subseteq$ FORINDL

if $A, B \in$ FORINDL then $\neg A$, $A \vee B$, $A \wedge B$, $A \rightarrow B$, $A \leftrightarrow B \in$ FORINDL

if $R \in$ CONREL and $A \in$ FORINDL then $\underline{R}A$, $\overline{R}A \in$ FORINDL.

Formulas of the form $\neg A$, $A \vee B$, and $A \wedge B$ are intended to represent complement, union, and intersection of sets of objects represented by A and B, respectively. Expression $A \rightarrow B$ represents the union of complement of a set corresponding to A and a set corresponding to B. Expression $A \leftrightarrow B$ represents the intersection of sets of objects determined by $A \rightarrow B$ and $B \rightarrow A$. Lastly, expressions $\underline{R}A$ and $\overline{R}A$ represent the lower and upper approximation of a set corresponding to A with respect to an indiscernibility relation R.

We define meaning of the formulas of the given language by means of notions of model and satisfiability of the formulas in a model. By a model we mean a triple

$M = (OB, m, v)$

where OB is a non-empty set of objects

m is a meaning function which assigns equivalence relations on set OB to constants from set CONREL

v is a function from set VAROB into set P(OB) of all the subsets of set OB.

By induction with respect to a structure of a formula we define the notion of satisfiability of the formulas in a model. We say that a formula A is satisfied in a model M by an object $o \in OB$ (M, o sat A) iff the following conditions are satisfied:

M, o sat p iff $o \in v(p)$ for $p \in$ VAROB

M, o sat $\neg A$ iff not M, o sat A

M, o sat $A \vee B$ iff M, o sat A or M, o sat B

M, o sat $A \wedge B$ iff M, o sat A and M, o sat B

M, o sat $A \rightarrow B$ iff not M, o sat A or M, o sat B

M, o sat $A \leftrightarrow B$ iff M, o sat $A \rightarrow B$ and M, o sat $B \rightarrow A$

M, o sat $\underline{R}A$ iff for all $o' \in OB$ if $(o, o') \in m(R)$ then M, o' sat A

M, o sat $\overline{R}A$ iff there is an $o' \in OB$ such that $(o, o') \in m(R)$ and M, o' sat A.

We say that a set T of formulas is satisfied in a model M by an object o (M, o sat T) iff for each formula $A \in T$ we have M, o sat A. A set T is satisfiable iff there is a model M and an object o such that M, o sat T.

According to the given semantics to each formula A of the language there is associated the set of those objects which satisfy the formula in a model; we call this set extension of formula in model

$ext_M A = \{o \in OB\colon M, o \text{ sat } A\}$

Extension of compound formulas depend on the extensions of their components in the following way

Fact 5.8.1

(a) $ext_M p = v(p)$ for $p \in$ VAROB

(b) $ext_M \neg A = -ext_M A$

(c) $ext_M A \vee B = ext_M A \cup ext_M B$

(d) $ext_M A \wedge B = ext_M A \cap ext_M B$

(e) $ext_M A \rightarrow B = -ext_M A \cup ext_M B$

(f) $ext_M A \leftrightarrow B = ext_M (A \rightarrow B) \cap ext_M (B \rightarrow A)$

(g) $ext_M \underline{R}A = \underline{m(R)}\ ext_M A$

(h) $ext_M \overline{R}A = \overline{m(R)}\ ext_M (A)$

Axioms of INDL

A1. All formulas having the form of a tautology of the classical propositional logic.

A2. $\underline{R}(A \rightarrow B) \rightarrow (\underline{R}A \rightarrow \underline{R}B)$

A3. $\underline{R}A \rightarrow A$

A4. $A \rightarrow \underline{R}\neg\underline{R}\neg A$

A5. $\underline{R}A \rightarrow \underline{R}\underline{R}A$

Rules of inference

R1 $\dfrac{A, A \rightarrow B}{B}$ R2 $\dfrac{A}{\underline{R}A}$

This axiomatization corresponds very closely to the axiomatization for modal logic S5 (Gabbay (1976)), however a difference consists in considering a family of equivalence relations in the language and in models.

The proof of completeness of logic INDL follows closely the earlier completeness proofs. A cononical model in set FT of all the maximal filters of Boolean algebra AINDL is defined as follows:

$M_o = (OB_o, m_o, v_o)$

where $OB_o = FT$

$$m_o(R) = \{(F_1, F_2) \in FT \times FT: \text{for any formula } A \text{ if } [\underline{R}A] \in F_1 \text{ then } [A] \in F_2\}$$

$$v_o(p) = \{F \in FT: [p] \in F\}$$

Fact 5.8.2

For any $R \in$ CONREL $m_o(R)$ is an equivalence relation.

Proof: By axioms A2 and 10.2 (b) we have $[\underline{R}A] \leq [A]$. Hence if $[\underline{R}A] \in F$ then $[A] \in F$, and so relation $m_o(R)$ is reflexive. Let us now assume that $(F_1, F_2) \in m_o(R)$, $[\underline{R}A] \in F_2$ and suppose $[A] \notin F_1$. Hence, since F_1 is a maximal filter, we have $[\neg A] \in F_1$. By axiom A4 we have $[\underline{R}\neg\underline{R}A] \in F_1$. Thus $[\neg\underline{R}A] \in F_2$, a contradiction. Hence relation $m_o(R)$ is symmetric.

Let us now assume that $(F_1, F_2) \in m_o(R)$, $(F_2, F_3) \in m_o(R)$, $[\underline{R}A] \in F_1$, and suppose $[A] \notin F_3$. By axiom A5 we have $[\underline{R}\ \underline{R}\ A] \in F_1$, and hence $[\underline{R}A] \in F_3$. It follows that $[A] \in F_3$, a contradiction. Hence relation $m_o(R)$ is transitive.

As in the previous completeness proofs the key lemma is as follows:

Fact 5.8.3

The following conditions are equivalent

(a) M_o, F sat A

(b) $[A] \in F$

Proof: The proof is by induction with respect to a structure of a formula. For variables and formulas of the form $\neg B$ and $B \rightarrow C$ the proof is easily obtained from the respective definitions. We prove the lemma for a formula of the form $\underline{R}B$. Let us assume that M_o, F sat $\underline{R}B$ and suppose $[\underline{R}B] \notin F$. We consider set $Z_{FR} = \{[C] : [\underline{R}C] \in F\}$. We now prove four properties of this set.

(1) Set Z_{FR} is non-empty

It follows from the fact that $[\underline{R}(A \vee \neg A)] \in Z_{FR}$

(2) Set Z_{FR} is a filter

We have $[B_1] \cap [B_2] \in Z_{FR}$ iff $[B_1 \wedge B_2] \in Z_{FR}$. Hence $[\underline{R}(B_1 \wedge B_2)] \in F$. Since $\vdash \underline{R}(A \wedge B) \leftrightarrow \underline{R}A \wedge \underline{R}B$, we have $[\underline{R}B_1 \wedge \underline{R}B_2] \in F$. It is equivalent to $[\underline{R}B_1] \in F$ and $[\underline{R}B_2] \in F$. Hence $[B_1] \in Z_{FR}$ and $[B_2] \in Z_{FR}$.

(3) Filter Z_{FR} is a proper filter

Let us suppose that $0 \in Z_{FR}$. Then we have $[\underline{R}(A \wedge \neg A)] \in F$ and hence $1 = [\underline{R}(A \vee \neg A)] \notin F$, a contradiction.

(4) Filter G generated by set $Z_{FR} \cup \{[\neg B]\}$ is a proper filter

We show that for any $[A_1], \ldots, [A_n] \in Z_{FR}$, for $n \geq 1$, we have $[A_1] \cap \ldots \cap [A_n] \cap [\neg B] \neq 0$. For suppose not, then we have $T \vdash A_1 \wedge \ldots \wedge A_n \wedge \neg B \rightarrow A \wedge \neg A$, and hence $T \vdash A_1 \wedge \ldots \wedge A_n \rightarrow B$. By rule R2 and axiom A2 we obtain $T \vdash \underline{R}A_1 \wedge \ldots \wedge \underline{R}A_n \rightarrow \underline{R}B$. Since $[A_1], \ldots, [A_n] \in Z_{FR}$, we have $[\underline{R}A_1], \ldots, [\underline{R}A_n] \in F$, and hence $[\underline{R}A_1 \wedge \ldots \wedge \underline{R}A_n] \in F$. So $[\underline{R}B] \in F$ and this is in conflict with the supposition.

It follows that filter G can be extended to a maximal filter H such that $[\neg B] \in H$ and for any formula C if $[\underline{R}C] \in F$ then $[C] \in H$. Hence $(F, H) \in m_o(R)$ and M_o, H sat $\neg B$, but this is a contradiction with the assumption. Let us now assume that $[\underline{R}B] \in F$ and consider set Z_{FR}. We have $[B] \in Z_{FR}$. By Kuratowski-Zorn lemma there is a maximal filter G which includes set Z_{FR}, and hence $(F,G) \in m_o(R)$ and $[B] \in G$. But Z_{FR} is included in every filter G such that $(F,G) \in m_o(R)$, thus $[B]$ belongs to every such filter. By the induction hypothesis we have M_o, G sat B for all G satisfying $(F,G) \in m_o(R)$. Hence M_o, F sat $\underline{R}B$.

The above lemma enables us to prove completeness and compactness of logic INDL.

5.9. Properties of KR systems expressible in logic INDL

In this section we show how formulas of the given language can be used to express properties of sets of objects and properties of knowledge representation systems.

Fact 5.9.1

(a) $\models_M A \to B$ iff $ext_M A \subseteq ext_M B$

(b) $\models_M A \leftrightarrow B$ iff $ext_M A = ext_M B$

(c) $\models_M \overline{R}A \to \underline{R}A$ iff $ext_M A$ is definable in a system S such that ind(S) = m(R)

(d) $\models_M \underline{R}A$ iff $ext_M A$ is internally nondefinable in a system S such that ind(S) = m(R)

(e) $\models_M \overline{R}A$ iff $ext_M A$ is externally nondefinable in a system S such that ind(S) = m(R)

(f) $\models_M \neg(\overline{R}A \to \underline{R}A)$ iff $ext_M A$ is totally nondefinable in a system S such that ind(S) = m(R).

The proof follows immediately from the definition of satisfiability.

In the next lemma we list some properties of a knowledge representation system related to a model. Let a model M = (OB, m, v) be given and let S be a system such that ind(S) = m(R) for a certain $R \in$ CONREL.

Fact 5.9.2

(a) $\models_M \overline{R}A \to \underline{R}A$ for every $A \in$ FOR iff system S is selective

(b) $\models_M \overline{R}(A \wedge B) \wedge \overline{R}(A \wedge \neg B)$ for every $A, B \in$ FOR iff equivalence classes of ind(S) have at least two elements

(c) $\models_M \overline{R}(A \wedge B) \wedge \overline{R}(A \wedge \neg B) \to \underline{R}(A)$ for every $A, B \in$ FOR iff each equivalence class of ind(S) has exactly two elements.

Proof: The formula in condition (a) says that for any A the upper

approximation of a set corresponding to A is included in its lower approximation. By 5.3.3 (b) and 5.3.4 (b) condition (a) holds. The formula in condition (b) says that in model M for any object o there are objects o_1 and o_2 such that $o_1 \in ext_M A$, $o_1 \in ext_M B$, $(o, o_1) \in m(R)$, $o_2 \in ext_M A$, $o_2 \in ext_M B$, and $(o, o_2) \in m(R)$. m(R) is an equivalence relation and we possibly have $o_1 = o$ or $o_2 = o$ but not $o_1 = o_2$, since o_1 and o_2 are separated by $ext_M B$. Hence condition (b) holds. In the formula from condition (c) the left hand side of implication guarantees the existence of an object o satisfying condition (b). The formula on the right hand side of this implication says that this object is the only one satisfying this condition.

It is easy to see that in the similar way we can define formulas expressing the fact that in a system related to a model each elementary set has at least or exactly n elements, for $n \geq 1$.

In the following we list some formulas which express relations between knowledge representation systems. Let a model M = (OB, m, v) be given and let S_1, S_2 and S_3 be the systems such that $ind(S_i) = m(R_i)$ for i = 1, 2, 3 for some constants R_1, R_2, and R_3.

Fact 5.9.3

(a) $\models_M \overline{R}_1 A \to \overline{R}_2 A$ for every $A \in$ FORINDL iff $S_1 \leq S_2$

(b) $\models_M \underline{R}_2 A \to \underline{R}_1 A$ for every $A \in$ FORINDL iff $S_1 \leq S_2$

(c) $\models_M (\underline{R}_1 A \to \underline{R}_3 A) \wedge (\underline{R}_2 A \to \underline{R}_3 A)$ for every $A \in$ FORINDL and $\models_M (\underline{R}_1 A \to \underline{R}A) \wedge (\underline{R}_2 A \to \underline{R}A) \to (\underline{R}_3 A \to \underline{R}A)$ for every $A \in$ FORINDL and every $R \in$ CONREL iff $ind(S_3) = ind(S_1) \cap ind(S_2)$

(d) $\models_M (\underline{R}_3 A \to \underline{R}_1 A) \wedge (\underline{R}_3 A \to \underline{R}_2 A)$ for every $A \in$ FORINDL and $\models_M (\underline{R}A \to R_1 A) \wedge (\underline{R}A \to \underline{R}_2 A) \to (\underline{R}A \to \underline{R}_3 A)$ for every $A \in$ FORINDL and every $R \in$ CONREL iff $ind(S_3) = (ind(S_1) \cup ind(S_2))^*$

Proof: The formula in condition (a) says that for any A the upper approximation of $ext_M A$ in system S_1 is included in its upper approximation in system S_2. By 5.5.1 condition (a) holds. Similarly, condition (b) follows from 5.5.2. The first formula in condition (c) says that relation $ind(S_3)$ is included both in $ind(S_1)$ and $ind(S_2)$. The second formula says that $ind(S_3)$ is the greatest relation with that property and hence condition (c) holds. The formulas given in condition (d) say that $ind(S_3)$ is the least relation containing both $ind(S_1)$

and ind(S_2). Since these relations are equivalences, ind(S_3) is the transitive closure of ind(S_1)∪ind(S_2). Thus condition (d) holds.

5.10. Summary

The central aim of this chapter has been to consider the broader implications that follow from the techniques of knowledge representation developed in chapters 2, 3 and 4. We investigated how the expressive power of any KR system is influenced by the indiscernibility of objects in the system. Proposals have been made for considering approximate definability of sets of objects to reflect deep structures of concepts which are meaningful for the system. We developed logical formalism providing tools for the examination of expressive power of KR systems in terms of indiscernibility relations.

We have also dealt with questions of what are the criteria for guiding the selection of attributes in KR systems.

The methodological problems attempted here can easily be formulated for KR systems of temporal information. It is only necessary to consider indiscernibility relations for each moment of time. In this way any KR system determines a family of indiscernibility relations and hence we can consider definability of concepts at a certain moment. Similarly, we can reconstruct dependencies of attributes with the reference to the time scale.

6. LOGIC REPRESENTATION OF INFORMATION

6.1. Information logic IL

In this section we introduce a language which is expressive enough to represent a wide variety of facts related to deterministic, many--valued and nondeterministic information about objects. We show how inferences can be made from sets of expressions in the language and we discuss how to deduce statements from the other statements. The basic concepts which have their counterparts in the language are object, attribute, value of attribute and fact. As previously, an object is anything we want to store information about. A property an object might have in a certain real world state is expressed by using the notion of an attribute (e.g. colour) and an attribute value (e.g. green). Any pair consisting of an attribute and a value of this attribute represents an atomic property of objects. From a logic point of view atomic properties are one-place predicates e.g. (colour, green) (x), where x is a variable ranging over a set of objects, is the predicate which results in a true predication whenever x takes a green object as its value. From atomic properties we form compound properties by using logical operations, e.g. (colour, green) (x) or (colour, black) (x) is the property which results in a true proposition whenever x takes a green or a black object as its value.

A sentence stating that a property holds or does not holds for an object is called fact. For instance, if objects we are interested in are plane figures F_1, F_2, and F_3 then the following sentence is a fact: (shape, oval) (F_1) and (shape, triangle) (F_2) and not(shape, ellipse) (F_3).

Since in almost all applications a huge number of possible real world facts is involved, it is impossible to specify a system by listing of all imaginable facts. One rather has to use an axiomatic definition of possible facts. Any collection of facts providing an information about the current state of given objects will be treated as an KR system. Facts listed in a system are called explicit facts; they play the role of axioms. To enable us inferring consequences of explicit facts we develop a logic called information logic, and the proof procedure for the logic which is based on the deduction methods in the ordinary predicate logic. The facts which can be derived from explicit facts are called implicit facts; they play the role of theorems.

Set of symbols of the language of logic IL is the union of the following nonempty, at most denumerable, and pairwise disjoint sets:

a set VAROB of object variables, denoted by $x, y, x_1, y_1, \ldots$

a set CONAT of attribute constants, denoted by $a, a_1, \ldots$

a set CONVAL of attribute value constants, denoted by $w, w_1, \ldots$

a set CONOB of object constants, denoted by $o, o_1, \ldots$

set $\{\neg, \vee, \wedge, \rightarrow, \leftrightarrow\}$ of propositional operations of negation, disjunction, conjunction, implication, and equivalence, respectively

set $\{\exists, \forall\}$ of quatifiers, called existential quantifier and universal quantifier, respectively.

Set FORIL of formulas of logic IL is the least set satisfying the following conditions:

if $x \in$ VAROB, $o \in$ CONOB, $a \in$ CONAT, and $w \in$ CONVAL then (x,a,w) and $(o,a,w) \in$ FORIL

if $A, B \in$ FORIL then $\neg A$, $A \vee B$, $A \wedge B$, $A \rightarrow B$, and $A \leftrightarrow B \in$ FORIL

if A FORIL then $\exists x\ A$ and $\forall x\ A \in$ FORIL

As usually we assume that formulas do not contain redundant or overlapping quantifiers. Moreover, we adopt the usual definition of free and bound variable. An object variable in a formula is said to have a bound occurrence if it stands within the scope of a quantifier with the same object variable; otherwise it is said to have a free occurrence. A formula without free variables is called a sentence.

Let DES be the set of all pairs (a,w) for $a \in$ CONAT and $w \in$ CONVAL. Elements of set DES are called descriptors.

It is easy to see that there is a correspondence between formulas of logic IL and formulas of the classical predicate calculus PC. We can treat descriptors as monadic predicate symbols and then formulas of logic IL can be considered as formulas of the monadic predicate calculus MPC.

In the language of logic IL we have introduced three kinds of constants. Thus semantics of the language should be defined by using a three-sorted universe and a meaning function, which assigns elements of the universe to constants of the language.

Let OB and AT be nonempty sets, called set of objects and set of attributes, respectively. For each attribute $a \in$ AT let VAL_a be a nonempty set called set of values of attribute a. Let $VAL = \bigcup_{a \in AT} VAL_a$.

The system U = (OB, AT, VAL) is called universe. Any mapping m: CONOB $\cup$ CONAT $\cup$ CONVAL$\rightarrow$OB $\cup$ AT $\cup$ VAL such that m(CONOB) = OB, m(CONAT) = AT and m(CONVAL) = VAL is called meaning function over the universe U. Given a universe U, by a valuation over U we mean any function v:VAROB $\rightarrow$OB.

By a model for logic IL we mean any system

M = (U, m, f)

where U is a universe, m is a meaning function over U and $f \subseteq$ OBxATxVAL is a nonempty relation such that if $(o,a,w) \in f$ then $w \in VAL_a$.

Given a model M = (U, m, f) and a valuation v over U, we say that a formula A is satisfied by v in M (M, v sat A) whenever the following conditions are satisfied:

M, v sat (x a w) iff $(v(x), m(a), m(w)) \in f$

M, v sat (o a w) iff $(m(o), m(a), m(w)) \in f$

M, v sat $\neg$A iff not M, v sat A

M, v sat $A \vee B$ iff M, v sat A or M, v sat B

M, v sat $A \wedge B$ iff M, v sat A and M, v sat B

M, v sat $A \rightarrow B$ iff not M, v sat A or M, v sat B

M, v sat $A \leftrightarrow B$ iff M, v sat $A \rightarrow B$ and M, v sat $B \rightarrow A$

M, v sat $\exists$ xA iff there is a $p \in$ OB such that M, v_p sat A

M, v sat $\forall$ xA iff for all $p \in$ OB we have M, v_p sat A

where v_p is the valuation over U such that $v_p(x) = p$ and $v_p(y) = v(y)$ for $y \neq x$.

A formula A is said to be true in a model M = (U, m, f) ($\models_M A$) iff for every valuation v over universe U we have M, v sat A. A formula A is said to be valid in logic IL ($\models A$) iff A is true in every model for IL. A set T of formulas is said to be true in a model M if every formula $A \in T$ is true in M. A set T is said to be satisfiable iff there is a model M such that T is true in M. We say that formula A is semantical consequence of a set T ($T \models A$) iff for every model M formula A is true in M whenever T is true in M.

We give a deductive structure to the language of IL in the usual way, first specifying a recursive set of axioms and inference rules and then defining a syntactical consequence operation.

The axiomatization of IL corresponds very closely to the axiomatization of the classical predicate calculus PC. Axioms of IL are substitutions of the axioms of PC, that is they are formulas of IL which have the form of theorems of PC. Similarly, rules of inference of IL are substitutions of the rules of PC.

Axioms of IL

A1. $A \rightarrow (B \rightarrow A)$

A2. $(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$

A3. $A \rightarrow (\neg A \rightarrow B)$

A4. $(\neg A \rightarrow A) \rightarrow A$

A5. $\forall x(A \rightarrow B(x)) \rightarrow (A \rightarrow \forall x B(x))$, where x is any object variable not occurring free in A

A6. $\forall x A(x) \rightarrow A(o)$, where $o \in$ CONOB.

The propositional operations $\vee, \wedge, \rightarrow$ and the existential quantifier $\exists$ can be defined by means of implication, negation and universal quantifier:

$A \vee B = \neg A \rightarrow B$

$A \wedge B = \neg(A \rightarrow \neg B)$

$A \leftrightarrow B = (A \rightarrow B) \wedge (B \rightarrow A)$

$\exists xA = \neg\forall x \neg A$.

Rules of inference of IL

$\dfrac{A,\ A \rightarrow B}{B}$ modus ponens

$\dfrac{A}{\forall xA}$ generalization

A derivation of a formula A from a set T of formulas is a finite sequence of formulas each of which is either an axiom or a formula from T or else is obtainable from earlier formulas by one of the rules of inference, and A is the last formula in the sequence. We now define the syntactical consequence operation. Formula A is said to be derivable from set T of formulas ($T \vdash A$) if there is a derivation of A from T. A formula A is a theorem of IL ($\vdash A$) if it is derivable merely from the axioms. A set of formulas is consistent if the formula of the form $A \wedge \neg A$ is not derivable from T. A set is inconsistent if it is not consistent.

The axioms of IL are easily seen to be valid in IL and the rules of inference clearly preserve validity. Hence we have the following theorem.

<u>Fact 6.1.1</u> (Soundness theorem)

(a) $\vdash A$ implies $\models A$

(b) $T \vdash A$ implies $T \models A$

(c) T satisfiable implies T consistent.

Given set T of formulas, let D(T) denote the set of all formulas derivable from T, that is $D(T) = \{A \in \text{FORIL}: T \vdash A\}$. We can treat set D(T) as the image of the set T under the operation D. The following properties of the operation D are well known.

<u>Fact 6.1.2</u>

(a) $T \subseteq D(T)$

(b) $D(D(T)) = D(T)$

(c) $T_1 \subseteq T_2$ implies $D(T_1) \subseteq D(T_2)$

(d) $D(T_1 \cup T_2) = D(D(T_1) \cup D(T_2))$.

In the following we list some other useful properties of the operation D.

<u>Fact 6.1.3.</u>

(a) $\vdash A$ implies $A \in D(T)$ for any $T \subseteq$ FORIL

(b) $A \in D(T)$ and $(A \rightarrow B) \in D(T)$ imply $B \in D(T)$

(c) $A \in D(T)$ implies $\forall xA \in D(T)$

(d) $B \in D(T \cup \{A\})$ iff $(A \rightarrow B) \in D(T)$

(e) $A \notin D(T)$ iff $T \cup \{\neg A\}$ inconsistent.

We now sketch the method of establishing completeness of logic IL. This method follows closely the earlier proofs of completeness given in chapter 2, 3 and 4.

Given a consistent set T of formulas of logic IL, we define a congruence in set FORIL determined by set T and we consider algebra AIL of all the equivalence classes of this congruence.

<u>Fact 6.1.4</u> (Completeness theorem)

(a) $\models A$ implies $\vdash A$

(b) $T \models A$ implies $T \vdash A$

(c) T consistent implies T satisfiable

Proof: Let us assume that $T \models A$ holds and suppose not $T \vdash A$. We conclude that $[\neg A] \neq 0$ and by Rasiowa - Sikorski lemma there is a Q-filter F in algebra AIL such that $[\neg A] \in F$. We then define a canonical universe $U_o = (OB_o, AT_o, VAL_o)$ and a canonical model $M_o = (U_o, m_o, f_o)$ where

OB_o = CONOB $\cup$ VAROB

AT_o = CONAT

VAL_o = CONVAL

$(p, a, w) \in f_o$ iff $[(p\ a\ w)] \in F$ for all $p \in OB_o$, $a \in AT_o$, and $w \in VAL_o$.

$m_o(o) = o$, $m_o(a) = a$, $m_o(w) = w$

Let v_o: VAROB $\rightarrow OB_o$ be a valuation over universe U_o such that $v_o(x) = x$ for any object variable x. We have the following lemma:

M_o, v_o sat A iff $[A] \in F$

We prove this condition for a formula A of the form $\exists xB(x)$. For the remaining formulas the proof is similar to that followed in previous chapters. Let us assume that M_o, v_o sat $\exists xB(x)$. Hence there is a $p \in OB_o$ such that M_o, v_{op} sat $B(x)$, where $v_{op}(x) = p$ and $v_{op}(y) =$

$v_o(y)$ for $y \neq x$. Hence M_o, v_o sat $B(p)$. Suppose now $[\exists x B(x)] \notin F$. We conclude that $[\forall x B(x)] \in F$ and by A6 $[\neg B(p)] \in F$. By the induction hypothesis M_o, v_o sat $\neg B(p)$, a contradiction. Let us then assume that $[\exists x B(x)] \in F$. Since F is a Q-filter, there is a $p \in OB_o$ such that $[B(p)] \in F$. By the induction hypothesis M_o, v_o sat $B(p)$. Hence M_o, v_o sat $\exists x B(x)$.

It is now easy to see that since $[\neg A] \in F$, we have M_o, v_o sat $\neg A$. But for each $B \in T$ we have $[B] = 1$. It follows that for any Q-filter F we have $[B] \in F$. Hence M_o, v_o sat B for all formulas $B \in T$. We conclude that M_o, v_o sat T and M_o, v_o sat $\neg A$, a contradiction.

We now consider implicational formulas of logic IL which can play a role of what is called production rule in AI systems. In fact we consider a slightly more general form of rules:

$A_1 \wedge \ldots \wedge A_n \to B_1 \vee \ldots \vee B_m$ for nonnegative n, m

The formulas $A_1, \ldots, A_n$ form the antecedent and the formulas $B_1, \ldots, B_m$, the succedent of the rule. Both expressions may be empty. If the antecedent is empty the rule reduces to the formula $B_1 \vee \ldots \vee B_m$. This means the same as if a valid formula stood in the antecedent. If the succedent is empty, the rule means the same as the formula $\neg(A_1 \wedge \ldots \wedge A_n)$. This means the same as if a negation of a valid formula stood in the succedent.

In the following we present some properties of rules which may be useful in deriving new rules. These properties correspond very closely to the Gentzen inference figure schemata for the classical logic (Szabo M.E. (ed) (1969)). Following Gentzen-style formalism, instead of formula $A_1 \wedge \ldots \wedge A_n \to B_1 \vee \ldots \vee B_m$ we write $A_1, \ldots, A_n \to B_1, \ldots, B_m$. The informal meaning of this expression is no different from that of the above formula; the expression differ merely in their formal structure. In what follows T_1, T_2, T_3, and T_4 denote finite or empty sequences of formulas.

Fact 6.1.5

($\to\wedge$) $\models_M T_1 \to T_2, A$ and $\models_M T_1 \to T_2, B$ imply $\models_M T_1 \to T_2, A \wedge B$

($\wedge\to$) $\models_M A, T_1 \to T_2$ or $\models_M B, T_1 \to T_2$ imply $A \wedge B, T_1 \to T_2$

($\to\vee$) $\models_M T_1 \to T_2, A$ or $\models_M T_1 \to T_2, B$ imply $\models_M T_1 \to T_2, A \vee B$

($\vee\to$) $\models_M A, T_1 \to T_2$ and $\models_M B, T_1 \to T_2$ imply $\models_M A \vee B, T_1 \to T_2$

($\to\neg$) $\models_M A, T_1 \to T_2$ implies $\models_M T_1 \to T_2, \neg A$

($\neg\to$) $\models_M T_1 \to T_2, A$ implies $\models_M \neg A, T_1 \to T_2$

($\to\forall$) $\models_M T_1 \to T_2, A(y)$ implies $\models_M T_1 \to T_2, \forall x A(x)$ and y must not occur neither in $A(x)$ nor in any formula of T_1 and T_2

($\forall\to$) $\models_M A(o), T_1 \to T_2$ implies $\models_M \forall x A(x), T_1 \to T_2$ for $o \in$ CONOB

($\to\exists$) $\models_M T_1 \to T_2, A(o)$ implies $\models_M T_1 \to T_2, \exists x A(x)$ for $o \in$ CONOB

($\exists\to$) $\models_M A(y), T_1 \to T_2$ implies $\models_M \exists x A(x), T_1 \to T_2$ and y must not occur neither in $A(x)$ nor in any formula of T_1 and T_2

(a) $\models_M T_1 \to T_2$ implies $\models_M A, T_1 \to T_2$ and $\models_M T_1 \to T_2, A$

(b) $\models_M A, A, T_1 \to T_2$ implies $\models_M A, T_1 \to T_2$

(c) $\models_M T_1 \to T_2, A, A$ implies $\models_M T_1 \to T_2, A$

(d) $\models_M T_1, A, B, T_2 \to T_3$ implies $\models_M T_1, B, A, T_2 \to T_3$

(e) $\models_M T_1 \to T_2, A, B, T_3$ implies $\models_M T_1 \to T_2, B, A, T_3$

(f) $\models_M T_1 \to T_2, A$ and $\models_M A, T_3 \to T_4$ imply $\models_M T_1, T_3 \to T_2, T_4$

6.2. Logic knowledge representation systems

The language defined in the previous section provides a means for representing information determined by a universe. Formulas of the language of IL can be treated as schemes of sentences which express knowledge about objects of the universe. Given a universe U = (OB, AT, VAL) and a meaning function m over U such that m(CONOB) = OB, m(CONAT) = AT and m(CONVAL) = VAL, we define set FORIL(U) of expressions which are obtained from formulas of IL through assignment of values of constants determined by the mapping m for these constants. The expressions from set FORIL(U) are referred to as U-formulas. U-formulas without free variables are called U-sentences. U-sentences express knowledge about universe U.

The concept of derivability can be extended in a natural way to U-formulas. We say that U-formula A is derivable from set T of U-formulas whenever it can be obtained from U-formulas having the form of axioms of IL or members of T by repeated application of inference rules of IL. We use the same notation as for IL, namely we write $T \vdash A$ whenever U-formula A is derivable from set T of U-formulas.

Given a universe U = (OB, AT, VAL) and a nonempty set T of U-sentences, by logic KR system over U we mean system

S = (U, T)

Thus a logic KR system consists of a nonempty set OB of objects, a nonempty set of characteristics of objects represented by attributes and values of attributes, and a nonempty set of sentences which express assumptions concerning properties of objects.

The following example explains how any system of deterministic information can be defined as a logic KR system.

Example 6.2.1

Consider a KR system which provides information about languages (Lan) (French (F), Hungarian (H), German (D), Swedish (S) and Romanian (R)) which persons $P_1, P_2, P_3, P_4, P_5, P_6$ speak, and about degrees (Deg) (bachelor of science (BS), master of science (MS) and philosophy doctor (PhD)) they have. The respective logic KR system S_1 is defined as follows:

$OB = \{P_1, P_2, P_3, P_4, P_5, P_6\}$

$AT = \{Lan, Deg\}$

$VAL_{Lan} = \{F, H, D, S, R\}$

$VAL_{Deg} = \{BS, MS, PhD\}$

$VAL = VAL_{Lan} \cup VAL_{Deg}$

$U = (OB, AT, VAL)$

Let T_1 be the following set of U-sentences

$(P_1\ Lan\ F) \wedge (P_5\ Lan\ F)$

$(P_2\ Lan\ H)$

$(P_3\ Lan\ D)$

$(P_4\ Lan\ S)$

$(P_6\ Lan\ R)$

$(P_1\ Deg\ PhD)$

$(P_2\ Deg\ BS) \wedge (P_5\ Deg\ BS) \wedge (P_6\ Deg\ BS)$

$(P_3\ Deg\ MS) \wedge (P_4\ Deg\ MS)$

Moreover, T_1 contains all the sentences of the following schemes:

$\forall x((x\ Lan\ p) \leftrightarrow \neg(x\ Lan\ p_1) \wedge \neg(x\ Lan\ p_2) \wedge \neg(x\ Lan\ p_3) \wedge \neg(x\ Lan\ p_4))$

where $p, p_1, p_2, p_3, p_4 \in \{F, H, D, S, R\}$, and for each $i, j \in \{1,2,3,4\}$ we have $p_i \neq p$ and $p_i \neq p_j$.

$\forall x((x\ Deg\ q) \leftrightarrow \neg(x\ Deg\ q_1) \wedge \neg(x\ Deg\ q_2))$

where $q, q_1, q_2 \in \{BS, MS, PhD\}$, and for each $i, j \in \{1, 2\}$ we have

$q \neq q_i$ and $q_i \neq q_j$.

Let $S_1 = (U, T_1)$. Set T_1 can be considered as a definition of information function f_1: $OB \times AT \to VAL$ given by the following table:

	Lan	Deg
P_1	F	PhD
P_2	H	BS
P_3	D	MS
P_4	S	MS
P_5	F	BS
P_6	R	BS

Obviously, system (U, f_1) is the system of deterministic information. It is easy to see that any deterministic KR system can be represented as a logic KR system.

We now consider an example of a logic KR system which corresponds to many-valued KR system.

Example 6.2.2

Consider universe U from example 6.2.1 and the following set T_2 of U-sentences:

$(P_1\ Lan\ F) \wedge (P_1\ Lan\ D)$

$(P_2\ Lan\ H) \wedge (P_2\ Lan\ R)$

$(P_3\ Lan\ D) \wedge (P_3\ Lan\ F) \wedge (P_3\ Lan\ S)$

$(P_4\ Lan\ F)$

$(P_5\ Lan\ F) \wedge (P_5\ Lan\ D)$

$(P_6\ Lan\ R)$

$(P_1\ Deg\ BS) \wedge (P_1\ Deg\ MS) \wedge (P_1\ Deg\ PhD)$

$(P_2\ Deg\ BS) \wedge (P_5\ Deg\ BS) \wedge (P_6\ Deg\ BS)$

$(P_3\ Deg\ BS) \wedge (P_3\ Deg\ MS)$

$(P_4\ Deg\ BS) \wedge (P_4\ Deg\ MS)$

Set T_2 can be considered as a definition of the information relation $f_2 \subseteq OB \times AT \times VAL$ given by the following table:

	Lan	Deg
P_1	F, D	BS, MS, PhD

P_2	H, R	BS
P_3	F, D, S	BS, MS
P_4	F,	BS, MS
P_5	F, D	BS
P_6	R	BS

System (U, f_2) is the system of many-valued information.

Example 6.2.3

Consider universe U from example 6.2.1 and the following set T_3 of U-sentences:

$(P_1$ Lan F) v $(P_1$ Lan D) v $(P_1$ Lan S)

$(P_2$ Lan H) v $(P_2$ Lan R) v $(P_2$ Lan F) v $(P_2$ Lan D)

$(P_3$ Lan F) v $(P_3$ Lan H)

$(P_4$ Lan F)

$(P_5$ Lan R)

$(P_6$ Lan D)

$\forall$x((x Deg BS) v (x Deg MS) v (x Deg PhD)

In system $S_3 = (U, T_3)$ we have incomplete information about languages persons P_1, P_2, and P_3 speak. The first formula says that P_1 speaks French or German or Swedish. This means that P_1 speaks at least one of these languages. Similarly, P_2 speaks Hungarian or Romanian or French or German, and P_3 speaks French or Hungarian. This is a kind of incomplete information when we can only know that values of a certain attribute for a certain object belong to a given subset of values.

The given formulas do not enable us to define an information relation. We can define only the family of sets $W_{oa} \subseteq VAL_a$ for $o \in OB$ and $a \in AT$ such that if $w \in W_{oa}$ then w is a possible value of attribute a for object o. We have:

$W_{P_1Lan} = \{F, D, S\}$

$W_{P_2Lan} = \{H, R, F, D\}$

$W_{P_3Lan} = \{F, H\}$

$W_{P_4Lan} = \{F\}$

$W_{P_5Lan} = \{R\}$

$W_{P_6Lan} = \{D\}$

$W_{P_iDeg} = \{BS, MS, PhD\}$ for i = 1,...,6

We can identify system S_3 with the system (OB, AT, $\{VAL_a\}_{a \in AT}$, f_3) of nondeterministic information where $f_3(o,a) = W_{oa}$ for $o \in OB$ and $a \in AT$.

Example 6.2.4

Let the universe U be defined as follows. Set OB of objects consists of five plane figures F_1, F_2, F_3, F_4, F_5. Set AT of attributes consists of two attributes a and b, determining a shape of a figure, namely $VAL_a = \{$oval, polygonal$\}$ and $VAL_b = \{$ellipse, triangle, square, rectangle$\}$. Assume we are given the following set T of sentences:

1. $\forall$x((x b ellipse)$\rightarrow$(x a oval))
2. $\forall$x((x b triangle) v (x b square) v (x b rectangle)$\rightarrow$(x a polygonal))
3. $\forall$x((x a oval) v (x a polygonal))
4. $(F_1$ b ellipse) $\wedge$ $(F_2$ b ellipse)
5. $(F_3$ b triangle)
6. $(F_4$ b square) v $(F_4$ b triangle)
7. $(F_5$ b rectangle) v $(F_5$ b ellipse)

The following sentences are examples of implicit facts derivable in in this system:

$(F_1$ a oval)

$(F_3$ a polygonal)

$\exists$x(x a polygonal)

$(F_4$ b ellipse)$\rightarrow$$(F_4$ a oval)

Sentences 1 and 2 describe dependence of attribute a on attribute b. They can be treated as a definition of the dependency function corresponding to b$\rightarrow$a.

The language of logic IL enables us to deal both with local problems concerning dependencies in a system, such as whether a set of formulas representing dependencies implies another dependency, and with global ones, such as whether a set of dependencies is redundant, that is whether it can be derived from the remaining facts of the system.

In the following sections we discuss methodological problems concerning logic representation of knowledge.

6.3. Equivalence of logic KR systems

Given a system $S = (U, T)$, set T can be considered as the set of explicit facts provided by system S, set $D(T) - T$ of all formulas derivable from T and not belonging to T is the set of implicit facts, and set $D(T)$ is the set of all the facts in S.

Let $ST = \{S_i\}_{i \in I}$ be a family of logic KR systems of the form $S_i = (U, T_i)$ such that all the systems have the same universe U = (OB, AT, VAL). We introduce an ordering relation $\leq$ in the family ST as follows:

$S_1 \leq S_2$ iff $D(T_1) \subseteq D(T_2)$

If $S_1 \leq S_2$ holds then S_1 is said to be a subsystem of S_2. Hence S_1 is a subsystem of S_2 iff the set $D(T_1)$ of facts in S_1 is included in the set $D(T_2)$ of facts in S_2. S_1 is said to be a proper subsystem of S_2 iff $D(T_1) \subsetneq D(T_2)$.

Systems $S_1, S_2 \in ST$ are said to be equivalent ($S_1 \sim S_2$) iff $S_1 \leq S_2$ and $S_2 \leq S_1$.

Example 6.3.1

Consider system $S = (U, T)$ from example 6.2.4 and a system with the same universe as S and with the following set T' of explicit facts:

formulas 1, 2, 3, 4, 5, from T

(F_4 b square)

(F_5 b rectangle)

System $S' = (U, T')$ is a subsystem of system S.

Examples of equivalent systems one can easily obtain by using the following theorem.

Fact 6.3.1

(a) $D(\{A, B\}) = D(\{A \wedge B\})$

(b) $D(T_1) = D(T_1')$ and $D(T_2) = D(T_2')$ imply $D(T_1 \cup T_2) = D(T_1' \cup T_2')$

Fact 6.3.2

(a) if $T_1 = D(T_1)$, $T_2 = D(T_2)$, and $T = D(T)$ then $T = T_1 \cup T_2$ implies $T = T_1$ or $T = T_2$

(b) if $T_1 = D(T_1)$, $T_2 = D(T_2)$, and $T = D(T)$ then $T \subseteq T_1 \cup T_2$ implies $T \subseteq T_1$ or $T \subseteq T_2$.

This theorem implies restrictions in obtaining equivalent systems or subsystems of a given system.

6.4. Properties of logic KR systems

In this section we investigate logic KR systems of the form (U, T) from the point of view of properties of set T of sentences. We consider systems in which set T of explicit facts is consistent or maximal or independent, and we list theorems which characterize such systems. These theorems are closely related to the well known theorems of the proof theory.

A system $S = (U, T)$ is consistent if set T of explicit facts is consistent. A system is said to be inconsistent if it is not consistent. The following theorems characterize systems from the point of view of their consistency.

Fact 6.4.1

The following conditions are equivalent:

(a) A system $S = (U, T)$ is consistent

(b) Set $D(T)$ is consistent

(c) For any U-formula A we have $A \notin D(T)$ or $\neg A \notin D(T)$

(d) There is an U-formula A such that $A \notin D(T)$

(e) Any subsystem $S' = (U, T')$ of S such that T' is a finite set is consistent.

It follows from condition (d) that if a system S is inconsistent then its set of all facts coincides with the whole set of U-formulas.

Fact 6.4.2

(a) If S_1 is consistent and $S_2 \leq S_1$ then S_2 is consistent

(b) If $S_1 \sim S_2$ and S_1 is consistent then S_2 is consistent

A system $S = (U, T)$ is maximal if for any U-sentence A we have $A \in D(T)$ or $\neg A \in D(T)$. This means that for every U-sentence A either A or $\neg A$ is a fact in the system. In other words, for any fact A, either A or $\neg A$ can be derived from the set of explicit facts of the system.

The following theorems characterize maximal systems.

Fact 6.4.3

The following conditions are equivalent:

(a) A system $S = (U, T)$ is maximal

(b) For any U-sentence A if $A \notin D(T)$ then system $S' = (U, T \cup \{A\})$ is inconsistent

(c) For any consistent system S' if $S \leq S'$ then $S \sim S'$.

It follows from this theorem that if a system is maximal then it is not possible to add new information without loosing consistency.

The following conditions are satisfied:

(a) If S_1 is maximal and $S_1 \leq S_2$ then S_2 is maximal

(b) If $S_1 \sim S_2$ and S_1 is maximal then S_2 is maximal

A system $S = (U, T)$ is independent if for any $A \in T$ we have $A \notin D(T - \{A\})$. Hence in an independent system none of its explicit facts is derivable from the remaining explicit facts. Thus, from the logic point of view, independent system has reasonably "small" set of explicit facts.

Fact 6.4.4

The following conditions are equivalent:

(a) A system $S = (U, T)$ is independent

(b) For any system S' such that $S' \leq S$ we have not $S \sim S'$

(c) For any $A \in T$ system $(U, T - \{A\} \cup \{\neg A\})$ is consistent.

Fact 6.4.5

For any system $S = (U, T)$ if T is finite then there is a subsystem S' of S which is independent and equivalent to S.

Fact 6.4.6

(a) If S_1 is independent and $S_2 \leq S_1$ then S_2 is independent

(b) If S_1 is independent and $S_1 \sim S_2$ then there is no proper subsystem of S_1 equivalent to S_2.

Example 6.4.1

Let universe U be given such that

$OB = \{o_1, o_2, o_3\}$

$AT = \{a, b\}$

$VAL_a = VAL_b = \{u, v\}$

$VAL = VAL_a \cup VAL_b$.

Let us consider all the atomic properties which can be defined for objects in universe U:

(a u) (a v) (b u) (b v)

Let system $S = (U, T)$ be given such that set T of explicit facts is as follows:

$\neg(o_1\ a\ u) \wedge (o_1\ a\ v) \wedge (o_1\ b\ u) \wedge \neg(o_1\ b\ v)$

$\neg(o_2\ a\ u) \wedge (o_2\ a\ v) \wedge (o_2\ b\ u) \wedge \neg(o_2\ b\ v)$

$\neg(o_3\ a\ u) \wedge (o_3\ a\ v) \wedge \neg(o_3\ b\ u) \wedge (o_3\ b\ v)$

These formulas provide a complete characterization of objects o_1, o_2, and o_3 in universe U. System S is consistent, maximal, and independent. U-formulas $\forall x(x\ a\ v)$ and $\exists x(x\ a\ v) \wedge (x\ b\ u)$ are examples of implicit facts in the system. System S_1 obtained from S by adding

U- formula $\exists x(x\ a\ u)$ is inconsistent since none of the objects in the system has property (a u). If we reject a formula from T or if we drop a conjunct in a formula from T then we will obtain the system which is not maximal. System S_2 obtained from S by adding U-formula $\forall x(x\ b\ u) \rightarrow \neg(x\ b\ v)$ is not independent.

6.5. Dynamics of logic KR systems

KR systems are not static objects, they interact with the environment. Usually they change their information content as the result of adding new facts or removing some of the existing ones. In this section we discuss the problem how the assimilation of new facts may change properties of the system.

Assume we are given a system $S = (U, T)$ and an U-sentence A not occurring in T. We treat A as a new fact which is to be added to system S. Hence we define the system $S' = (U, T \cup \{A\})$, and we have to consider the following cases.

Case 6.5.1. System $(U, T \cup \{A\})$ is inconsistent.

If we accept A then we should restore consistency. We have to remove from T those facts which are in conflict with A. By confronting information A with facts from D(T) we can improve information content of the system.

If we treat D(T) as a set of assumptions, then we should reject A, as information which is not confirmed by facts of the system.

Example 6.5.1

Consider system S from example 6.2.4. The system obtained from S by adding sentence $A = (F_2\ a\ \text{polygonal})$ to T is inconsistent, since sentence $\neg(F_2\ a\ \text{polygonal})$ can be derived from T. This derivation is as follows:

8. $(F_2\ b\ \text{ellipse}) \rightarrow (F_2\ a\ \text{oval})$ from 1 and A6

9. $(F_2\ a\ \text{oval}) \vee (F_2\ a\ \text{polygonal})$ from 3 and A6

10. $(F_2\ a\ \text{polygonal}) \rightarrow \neg(F_2\ a\ \text{oval})$ from 9

11. $\neg(F_2\ a\ \text{oval})$.

Case 6.5.2 Set D(T) contains information A.

This means that A is an implicit fact in system S, and sets of facts D(T) and $D(T \cup \{A\})$ coincide. In this case systems (U, T) and $(U, T \cup \{A\})$ are equivalent and A is the redundant fact.

Example 6.5.2

Consider system S from example 6.2.4 and sentence $A = \exists x(x\ b\ \text{ellipse}) \wedge \exists x(x\ b\ \text{triangle})$. This sentence can be derived from T. By axioms A6 we have $\vdash A(o) \to \exists x A(x)$, and by formulas 4 and 5 we obtain $T_4 \vdash A$.

Case 6.5.3. Systems $(U, T \cup \{A\})$ and $(U, T \cup \{\neg A\})$ are consistent.

This means that both facts A and $\neg A$ can be assimilated by system S. On other words A is independent from S.

Example 6.5.3

Sentence (F_5 b rectangle) is independent from system S given in example 6.2.4.

Case 6.5.4. Some explicit facts in system S can be derived from A and the rest of explicit facts.

In this case we treat set T as the union of sets T_1 and T_2 such that $T_2 \subseteq D(T_1 \cup \{A\})$. We might consider system $(U, T_1 \cup \{A\})$ as the more useful than the original one. By theorem 6.1.2 we have $D(T) \subseteq D(T_1 \cup \{A\})$ and hence system (U, T) is a subsystem of $(U, T_1 \cup \{A\})$.

Example 6.5.4

Consider system S from example 6.2.4 and sentence A = (F_4 b square) $\wedge$ (F_5 b rectangle). It is easy to see that formulas 6 and 7 from T can be derived from A. Hence if we use sentence A instead of sentences 6 and 7 then the set of all facts of the obtained system will contain the set of all facts of system S.

6.6. Summary

The primary purpose of this chapter was to introduce a language whose expressive power was sufficient to represent deterministic, non-deterministic and many-valued information. We developed deduction method for the language and we prove completeness of the method. We discussed methodological problems specific for logic representation of knowledge, namely consistency, maximality, and independence of knowledge expressed in the given language. We also investigated how consistency is influenced by dynamic changes of information content of KR systems.

REFERENCES

Aikins, J.S., (1983) Prototypical Knowledge for Expert Systems. Artificial Intelligence 20, 163 - 210

Ajdukiewicz, K., (1974) Pragmatic Logic. Translated from the Polish by Olgierd Wojtasiewicz, Reidel, Dordrecht, Polish Scientific Publishers, Warsaw

Ashany, R., (1976) SPARCOM: A Sparse Matrix. Associative Relational Approach to Dynamic Data Structuring and Data Retrieval. IBM Technical Report TR OO 2774

Banerji, R.B., (1980) Artificial Intelligence: A Theoretical Perspective. Elsevier North Holland, New York

Black, M., (1970) Margins of Precision. Cornell University Press, Ithaca New York

Bobrow, D.G., (1975) Dimensions of Representation. In: Bobrow, D.G., Collins, A. (eds.) Representation and Understanding. Academic Press, New York

Bobrow, D.G., (1977) A Panel on Knowledge Representation. Proc. Fifth International Joint Conference on Artificial Intelligence, Carnegie-Melon University, Pittsburgh, PA

Bobrow, D.G., Collins, A., (eds) (1975) Representation and Understanding. Academic Press, New York

Bobrow, D.G., Kaplan, R., Kay, M., Norman, D., Thompson, H., Winograd, T., (1977) GUS, a Framework-Driven Dialog System. Artificial Intelligence 8, 155 - 173

Bobrow, D.G., Winograd, T., (1977) An Overview of KRL: A Knowledge Representation Language. Journal of Cognitive Science 1, 3 - 46

Brachman, R.J., Smith, B.C., (1980) Special Issue of Knowledge Representation. SIGART Newsletter 70, 1 - 138

Bruce, B., (1972) A Model for Temporal References and its Application in a Question Answering Program. Artificial Intelligence 3, 1 - 26

Bubenko, J., (1977) The Temporal Dimension in Information Modelling. In: Nijssen, G.M., (ed) Architecture and Models in Data Base Management Systems. North Holland, 93 - 118

Buchanan, B.G., Feigenbaum, E.A., (1978) DENDRAL and Meta-DENDRAL: Their Applications Dimension. Artificial Intelligence 11, 5 - 24

Buchanan, B.G., Feigenbaum, E., Sridharan, (1972) Heuristic Theory Formation. Machine Intelligence 7, 267 - 290

Buchanan, B.G., Lederberg, J., (1971) The Heuristic DENDRAL Program for Explaining Empirical Data. Proc. IFIP Congress 71 Ljubljana, Yugoslavia, 178 - 188

Buchanan, B.G., Sutherland, G., Feigenbaum, E., (1969) A Program for Generating Explanatory Hypotheses in Organic Chemistry. Machine Intelligence 4, 209 - 254

Burgess, J.P., (1979) Logic and Time. Journal of Symbolic Logic, 566-581

Davis, R., Lenat, D., (1982) Knowledge - Based Systems in Artificial Intelligence. McGraw-Hill

Feigenbaum, E., Buchanan, B., Ledeberg, J., (1971) On Generality and Problem Solving: A Case Study Using The DENDRAL Program. Machine Intelligence 6, 165 - 190

Feigenbaum, E.A., Feldman, J., (eds) (1963) Computers and Thought. McGraw-Hill, New York

Gabbay, D.M., (1976) Investigations in Modal and Tense Logics with Applications to Problems in Philosophy and Linguistics. Reidel Synthese Library, vol. 92

Goldstein, I.P., Roberts, R.B., (1977) NUDGE, A Knowledge - Based Scheduling Program. Fifth International Joint Conference on Artificial Intelligence, 257 - 263

Hempel, C.G., (1952) Fundamentals of Concept Formation in Empirical Sciences. University of Chicago Press, Chicago

Hewitt, C., (1973) A Universal Modular ACTOR Formalism for Artificial Intelligence. Third International Joint Conference on Artificial Intelligence, 235 - 245

Hintikka, J., (1962) Knowledge and Belief. Cornell University Press, Ithaca, N.Y.

Hunt, E.B., (1974) Concept Formation. John Wiley and Sons, New York

Hunt, E.B., Marin, J., Stone, P.J., (1966) Experiments in Induction. Academic Press, New York

Kaplan, R.M., (1972) Augmented Transition Networks as Psychological Models of Sentence Comprehension. Artificial Intelligence 3, 77 - 100

Kulikowski, C.A., Weiss, S., (1972) The Medical Consultant Program - Glaucoma. Dept. of Comp. Sci., Rutgers Univ., New Brunswick, NJ

Kunz, J., Fallat, R., Mellung, D., Osborn, J., Votteri, B., Nii, H., Aikins, J., Fogan, L., Feigenbaum, E., (1978) A Physiological Rule Based System for Interpreting Pulmonary Function Test Results. Working Paper HPP-78-19, Heuristic Programming Project, Dept. of Computer Science, Stanford University

Marek, W., Pawlak, Z., (1976) Information Storage and Retrieval Systems. Theoretical Computer Science 1, 331 - 357

McDermott, D., (1978) The Last Survey of Representation of Knowledge. Proc. of the AISB/GI Conference on AI, Hamburg, 206 - 221

Michalski, R., (1980) Pattern Recognition as Rule-Guided Inductive Inference. IEEE Transaction on Pattern Analysis and Machine Intelligence 4, 349 - 361

Michalski, R., Stepp, R., Diday, E., (1981) A Recent Advance in Data Analysis: Clustering Objects into Classes Characterized by Conjunctive Concepts. In: Kanal, L., Rosenfeld, A. (eds) Progress in Pattern Recognition

Minsky, M., (ed) (1968) Semantic Information Processing. MIT Press, MIT

Minsky, M., (1975) A Framework for Representing Knowledge. In: Winston, P. (ed) The Psychology of Computer Vision. McGraw-Hill, New York, 211-277

Moore, G.W., Riede, U.N., Sandritter, G., (1977) Application of Quine's Nullities to a Quantitive Organelle Pathology. Journal of Theoretical Biology 65, 633 - 657

Newell, A., (1982) The Knowledge Level. Artificial Intelligence 18, 87 - 127

Nilsson, N.J., (1971) Problem-Solving Methods in Artificial Intelligence McGraw-Hill, New York

Nilsson, N.J., (1980) Principles of Artificial Intelligence. Tioga,

Palo Alto

Orłowska, E., (1982) Dynamic Information Systems. Fundamenta Informaticae 5, 101 - 118

Orłowska, E., (1983 (a)) Representation of Vague Information. ICS PAS Reports 503

Orłowska, E., (1983 (b)) Semantics of Vague Concepts. 7th International Congress of Logic Methodology and Philosophy of Science, Salzburg, Austria, vol 2, 127 - 130

Orłowska, E., (1983 (c)) Representation of Temporal Information. Journal of Computer and Information Sciences 11

Orłowska, E., Pawlak, Z., (1984 (a)) Expressive Power of Knowledge Representation Systems. International Journal of Man-Machine Studies (to appear)

Orłowska, E., Pawlak, Z., (1984 (b)) Representation of Nondeterministic Information. Theoretical Computer Science (to appear)

Pople, H., Meyers, J., Miller, R., (1975) DIALOG, a Model of Diagnostic Logic for Internal Medicine. Fourth International Joint Conference on Artificial Intelligence, 848 - 855

Pauker, S.G., Szalovits P., (1977) Analyzing and Simulating Taking the History of the Present Illness: Context Formation. In: Schneider and Sagvall Hein (eds) Computational Linguistics in Medicine. North-Holland, Amsterdam, 109 - 118

Pawlak, Z., (1981) Information Systems, Theoretical Foundations. Information Systems 3, 205 - 218

Pawlak, Z., (1982) Rough Sets. International Journal of Computer and Information Sciences 11, 341 - 350

Pawlak, Z., (1983) Information Systems, Theoretical Foundations (in Polish). WNT, Warsaw

Pawlak, Z., (1984) Rough Classification. International Journal of Man-Machine Studies (to appear)

Rasiowa, H., Sikorski, R., (1970) Mathematics of Metamathematics. Polish Scientific Publishers, Warsaw

Robinson, A., (1966) Non-Standard Analysis. North Holland, Amsterdam

Russel, B., (1923) Vagueness. Australasian Journal of Philosophy 1, 84 - 92

Shortliffe, E.H., (1976) MYCIN: Computer-Based Medical Consultations. American Elsevier, New York

Sloman, A., (1971) Interactions Between Philosophy and Artificial Intelligence: The Role of Intuition and Non-Logical Reasoning in Intelligence. Artificial Intelligence 2, 209 - 225

Szabo, M.E., (ed. 1969) The Collected Papers of Gerhard Gentzen. North-Holland

Tou, J.T., (1980) Knowledge Engineering. International Journal of Computer and Information Sciences 9, 275 - 285

Van Melle, W., (1980) EMYCIN: A Domain - Independent Production - Rule System for Consultation Programs. Heuristic Programming Project, Dept. of Computer Science, Stanford University

Vopenka, P., (1979) Mathematics in the Alternative Set Theory. Teubner -Texte zur Mathematic, Leipzig

Warmus, M., (1983) Personal Communication

Winograd, T., (1972) Understanding Natural Language. Academic Press, New York

Winograd, T., (1975) Frame Representation and Declarative Procedural Controversy. In: Bobrow, D.G., Collins, A. (eds) Representation and Understanding. Academic Press

Winston, P., (ed) (1975) The Psychology of Computer Vision. McGraw-Hill New York

Wójcicki, R., (1982) Wykłady z metodologii nauk (Lectures in Methodology of Sciences), Polish Scientific Publishers, Warsaw

Zadeh, L.A., (1975) Fuzzy Logic and Approximate Reasoning. Synthese 30, 407 - 428

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