Available online at www.sciencedirect.com
ScienceDirect
Information Sciences 177 (2007) 3–27
INFORMATION SCIENCES
AN INTERNATIONAL JOURNAL
www.elsevier.com/locate/ins

Rudiments of rough sets

Zdzisław Pawlak ✠, Andrzej Skowron *

Institute of Mathematics, Warsaw University, Banacha 2, 02-097 Warsaw, Poland

Received 23 February 2006; received in revised form 7 June 2006; accepted 7 June 2006

Commemorating the life and work of Zdzisław Pawlak

Abstract

Worldwide, there has been a rapid growth in interest in rough set theory and its applications in recent years. Evidence of this can be found in the increasing number of high-quality articles on rough sets and related topics that have been published in a variety of international journals, symposia, workshops, and international conferences in recent years. In addition, many international workshops and conferences have included special sessions on the theory and applications of rough sets in their programs. Rough set theory has led to many interesting applications and extensions. It seems that the rough set approach is fundamentally important in artificial intelligence and cognitive sciences, especially in research areas such as machine learning, intelligent systems, inductive reasoning, pattern recognition, mereology, knowledge discovery, decision analysis, and expert systems. In the article, we present the basic concepts of rough set theory and point out some rough set-based research directions and applications.

Keywords: Vague concepts; Information and decision systems; (in)Discernibility; Approximation spaces; Set approximations; Rough sets; Rough membership functions; Reducts; Decision rules; Dependencies of attributes; Boolean reasoning

Besides known and unknown what else is there?
Harold Pinter (1965). The Homecoming.
Methuen, London

1. Introduction

The basic ideas of rough set theory and its extensions as well as many interesting applications can be found in a number of books (see, e.g., [28,33,44,48,74,97,112,113,131,182,194,195,206,237,241,244,245,272,305,374]), issues of the Transactions on Rough Sets [225–228], special issues of other journals (see, e.g., [25,129,218,

* Corresponding author.
E-mail address: skowron@mimuw.edu.pl (A. Skowron).
✠ Professor Zdzisław Pawlak passed away on 7 April 2006.

0020-0255/$ - see front matter © 2006 Elsevier Inc. All rights reserved.
doi:10.1016/j.ins.2006.06.003

193,281,306,377,378]), proceedings of international conferences (see, e.g., [1,88,130,243,280,293,300,301,331, 336,337,350,376,380]), tutorials (see, e.g., [110]). For more information one can also visit web pages www.roughsets.org and logic.mimuw.edu.pl.

The basic notions of rough sets and approximation spaces were introduced during the early 1980s (see, e.g., [199,201,202]). In this paper, the basic concepts of rough set theory are presented. We also point out some research directions and applications based on rough sets. In articles [210,214], we discuss in more detail two selected topics, namely, extensions of the rough set approach and the combination of rough sets and Boolean reasoning with applications in pattern recognition, machine learning, data mining and conflict analysis.

2. Sets and vague concepts

In this section, we give some general remarks on the concept of a set and the place of rough sets in set theory.

The concept of a set is fundamental for the whole of mathematics. Modern set theory was formulated by Cantor [19]. Bertrand Russell discovered that the intuitive notion of a set proposed by Cantor leads to antinomies [266]. Two kinds of remedy for this problem have been proposed: axiomatization of Cantorian set theory and alternative set theories.

Another issue discussed in connection with the notion of a set or a concept is vagueness (see, e.g., [15,53,87,105,106,267]). Mathematics requires that all mathematical notions (including set) must be exact [55]. However, philosophers (see, e.g., [105,106,262]) and recently computer scientists (see, e.g., [206,370]) have become interested in vague concepts.

2.1. Sets

The notion of a set is a basic one of mathematics. Most mathematical structures refer to it.

The definition of this notion and the creation of set theory are due to German mathematician Georg Cantor (1845–1918) [19], who laid the foundations of contemporary set theory over 100 years ago.

The birth of set theory can be traced back to Georg Cantor's 1873 proof of the uncountability of real line (i.e., the set of all real numbers is not countable) [18]. It was Bernhard Bolzano (1781–1848) who coined the term *Menge* ("set"), which Cantor used to refer to objects in his theory. According to Cantor, a set is a collection of any objects, which can be considered as a whole according to some law. As one can see, the notion of set is very intuitive and simple.

Mathematical objects such as relations, functions, numbers, are examples of sets. In fact, set theory is needed in mathematics to provide rigor.

The notion of a set is not only fundamental for the whole of mathematics but it also plays an important role in natural language. We often speak about sets (collections) of various objects of interest such as collection of books, paintings and people.

> The intuitive meaning of a set according to some dictionaries is the following: "A number of things of the same kind that belong or are used together."
>
> *Webster's Dictionary*

> "Number of things of the same kind, that belong together because they are similar or complementary to each other."
>
> *The Oxford English Dictionary*

Thus a set is a collection of things which are somehow related to each other but the nature of this relationship is not specified in these definitions.

In fact, these definitions are due to the original definition given by Cantor.

2.2. Antinomies

In 1903, the renowned English philosopher Bertrand Russell (1872–1970) observed [266] that the intuitive notion of a set given by Cantor leads to logical *antinomies* (contradictions), i.e., Cantor set theory is contradictory (there are other kinds of antinomies, which are outside the scope of this paper). A logical antinomy (for simplicity, we refer to "antinomy" in the rest of this paper) arises whenever correct logical reasoning leads to a contradiction, i.e., to propositions A and *non*-A, which is not allowed in logic.

As an example let us discuss briefly the so-called Russell's antinomy. Consider the set X containing all the sets Y, which are not the elements of themselves. If we assume that X is its own element then X, by definition, cannot be its own element; while if we assume that X is not its own element then, according to the definition of the set X, it must be its own element. Thus while applying each assumption we obtain contradiction.

Antinomies show that a set cannot be a collection of arbitrary elements, as was stipulated by Cantor.

One could think that antinomies are ingenuous logical play, but it is not so. They question the essence of logical reasoning. That is why there have been attempts to "repair" Cantor's theory for over 100 years or to substitute another set theory for it but the results have not been good so far. Is then all mathematics based on doubtful foundations?

As a remedy for this defect several axiomatizations of set theory have been proposed (e.g., [101]).

Instead of improvements of Cantors' set theory by its axiomatization, some mathematicians proposed escape from classical set theory by creating a completely new idea of a set, which would free the theory from antinomies [3,126,347].

No doubt the most interesting proposal was given by Polish logician Stanisław Leśniewski, who introduced the relation of "being a part" instead of the membership relation between elements and sets employed in classical set theory. In his set theory called mereology, *being a part* is a fundamental relation [126]. Mereology is a significant part of recent studies on the foundations of mathematics (see, e.g., [20,54,124,271,345]), artificial intelligence [314], cognitive science [311], natural language [52], and research in rough set theory (see, e.g., [237,242,246]).

The problem of finding an alternative to classical set theory has failed to be solved until now.

The deficiency of sets, mentioned above, has rather philosophical than practical meaning, since sets used practically in mathematics are free from the above discussed faults. Antinomies are associated with very "artificial" sets constructed in logic but not found in sets used in "everyday" mathematics. That is why we can use mathematics safely.

2.3. Vagueness

Another issue discussed in connection with the notion of a set is vagueness. Mathematics requires that all mathematical notions (including set) must be exact, otherwise precise reasoning would be impossible. However, philosophers [105,106,262,268] and recently computer scientists [140,177,179,278] as well as other researchers have become interested in *vague* (imprecise) concepts.

In classical set theory a set is uniquely determined by its elements. In other words, this means that every element must be uniquely classified as belonging to the set or not. That is to say the notion of a set is a *crisp* (precise) one. For example, the set of odd integers is crisp because every integer is either odd or even.

In contrast to odd integers, the notion of a beautiful painting is vague, because we are unable to classify uniquely all paintings into two classes: beautiful and not beautiful. Some paintings cannot be decided whether they are beautiful or not and thus they remain in the doubtful area. Thus, *beauty* is not a precise but a vague concept.

Almost all concepts we are using in natural language are vague. Therefore, common sense reasoning based on natural language must be based on vague concepts and not on classical logic. Interesting discussion of this issue can be found in [262].

The idea of vagueness can be traced back to the ancient Greek philosopher Eubulides of Megara (ca. 400BC) who first formulated so-called "sorites" (heap) and "falakros" (bald man) paradoxes (see, e.g., [105,106]). The bald man paradox goes as follows: suppose a man has 100,000 hairs on his head. Removing one hair from his head surely cannot make him bald. Repeating this step we arrive at the conclusion that a man without any hair is not bald. Similar reasoning can be applied to a hip of stones.

Vagueness is usually associated with the boundary region approach (i.e., existence of objects which cannot be uniquely classified relative to a set or its complement) which was first formulated in 1893 by the father of modern logic, German logician, Gottlob Frege (1848–1925) (see [55]).

According to Frege the concept must have a sharp boundary. To the concept without a sharp boundary there would correspond an area that would not have any sharp boundary-line all around. It means that mathematics must use crisp, not vague concepts, otherwise it would be impossible to reason precisely.

Summing up, vagueness is

- not allowed in mathematics;
- interesting for philosophy;
- a nettlesome problem for natural language, cognitive science, artificial intelligence, machine learning, philosophy, and computer science.

3. Rough sets

This section briefly delineates basic concepts in rough set theory.

3.1. Rough sets: an introduction

Rough set theory, proposed by Pawlak in 1982 [202,206] can be seen as a new mathematical approach to vagueness.

The rough set philosophy is founded on the assumption that with every object of the universe of discourse we associate some information (data, knowledge). For example, if objects are patients suffering from a certain disease, symptoms of the disease form information about patients. Objects characterized by the same information are indiscernible (similar) in view of the available information about them. The indiscernibility relation generated in this way is the mathematical basis of rough set theory. This understanding of indiscernibility is related to the idea of Gottfried Wilhelm Leibniz that objects are indiscernible if and only if all available functionals take on them identical values (Leibniz's Law of Indiscernibility: The Identity of Indiscernibles) [4,125]. However, in the rough set approach indiscernibility is defined relative to a given set of functionals (attributes).

Any set of all indiscernible (similar) objects is called an elementary set, and forms a basic granule (atom) of knowledge about the universe. Any union of some elementary sets is referred to as crisp (precise) set – otherwise the set is rough (imprecise, vague).

Consequently, each rough set has boundary-line cases, i.e., objects which cannot with certainty be classified either as members of the set or of its complement. Obviously crisp sets have no boundary-line elements at all. This means that boundary-line cases cannot be properly classified by employing available knowledge.

Thus, the assumption that objects can be "seen" only through the information available about them leads to the view that knowledge has granular structure. Due to the granularity of knowledge, some objects of interest cannot be discerned and appear as the same (or similar). As a consequence, vague concepts in contrast to precise concepts, cannot be characterized in terms of information about their elements. Therefore, in the proposed approach, we assume that any vague concept is replaced by a pair of precise concepts – called the lower and the upper approximation of the vague concept. The lower approximation consists of all objects which surely belong to the concept and the upper approximation contains all objects which possibly belong to the concept. The difference between the upper and the lower approximation constitutes the boundary region of the vague concept. Approximations are two basic operations in rough set theory.

Hence, rough set theory expresses vagueness not by means of membership, but by employing a boundary region of a set. If the boundary region of a set is empty it means that the set is crisp, otherwise the set is rough (inexact). A non-empty boundary region of a set means that our knowledge about the set is not sufficient to define the set precisely.

Rough set theory it is not an alternative to classical set theory but it is embedded in it. Rough set theory can be viewed as a specific implementation of Frege's idea of vagueness, i.e., imprecision in this approach is expressed by a boundary region of a set.

Rough set theory has attracted attention of many researchers and practitioners all over the world, who have contributed essentially to its development and applications. Rough set theory overlaps with many other theories. Despite this overlap, rough set theory may be considered as an independent discipline in its own right. The rough set approach seems to be of fundamental importance in artificial intelligence and cognitive sciences, especially in research areas such as machine learning, intelligent systems, inductive reasoning, pattern recognition, mereology, knowledge discovery, decision analysis, and expert systems. The main advantage of rough set theory in data analysis is that it does not need any preliminary or additional information about data like probability distributions in statistics, basic probability assignments in Dempster–Shafer theory, a grade of membership or the value of possibility in fuzzy set theory (see, e.g., [48] where some combinations of rough sets with non-parametric statistics are studied). One can observe the following about the rough set approach:

- introduction of efficient algorithms for finding hidden patterns in data,
- determination of optimal sets of data (data reduction),
- evaluation of the significance of data,
- generation of sets of decision rules from data,
- easy-to-understand formulation,
- straightforward interpretation of obtained results,
- suitability of many of its algorithms for parallel processing.

3.2. Indiscernibility and approximation

The starting point of rough set theory is the indiscernibility relation, which is generated by information about objects of interest (see Section 3.1). The indiscernibility relation expresses the fact that due to a lack of information (or knowledge) we are unable to discern some objects employing available information (or knowledge).

This means that, in general, we are unable to deal with each particular object but we have to consider granules (clusters) of indiscernible objects as a fundamental basis for our theory.

From a practical point of view, it is better to define basic concepts of this theory in terms of data. Therefore we will start our considerations from a data set called an *information system*. An information system is a data table containing rows labeled by objects of interest, columns labeled by attributes and entries of the table are attribute values. For example, a data table can describe a set of patients in a hospital. The patients can be characterized by some attributes, like *age*, *sex*, *blood pressure*, *body temperature*, etc. With every attribute a set of its values is associated, e.g., values of the attribute *age* can be *young*, *middle*, and *old*. Attribute values can be also numerical. In data analysis the basic problem we are interested in is to find patterns in data, i.e., to find a relationship between some set of attributes, e.g., we might be interested whether *blood pressure* depends on *age and sex*.

Suppose we are given a pair $\mathscr{A} = (U, A)$ of non-empty, finite sets U and A, where U is the *universe* of *objects*, and A – a set consisting of *attributes*, i.e. functions $a : U \rightarrow V_a$, where V_a is the set of values of attribute a, called the *domain* of a. The pair $\mathscr{A} = (U, A)$ is called an *information system* (see, e.g., [200]). Any information system can be represented by a data table with rows labeled by objects and columns labeled by attributes. Any pair (x, a), where $x \in U$ and $a \in A$ defines the table entry consisting of the value $a(x)$.[1]

Any subset B of A determines a binary relation $I(B)$ on U, called an *indiscernibility relation*, defined by

$$xI(B)y \text{ if and only if } a(x) = a(y) \text{ for every } a \in B, \tag{1}$$

where $a(x)$ denotes the value of attribute a for object x.

Obviously, $I(B)$ is an equivalence relation. The family of all equivalence classes of $I(B)$, i.e., the partition determined by B, will be denoted by $U/I(B)$, or simply U/B; an equivalence class of $I(B)$, i.e., the block of the partition U/B, containing x will be denoted by $B(x)$ (other notation used: $[x]_B$ or $[x]_{I(B)}$). Thus in view of the data we are unable, in general, to observe individual objects but we are forced to reason only about the accessible granules of knowledge (see, e.g., [194,206,247]).

If $(x, y) \in I(B)$ we will say that x and y are *B-indiscernible*. Equivalence classes of the relation $I(B)$ (or blocks of the partition U/B) are referred to as *B-elementary sets* or *B-elementary granules*. In the rough set approach

[1] Note, that in statistics or machine learning such a data table is called a sample [56].

the elementary sets are the basic building blocks (concepts) of our knowledge about reality. The unions of *B-elementary sets* are called *B-definable sets*.[2]

For $B \subseteq A$ we denote by $\mathrm{Inf}_B(x)$ the B-signature of $x \in U$, i.e., the set $\{(a, a(s)) : a \in A\}$. Let $\mathrm{Inf}(B) = \{\mathrm{Inf}_B(s) : s \in U\}$. Then for any objects $x, y \in U$ the following equivalence holds: $xI(B)y$ if and only if $\mathrm{Inf}_B(x) = \mathrm{Inf}_B(y)$.

The indiscernibility relation will be further used to define basic concepts of rough set theory. Let us define now the following two operations on sets $X \subseteq U$

$$B_*(X) = \{x \in U : B(x) \subseteq X\}, \tag{2}$$
$$B^*(X) = \{x \in U : B(x) \cap X \neq \emptyset\}, \tag{3}$$

assigning to every subset X of the universe U two sets $B_*(X)$ and $B^*(X)$ called the *B-lower* and the *B-upper approximation* of X, respectively. The set

$$BN_B(X) = B^*(X) - B_*(X), \tag{4}$$

will be referred to as the *B-boundary region* of X.

From the definition we obtain the following interpretation:

- The *lower approximation* of a set X with respect to B is the set of all objects, which can be for *certain* classified as X using B (are *certainly* X in view of B).
- The *upper approximation* of a set X with respect to B is the set of all objects which can be *possibly* classified as X using B (are *possibly* X in view of B).
- The *boundary region* of a set X with respect to B is the set of all objects, which can be classified neither as X nor as not-X using B.

In other words, due to the granularity of knowledge, rough sets cannot be characterized by using available knowledge. Therefore with every rough set we associate two *crisp* sets, called *lower* and *upper approximation*. Intuitively, the lower approximation of a set consists of all elements that *surely* belong to the set, whereas the upper approximation of the set constitutes of all elements that *possibly* belong to the set, and the *boundary region* of the set consists of all elements that cannot be classified uniquely to the set or its complement, by employing available knowledge. The approximation definition is clearly depicted in Fig. 1.

The approximations have the following properties:

$$\begin{aligned}
&B_*(X) \subseteq X \subseteq B^*(X)\\
&B_*(\emptyset) = B^*(\emptyset) = \emptyset, \quad B_*(U) = B^*(U) = U\\
&B^*(X \cup Y) = B^*(X) \cup B^*(Y)\\
&B_*(X \cap Y) = B_*(X) \cap B_*(Y)\\
&X \subseteq Y \text{ implies } B_*(X) \subseteq B_*(Y) \text{ and } B^*(X) \subseteq B^*(Y)\\
&B_*(X \cup Y) \supseteq B_*(X) \cup B_*(Y)\\
&B^*(X \cap Y) \subseteq B^*(X) \cap B^*(Y)\\
&B_*(-X) = -B^*(X)\\
&B^*(-X) = -B_*(X)\\
&B_*(B_*(X)) = B^*(B_*(X)) = B_*(X)\\
&B^*(B^*(X)) = B_*(B^*(X)) = B^*(X).
\end{aligned} \tag{5}$$

Let us note that the inclusions in (5) cannot be in general substituted by the equalities. This has some important algorithmic and logical consequences.

Now we are ready to give the definition of rough sets.

[2] One can compare data tables corresponding to information systems with relations in relational databases [58].

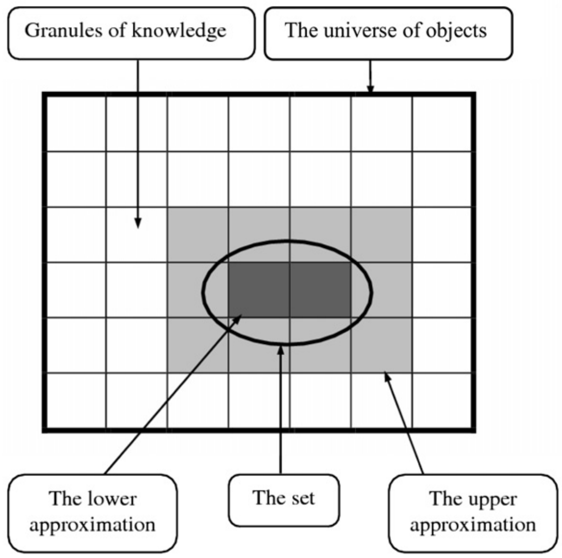

Fig. 1. A rough set.

If the boundary region of X is the empty set, i.e., $BN_B(X) = \emptyset$, then the set X is *crisp* (*exact*) with respect to B; in the opposite case, i.e., if $BN_B(X) \neq \emptyset$, the set X is referred to as *rough* (*inexact*) with respect to B. Thus any rough set, in contrast to a crisp set, has a non-empty boundary region.

One can define the following four basic classes of rough sets, i.e., four categories of vagueness:

$$\begin{aligned}
&B_*(X) \neq \emptyset \quad \text{and} \quad B^*(X) \neq U, \ \text{iff } X \text{ is roughly } B\text{-definable}\\
&B_*(X) = \emptyset \quad \text{and} \quad B^*(X) \neq U, \ \text{iff } X \text{ is internally } B\text{-indefinable}\\
&B_*(X) \neq \emptyset \quad \text{and} \quad B^*(X) = U, \ \text{iff } X \text{ is externally } B\text{-indefinable}\\
&B_*(X) = \emptyset \quad \text{and} \quad B^*(X) = U, \ \text{iff } X \text{ is totally } B\text{-indefinable.}
\end{aligned} \tag{6}$$

The intuitive meaning of this classification is the following.

If X is roughly *B-definable*, this means that we are able to decide for some elements of U that they belong to X and for some elements of U we are able to decide that they belong to $-X$, using B.

If X is internally B-indefinable, this means that we are able to decide about some elements of U that they belong to $-X$, but we are unable to decide for any element of U that it belongs to X, using B.

If X is externally B-indefinable, this means that we are able to decide for some elements of U that they belong to X, but we are unable to decide, for any element of U that it belongs to $-X$, using B.

If X is totally B-indefinable, we are unable to decide for any element of U whether it belongs to X or $-X$, using B.

Thus a set is *rough* (imprecise) if it has non-empty boundary region; otherwise the set is crisp (precise). This is exactly the idea of vagueness proposed by Frege.

Let us observe that the definition of rough sets refers to data (knowledge), and is *subjective*, in contrast to the definition of classical sets, which is in some sense an *objective* one.

A rough set can also be characterized numerically by the following coefficient:

$$\alpha_B(X) = \frac{\text{card}(B_*(X))}{\text{card}(B^*(X))}, \tag{7}$$

called the *accuracy of approximation*, where card(X) denotes the cardinality of $X \neq \emptyset$.[3] Obviously $0 \leqslant \alpha_B(X) \leqslant 1$. If $\alpha_B(X) = 1$ then X is *crisp* with respect to B (X is *precise* with respect to B), and otherwise, if $\alpha_B(X) < 1$ then X is *rough* with respect to B (X is *vague* with respect to B). The accuracy of approximation can be used to

[3] card (X) is also denoted by 1×1.

measure the quality of approximation of decision classes on the universe U. One can use another measure of accuracy defined by $1-\alpha_B(X)$ or by $1-\frac{\mathrm{card}(BN_B(X))}{\mathrm{card}(U)}$. Some other measures of approximation accuracy are also used, e.g., based on entropy or some more specific properties of boundary regions (see, e.g., [59,279, 297]). The choice of a relevant accuracy of approximation depends on a particular data set. Observe that the accuracy of approximation of X can be tuned by B. Another approach to accuracy of approximation can be based on the Variable Precision Rough Set Model (VPRSM) [375].

In the next section, we discuss decision rules (constructed over a selected set B of features or a family of sets of features) which are used in inducing classification algorithms (classifiers) making it possible to classify to decision classes unseen objects. Parameters which are tuned in searching for a classifier with the high quality are its description size (defined using decision rules) and its quality of classification (measured by the number of misclassified objects on a given set of objects). By selecting a proper balance between the accuracy of classification and the description size we expect to find the classifier with the high quality of classification also on unseen objects. This approach is based on the minimal description length principle [263,264,298].

3.3. Decision systems and decision rules

Sometimes we distinguish in an information system $\mathcal{A}=(U,A)$ a partition of A into two classes $C,D\subseteq A$ of attributes, called *condition* and *decision* (*action*) attributes, respectively. The tuple $\mathcal{A}=(U,C,D)$ is called a *decision system.*

Let $V=\bigcup\{V_a|a\in C\}\bigcup\{V_d|d\in D\}$. Atomic formulae over $B\subseteq C\cup D$ and V are expressions $a=v$ called *descriptors* (*selectors*) over B and V, where $a\in B$ and $v\in V_a$. The set $\mathcal{F}(B,V)$ of formulae over B and V is the least set containing all atomic formulae over B and V and closed with respect to the propositional connectives $\wedge$ (conjunction), $\vee$ (disjunction) and $\neg$ (negation).

By $\|\varphi\|_{\mathcal{A}}$ we denote the meaning of $\varphi\in\mathcal{F}(B,V)$ in the decision table $\mathcal{A}$ which is the set of all objects in U with the property φ. These sets are defined by $\|a=v\|_{\mathcal{A}}=\{x\in U|a(x)=v\}$, $\|\varphi\wedge\varphi'\|_{\mathcal{A}}=\|\varphi\|_{\mathcal{A}}\cap\|\varphi'\|_{\mathcal{A}}$; $\|\varphi\vee\varphi'\|_{\mathcal{A}}=\|\varphi\|_{\mathcal{A}}\cup\|\varphi'\|_{\mathcal{A}}$; $\|\neg\varphi\|_{\mathcal{A}}=U-\|\varphi\|_{\mathcal{A}}$. The formulae from $\mathcal{F}(C,V)$, $\mathcal{F}(d,V)$ are called *condition formulae of* $\mathcal{A}$ and *decision formulae of* $\mathcal{A}$, respectively.

Any object $x\in U$ belongs to the *decision class* $\|\bigwedge_{d\in D}d=d(x)\|_{\mathcal{A}}$ of $\mathcal{A}$. All decision classes of $\mathcal{A}$ create a partition U/D of the universe U.

A *decision rule* for $\mathcal{A}$ is any expression of the form $\varphi\Rightarrow\psi$, where $\varphi\in\mathcal{F}(C,V)$, $\psi\in\mathcal{F}(D,V)$, and $\|\varphi\|_{\mathcal{A}}\neq\emptyset$. Formulae φ and ψ are referred to as the *predecessor* and the *successor* of decision rule $\varphi\Rightarrow\psi$. Decision rules are often called "*IF* ... *THEN* ..." rules. Such rules are used in machine learning (see, e.g., [56]).

Decision rule $\varphi\Rightarrow\psi$ is *true* in $\mathcal{A}$ if and only if $\|\varphi\|_{\mathcal{A}}\subseteq\|\psi\|_{\mathcal{A}}$. Otherwise, one can measure its *truth degree* by introducing some inclusion measure of $\|\varphi\|_{\mathcal{A}}$ in $\|\psi\|_{\mathcal{A}}$.

Given two unary predicate formulae $\alpha(x),\beta(x)$, where x runs over a finite set U, Łukasiewicz [137] proposes to assign to $\alpha(x)$ the value $\frac{\mathrm{card}(\|\alpha(x)\|)}{\mathrm{card}(U)}$, where $\|\alpha(x)\|=\{x\in U: x \text{ satisfies } \alpha\}$. The fractional value assigned to the implication $\alpha(x)\Rightarrow\beta(x)$ is then $\frac{\mathrm{card}(\|\alpha(x)\wedge\beta(x)\|)}{\mathrm{card}(\|\alpha(x)\|)}$ under the assumption that $\|\alpha(x)\|\neq\emptyset$. Proposed by Łukasiewicz, that fractional part was much later adapted by machine learning and data mining literature.

Each object x of a decision system determines a *decision rule*

$$\bigwedge_{a\in C}a=a(x)\Rightarrow\bigwedge_{d\in D}d=d(x). \tag{8}$$

For any decision table $\mathcal{A}=(U,C,d)$ one can consider a *generalized decision* function $\partial_A: U\to \mathrm{Pow}(\times_{d\in D}V_d)$ defined by

$$\partial_A(x)=\{i:\exists x'\in U[(x',x)\in I(A) \text{ and } d(x')=i]\}, \tag{9}$$

where $\mathrm{Pow}(V_d)$ is the powerset of the Cartesian product $\times_{d\in D}V_d$ of the family $\{V_d\}_{d\in D}$.

$\mathcal{A}$ is called *consistent (deterministic)*, if $\mathrm{card}(\partial_A(x))=1$, for any $x\in U$. Otherwise $\mathcal{A}$ is said to be *inconsistent (non-deterministic)*. Hence, a decision table is inconsistent if it consists of some objects with different decisions but indiscernible with respect to condition attributes. Any set consisting of all objects with the same generalized decision value is called a *generalized decision class*. Now, one can consider certain (possible) rules (see, e.g. [75,80]) for decision classes defined by the lower (upper) approximations of such generalized decision classes of $\mathscr{A}$. This approach can be extend, using the relationships of rough sets with the Dempster–Shafer theory (see, e.g., [273,279]), by considering rules relative to decision classes defined by the lower approximations of unions of decision classes of $\mathscr{A}$.

Numerous methods have been developed for different decision rule generation that the reader can find in the literature on rough sets. Usually, one is searching for decision rules (semi) optimal with respect to some optimization criteria describing quality of decision rules in concept approximations.

In the case of searching for concept approximation in an extension of a given universe of objects (sample), the following steps are typical. When a set of rules has been induced from a decision table containing a set of training examples, they can be inspected to see if they reveal any novel relationships between attributes that are worth pursuing for further research. Furthermore, the rules can be applied to a set of unseen cases in order to estimate their classificatory power. For a systematic overview of rule application methods the reader is referred to the literature (see, e.g., [9,147]).

3.4. Dependency of attributes

Another important issue in data analysis is discovering dependencies between attributes in a given decision system $\mathscr{A} = (U, C, D)$. Intuitively, a set of attributes D depends totally on a set of attributes C, denoted $C \Rightarrow D$, if the values of attributes from C uniquely determine the values of attributes from D. In other words, D depends totally on C, if there exists a functional dependency between values of C and D. Hence, $C \Rightarrow D$ if and only if the rule (8) is true on $\mathscr{A}$ for any $x \in U$. D can depend partially on C. Formally such a dependency can be defined in the following way.

We will say that D *depends on* C to a *degree* k $(0 \leqslant k \leqslant 1)$, denoted $C \Rightarrow_k D$, if

$$k = \gamma(C, D) = \frac{\text{card}(\text{POS}_C(D))}{\text{card}(U)}, \tag{10}$$

where

$$\text{POS}_C(D) = \bigcup_{X \in U/D} C_*(X), \tag{11}$$

called a *positive region* of the partition U/D with respect to C, is the set of all elements of U that can be uniquely classified to blocks of the partition U/D, by means of C.

If $k = 1$ we say that D *depends totally* on C, and if $k < 1$, we say that D *depends partially* (to *degree* k) on C. If $k = 0$ then the *positive region* of the partition U/D with respect to C is empty.

The coefficient k expresses the ratio of all elements of the universe, which can be properly classified to blocks of the partition U/D, employing attributes C and will be called the *degree of the dependency*.

It can be easily seen that if D depends totally on C then $I(C) \subseteq I(D)$. It means that the partition generated by C is finer than the partition generated by D. Notice, that the concept of dependency discussed above corresponds to that considered in relational databases.

Summing up: D is *totally* (*partially*) dependent on C, if *all* (*some*) elements of the universe U can be uniquely classified to blocks of the partition U/D, employing C.

Observe, that (10) defines only one of possible measures of dependency between attributes (see, e.g., [296]). One also can compare the dependency discussed in this section with dependencies considered in databases [58].

3.5. Reduction of attributes

We often face a question whether we can remove some data from a data-table preserving its basic properties, that is – whether a table contains some superfluous data.

Let us express this idea more precisely.

Let $C, D \subseteq A$, be sets of condition and decision attributes respectively. We will say that $C' \subseteq C$ is a *D-reduct* (reduct with *respect* to D) of C, if C' is a minimal subset of C such that

$$\gamma(C, D) = \gamma(C', D). \tag{12}$$

The intersection of all D-reducts is called a *D-core* (core with *respect* to D). Because the core is the intersection of all reducts, it is included in every reduct, i.e., each element of the core belongs to some reduct. Thus, in a sense, the core is the most important subset of attributes, since none of its elements can be removed without affecting the classification power of attributes. Certainly, the geometry of reducts can be more compound. For example, the core can be empty but there can exist a partition of reducts into a few sets with non-empty intersection.

Many other kinds of reducts and their approximations are discussed in the literature (see, e.g., [12,164,165,274,294,297,298]). For example, if one change the condition (12) to $\partial_A(x) = \partial_B(x)$, then the defined reducts are preserving the generalized decision. Other kinds of reducts are preserving, e.g.,: (i) the distance between attribute value vectors for any two objects, if this distance is greater than a given threshold [274], (ii) the distance between entropy distributions between any two objects, if this distance exceeds a given threshold [294,297], or (iii) the so-called reducts relative to object used for generation of decision rules [12]. There are some relationships between different kinds of reducts. If B is a reduct preserving the generalized decision, than in B is included a reduct preserving the positive region. For mentioned above reducts based on distances and thresholds one can find analogous dependency between reducts relative to different thresholds. By choosing different kinds of reducts we select different degrees to which information encoded in data is preserved. Reducts are used for building data models. Choosing a particular reduct or a set of reducts has impact on the model size as well as on its quality in describing a given data set. The model size together with the model quality are two basic components tuned in selecting relevant data models. This is known as the minimal length principle (see, e.g., [263,264,297,298]). Selection of relevant kinds of reducts is an important step in building data models. It turns out that the different kinds of reducts can be efficiently computed using heuristics based, e.g., on the Boolean reasoning approach [16].

3.6. Discernibility and Boolean reasoning

Methodologies devoted to data mining, knowledge discovery, decision support, pattern classification, approximate reasoning require tools for discovering *templates (patterns)* in data and classifying them into certain *decision classes*. Templates are in many cases most frequent sequences of events, most probable events, regular configurations of objects, the decision rules of high quality, standard reasoning schemes. Tools for discovering and classifying of templates are based on *reasoning schemes* rooted in various paradigms [43]. Such patterns can be extracted from data by means of methods based, e.g., on Boolean reasoning and discernibility.

The discernibility relations are closely related to indiscernibility and belong to the most important relations considered in rough set theory.

The ability to discern between perceived objects is important for constructing many entities like reducts, decision rules or decision algorithms. In the classical rough set approach the discernibility relation $\mathrm{DIS}(B) \subseteq U \times U$ is defined by $x\mathrm{DIS}(B)y$ if and only if $\mathrm{non}(xI(B)y)$. However, this is, in general, not the case for the generalized approximation spaces (one can define indiscernibility by $x \in I(y)$ and discernibility by $I(x) \cap I(y) = \emptyset$ for any objects x, y where $I(x) = B(x), I(y) = B(y)$ in the case of the indiscernibility relation defined in Section 3.2 and in a more general case $I(x), I(y)$ are neighborhoods of objects not necessarily defined by the equivalence relation.

The idea of Boolean reasoning is based on construction for a given problem P of a corresponding Boolean function f_P with the following property: the solutions for the problem P can be decoded from prime implicants of the Boolean function f_P. Let us mention that to solve real-life problems it is necessary to deal with Boolean functions having large number of variables.

A successful methodology based on the discernibility of objects and Boolean reasoning has been developed for computing of many important ingredients for applications. These applications include generation of reducts and their approximations, decision rules, association rules, discretization of real value attributes, symbolic value grouping, searching for new features defined by oblique hyperplanes or higher order surfaces, pattern extraction from data as well as conflict resolution or negotiation.

Most of the problems related to generation of the above mentioned entities are NP-complete or NP-hard. However, it was possible to develop efficient heuristics returning suboptimal solutions of the problems. The results of experiments on many data sets are very promising. They show very good quality of solutions generated by the heuristics in comparison with other methods reported in literature (e.g., with respect to the classification quality of unseen objects). Moreover, they are very efficient from the point of view of time necessary for computing of the solution. Many of these methods are based on discernibility matrices. Note, that it is possible to compute the necessary information about these matrices using directly[4] information or decision systems (e.g., sorted in preprocessing [9,163,168,359]) which significantly improves the efficiency of algorithms.

It is important to note that the methodology makes it possible to construct heuristics having a very important *approximation property* which can be formulated as follows: expressions generated by heuristics (i.e., implicants) *close* to prime implicants define approximate solutions for the problem.

3.7. Rough membership

Let us observe that rough sets can be also defined employing the rough membership function (see Eq. (13)) instead of approximation [213]. That is, consider

$$\mu_X^B : U \to \langle 0, 1 \rangle,$$

defined by

$$\mu_X^B(x) = \frac{\mathrm{card}(B(x) \cap X)}{\mathrm{card}(X)}, \tag{13}$$

where $x \in X \subseteq U$.

The value $\mu_X^B(x)$ can be interpreted as the degree that x belongs to X in view of knowledge about x expressed by B or the degree to which the elementary granule $B(x)$ is included in the set X. This means that the definition reflects a subjective knowledge about elements of the universe, in contrast to the classical definition of a set.

The rough membership function can also be interpreted as the conditional probability that x belongs to X given B. This interpretation was used by several researchers in the rough set community (see, e.g., [78,297,338,358,379,367,375]). Note also that the ratio on the right hand side of Eq. (13) is known as the confidence coefficient in data mining [56,109]. It is worthwhile to mention that set inclusion to a degree has been considered by Łukasiewicz [137] in studies on assigning fractional truth values to logical formulas.

It can be shown that the rough membership function has the following properties [213]:

(1) $\mu_X^B(x) = 1$ iff $x \in B_*(X)$
(2) $\mu_X^B(x) = 0$ iff $x \in U - B^*(X)$
(3) $0 < \mu_X^B(x) < 1$ iff $x \in BN_B(X)$
(4) $\mu_{U-X}^B(x) = 1 - \mu_X^B(x)$ for any $x \in U$
(5) $\mu_{X \cup Y}^B(x) \geqslant \max(\mu_X^B(x),\ \mu_Y^B(x))$ for any $x \in U$
(6) $\mu_{X \cap Y}^B(x) \leqslant \min(\mu_X^B(x),\ \mu_Y^B(x))$ for any $x \in U$.

From the properties it follows that the rough membership differs essentially from the fuzzy membership [370], for properties (5) and (6) show that the membership for union and intersection of sets, in general, cannot be computed – as in the case of fuzzy sets – from their constituents membership. Thus formally the rough membership is more general from fuzzy membership. Moreover, the rough membership function depends on an available knowledge (represented by attributes from B). Besides, the rough membership function, in contrast to fuzzy membership function, has a probabilistic flavor.

Let us also mention that rough set theory, in contrast to fuzzy set theory, clearly distinguishes two very important concepts, vagueness and uncertainty, very often confused in the AI literature. Vagueness is the property of sets and can be described by approximations, whereas uncertainty is the property of elements of a set and can be expressed by the rough membership function.

[4] i.e., without the necessity of generation and storing of the discernibility matrices.

Both fuzzy and rough set theory represent two different approaches to vagueness. Fuzzy set theory addresses *gradualness* of knowledge, expressed by the fuzzy membership, whereas rough set theory addresses *granularity* of knowledge, expressed by the indiscernibility relation. A nice illustration of this difference has been given by Dider Dubois and Henri Prade [41] in the following example. In image processing fuzzy set theory refers to gradualness of gray level, whereas rough set theory is about the size of pixels.

Consequently, both theories are not competing but are rather complementary. In particular, the rough set approach provides tools for approximate construction of fuzzy membership functions. The rough-fuzzy hybridization approach proved to be successful in many applications (see, e.g., [192,195]).

Interesting discussion of fuzzy and rough set theory in the approach to vagueness can be found in [262]. Let us also observe that fuzzy set and rough set theory are not a remedy for classical set theory difficulties.

One of the consequences of perceiving objects by information about them is that for some objects one cannot decide if they belong to a given set or not. However, one can estimate the degree to which objects belong to sets. This is a crucial observation in building foundations for approximate reasoning. Dealing with imperfect knowledge implies that one can only characterize satisfiability of relations between objects to a degree, not precisely. One of the fundamental relations on objects is a rough inclusion relation describing that objects are parts of other objects to a degree. The rough mereological approach [194,241,242,244] based on such a relation is an extension of the Leśniewski mereology [126].

4. Rough sets and logic

The father of contemporary logic is a German mathematician Gottlob Frege (1848–1925). He thought that mathematics should not be based on the notion of set but on the notions of logic. He created the first axiomatized logical system but it was not understood by the logicians of those days.

During the first three decades of the 20th century, there was a rapid development in logic bolstered to a great extent by Polish logicians, especially Alfred Tarski (1901–1983) (see,e.g., [330]).

Development of computers and their applications stimulated logical research and widened their scope.

When we speak about logic, we generally mean *deductive logic*. It gives us tools designed for deriving true propositions from other true propositions. Deductive reasoning always leads to true conclusions. The theory of deduction has well established generally accepted theoretical foundations. Deductive reasoning is the main tool used in mathematical reasoning and found no application beyond it.

Rough set theory has contributed to some extent to various kinds of deductive reasoning. Particularly, various kinds of logics based on the rough set approach have been investigated, rough set methodology contributed essentially to modal logics, many valued logic, intuitionistic logic and others (see, e.g., [5,6,45,47,50,64,63,141,142,160,159,161,178,180,181,184,204,205,238,239,255–261,341,340,342,343]).

A summary of this research can be found in [237] and interested reader is advised to consult this volume.

In natural sciences (e.g., in physics) *inductive reasoning* is of primary importance. The characteristic feature of such reasoning is that it does not begin from axioms (expressing general knowledge about the reality) like in deductive logic, but some partial knowledge (examples) about the universe of interest are the starting point of this type of reasoning, which are generalized next and they constitute the knowledge about wider reality than the initial one. In contrast to deductive reasoning, inductive reasoning does not lead to true conclusions but only to probable (possible) ones. Also in contrast to the logic of deduction, the logic of induction does not have uniform, generally accepted, theoretical foundations as yet, although many important and interesting results have been obtained, e.g., concerning statistical and computational learning and others.

Verification of validity of hypotheses in the logic of induction is based on experiment rather than the formal reasoning of the logic of deduction. Physics is the best illustration of this fact.

The research on inductive logic has a few centuries' long history and outstanding English philosopher John Stuart Mill (1806–1873) is considered its father [146].

The creation of computers and their innovative applications essentially contributed to the rapid growth of interest in inductive reasoning. This domain develops very dynamically thanks to computer science. Machine learning, knowledge discovery, reasoning from data, expert systems and others are examples of new directions in inductive reasoning. It seems that rough set theory is very well suited as a theoretical basis for inductive reasoning. Basic concepts of this theory fit very well to represent and analyze knowledge acquired from examples, which can be next used as starting point for generalization. Besides, in fact rough set theory has been successfully applied in many domains to find patterns in data (data mining) and acquire knowledge from examples (learning from examples). Thus, rough set theory seems to be another candidate as a mathematical foundation of inductive reasoning [14,167,290].

The most interesting from computer science point of view is *common sense* reasoning. We use this kind of reasoning in our everyday life, and examples of such kind of reasoning we face in news papers, radio TV etc., in political, economic etc., debates and discussions.

The starting point to such reasoning is the knowledge possessed by the specific group of people (*common knowledge*) concerning some subject and intuitive methods of deriving conclusions from it. We do not have here possibilities of resolving the dispute by means of methods given by deductive logic (reasoning) or by inductive logic (experiment). So the best known methods of solving the dilemma is voting, negotiations or even war. See e.g., Gulliver's Travels [319], where the hatred between Tramecksan (high-heels) and Slamecksan (low-heels) or disputes between Big-Endians and Small-Endians could not be resolved without a war.

These methods do not reveal the truth or falsity of the thesis under consideration at all. Of course, such methods are not acceptable in mathematics or physics. Nobody is going to solve by voting, negotiations or declare a war – the truth of Fermat's theorem or Newton's laws.

Reasoning of this kind is the least studied from the theoretical point of view and its structure is not sufficiently understood, in spite of many interesting theoretical research in this domain [57]. The meaning of common sense reasoning, considering its scope and significance for some domains, is fundamental and rough set theory can also play an important role in it but more fundamental research must be done to this end [283].

In particular, the rough truth introduced in [204] and studied, e.g., in [6] seems to be important for investigating commonsense reasoning in the rough set framework.

Let us consider a simple example. In the considered decision table we assume $U = \text{Birds}$ is a set of birds that are described by some condition attributes from a set A. The decision attribute is a binary attribute *Files* with possible values *yes* if the given bird flies and *no*, otherwise. Then, we define the set of abnormal birds by $\mathrm{Ab}_A(\mathit{Birds}) = A_*(\{x \in \mathit{Birds} : \mathit{Flies}(x) = \mathit{no}\})$. Hence, we have, $\mathrm{Ab}_A(\mathit{Birds}) = \mathit{Birds} - A^*(\{x \in \mathit{Birds} : \mathit{Flies}(x) = \mathit{yes}\})$ and $\mathit{Birds} - \mathrm{Ab}_A(\mathit{Birds}) = A^*(\{x \in \mathit{Birds} : \mathit{Flies}(x) = \mathit{yes}\})$. It means that for normal birds it is consistent, with knowledge represented by A, to assume that they can fly, i.e., it is possible that they can fly. One can optimize $\mathrm{Ab}_A(\mathit{Birds})$ using A to obtain minimal boundary region in the approximation of $\{x \in \mathit{Birds} : \mathit{Flies}(x) = \mathit{no}\}$.

It is worthwhile to mention that in [38] has been presented an approach combining the rough sets with nonmonotonic reasoning. There are distinguished some basic concepts that can be approximated on the basis of sensor measurements and more complex concepts that are approximated using so-called transducers defined by first order theories constructed overs approximated concepts. Another approach to commonsense reasoning has been developed in a number of papers (see, e.g., [14,167,194,247,283]). The approach is based on an ontological framework for approximation. In this approach approximations are constructed for concepts and dependencies between the concepts represented in a given ontology expressed, e.g., in natural language. Still another approach combining rough sets with logic programming is discussed in [346].

To recapitulate, the characteristics of the three above mentioned kinds of reasoning are given below:

(1) Deductive:
- reasoning method: axioms and rules of inference;
- applications: mathematics;
- theoretical foundations: complete theory;
- conclusions: true conclusions from true premisses;
- hypotheses verification: formal proof.

(2) Inductive:
- reasoning method: generalization from examples;
- applications: natural sciences (physics);
- theoretical foundation: lack of generally accepted theory;
- conclusions: not true but probable (possible);
- hypotheses verification – experiment.

(3) Common sense:
- reasoning method based on common sense knowledge with intuitive rules of inference expressed in natural language;
- applications: every day life, humanities;
- theoretical foundation: lack of generally accepted theory;
- conclusions obtained by mixture of deductive and inductive reasoning based on concepts expressed in natural language, e.g., with application of different inductive strategies for conflict resolution (such as voting, negotiations, cooperation, war) based on human behavioral patterns;
- hypotheses verification – human behavior.

5. Exemplary research directions and applications

In the article we have discussed some basic issues and methods related to rough sets. For more detail the reader is referred to the literature cited at the beginning of this article (see also rsds.wsiz.rzeszow.pl).

We are now observing a growing research interest in the foundations of rough sets (see, e.g., [13,17,62,86,93,94,104,116–119,143,153,176,171,197,203,209,211–213,235,240,249,273,277,278,283–285, 289–292,307,312,308,310,315,328,352,366,364,369,371,372]), including the various logical (see, e.g., [5,6,45,47,50, 63,64,141,142,160,159,161,178,180,181,183,184,204,205,238,239,255–261,341,340,342,343]), algebraic (see, e.g., [7,21–24,29,49,98–100,173–175,185–187]), [248,252,353–355], philosophical aspects of rough sets (see, e.g., [3,33,182,237,262,278]), and complexity issues (see, e.g.,[26,34,35,154,155,162,168,298]).

Some relationships have already been established between rough sets and other approaches as well as a wide range of hybrid systems have been developed (see, e.g., [8,37,60,39,40,72,46,48,59,66,67,96,128,103, 127,134,136,139,148,149,157,189–196,207,208,216,219,229,230,232,236,244,245,251,265,188,279,281,286,288, 295,299,302,317,318,321–324,326,327,329,358–363,365,368]).

As a result, rough sets are linked with decision system modeling and analysis of complex systems, fuzzy sets, neural networks, evolutionary computing, data mining and knowledge discovery, pattern recognition, machine learning, data mining, and approximate reasoning, multicriteria decision making. In particular, rough sets are used in probabilistic reasoning, granular computing (including information granule calculi based on rough mereology), intelligent control, intelligent agent modeling, identification of autonomous systems, and process specification.

A wide range of applications of methods based on rough set theory alone or in combination with other approaches have been discovered in the following areas: acoustics, bioinformatics, business and finance, chemistry, computer engineering (e.g., data compression, digital image processing, digital signal processing, parallel and distributed computer systems, sensor fusion, fractal engineering), decision analysis and systems, economics, electrical engineering (e.g., control, signal analysis, power systems), environmental studies, digital image processing, informatics, medicine, molecular biology, musicology, neurology, robotics, social science, software engineering, spatial visualization, Web engineering, and Web mining. Below we list some suggestions for readings in selected application domains:

- machine learning, pattern recognition, data mining and knowledge discovery (see, e.g., [2,8,12,27,36, 42,48,59,61,65,67,76–82,85,90–92,107,108,221,120,122,123,127,132,133,135,148,149,166,190–195,211,231, 241,244,245,250,265,269,270,275,276,281,282,316,321–323,325,338,351,357]);
- bioinformatics (see, e.g.,[95,121,144,145,150,151,172,301,344]);
- multicriteria decision making (see, e.g.,[68–71,73,234]);
- medicine (see, e.g.,[10,51,83,84,89,102,215,301,303,304,309,320,332–335,338],[339,348,349,373]);
- signal processing and image processing (see, e.g.,[30–32,111–115,220,356]);
- hierarchical learning, ontology approximation (see, e.g.,[11,14,167,170,169,277,283,286,287]);
- other domains (see, e.g., [152,156–158,198,217,222–224,233,253,254,313]).

6. Conclusions

In this article we have presented basic concepts of rough set theory. We have also listed some research directions and exemplary applications based on the rough set approach.

A variety of methods for decision rules generation, reducts computation and continuous variable discretization are very important issues not discussed here. We have only mentioned the methodology based on discernibility and Boolean reasoning for efficient computation of different entities including reducts and decision rules. For more details the reader is referred to [214]. Several extensions of the rough set approach have been proposed in the literature [210]. In particular, it has been shown that the rough set approach can be used for synthesis and analysis of concept approximations in the distributed environment of intelligent agents. The relationships of rough set theory to many other theories has been extensively investigated. In particular, its relationships to fuzzy set theory, the theory of evidence, Boolean reasoning methods, statistical methods, and decision theory have been clarified and seem now to be thoroughly understood. There are reports on many hybrid methods obtained by combining the rough set approach with other approaches such as fuzzy sets, neural networks, genetic algorithms, principal component analysis, and singular value decomposition. Many important research topics in rough set theory such as various logics related to rough sets and many advanced algebraic properties of rough sets were only mentioned in the article. The reader can find details in the books, articles and journals cited in this paper.

Acknowledgements

The research of Andrzej Skowron has been supported by the grant 3 T11C 002 26 from Ministry of Scientific Research and Information Technology of the Republic of Poland.

Many thanks to Professors James Peters and Dominik Ślęzak for their incisive comments and for suggesting many helpful ways to improve this article.

References

[1] J.J. Alpigini, J.F. Peters, A. Skowron, N. Zhong (Eds.), Third International Conference on Rough Sets and Current Trends in Computing (RSCTC'2002), Malvern, PA, October 14–16, 2002, Lecture Notes in Artificial Intelligence, vol. 2475, Springer-Verlag, Heidelberg, 2002.
[2] A. An, Y. Huang, X. Huang, N. Cercone, Feature selection with rough sets for web page classification. In: Peters et al. [228], pp. 1–13.
[3] P. Apostoli, A. Kanda, Parts of the continuum: Towards a modern ontology of sciences, Technical Reports in Philosophical Logic, vol. 96 (1). The University of Toronto, Department of Philosophy, Toronto, Canada, 1999, Revised March, 1999.
[4] R. Ariew, D. Garber, G.W. Leibniz (Eds.), Philosophical Essays, Hackett Publishing Company, Indianapolis, 1989.
[5] P. Balbiani, D. Vakarelov, A modal logic for indiscernibility and complementarity in information systems, Fundamenta Informaticae 50 (3–4) (2002) 243–263.
[6] M. Banerjee, Logic for rough truth, Fundamenta Informaticae 71 (2–3) (2006) 139–151.
[7] M. Banerjee, M.K. Chakraborty, Rough set algebras. In: Pal et al. [194], pp. 157–184.
[8] M. Banerjee, S.K. Pal, Roughness of a fuzzy set, Information Sciences 93 (3–4) (1996) 235–246.
[9] J. Bazan, H.S. Nguyen, S.H. Nguyen, P. Synak, J. Wróblewski, Rough set algorithms in classification problems. In: Polkowski et al. [241], pp. 49–88.
[10] J. Bazan, A. Osmólski, A. Skowron, D. Ślęzak, M. Szczuka, J. Wróblewski, Rough set approach to the survival analysis. In: Alpigini et al. [1], pp. 522–529.
[11] J. Bazan, A. Skowron, On-line elimination of non-relevant parts of complex objects in behavioral pattern identification. In: Pal et al. [189], pp. 720–725.
[12] J.G. Bazan, A comparison of dynamic and non-dynamic rough set methods for extracting laws from decision tables. In: Polkowski and Skowron [244], pp. 321–365.
[13] J.G. Bazan, H.S. Nguyen, A. Skowron, M. Szczuka, A view on rough set concept approximation. In: Wang et al. [350], pp. 181–188.
[14] J.G. Bazan, J.F. Peters, A. Skowron, Behavioral pattern identification through rough set modelling. In: Ślęzak et al. [301], pp. 688–697.
[15] M. Black, Vagueness: an exercise in logical analysis, Philosophy of Science 4 (4) (1937) 427–455.
[16] F. Brown, Boolean Reasoning, Kluwer Academic Publishers, Dordrecht, 1990.
[17] E. Bryniarski, U. Wybraniec-Skardowska, Generalized rough sets in contextual spaces. In: Rough Sets and Data Mining – Analysis of Imperfect Data. pp. 339–354.
[18] G. Cantor, Über eine Eigenschaft des Inbegriffes aller reellen algebraischen Zahlen, Crelle's Journal für Mathematik 77 (1874) 258–263.
[19] G. Cantor, Grundlagen einer allgemeinen Mannigfaltigkeitslehre, B.G. Teubner, Leipzig, 1883.
[20] R. Casti, A. Varzi (Eds.), Parts and Places. The Structures of Spatial Representation, The MIT Press, Cambridge, MA, 1999.
[21] G. Cattaneo, Abstract approximation spaces for rough theories. In: Polkowski and Skowron [244], pp. 59–98.
[22] G. Cattaneo, D. Ciucci, Algebraic structures for rough sets. In: Peters et al. [228], pp. 208–252.

[23] G. Cattaneo, D. Ciucci, R. Giuntini, M. Konig, Algebraic structures related to many valued logical systems. Part I: Heyting–Wajsberg algebras, Fundamenta Informaticae 63 (4) (2004) 331–355.
[24] G. Cattaneo, D. Ciucci, R. Giuntini, M. Konig, Algebraic structures related to many valued logical systems. Part II: Equivalence among some widespread structures, Fundamenta Informaticae 63 (4) (2004) 357–373.
[25] N. Cercone, A. Skowron, N. Zhong (Eds.) Computational Intelligence: An International Journal vol. 17 (3) (2001) (Special issue).
[26] B.S. Chlebus, S.H. Nguyen, On finding optimal discretizations for two attributes. In: Polkowski and Skowron [243], pp. 537–544.
[27] M.R. Chmielewski, J.W. Grzymała-Busse, Global discretization of continuous attributes as preprocessing for machine learning, International Journal of Approximate Reasoning 15 (4) (1996) 319–331.
[28] K. Cios, W. Pedrycz, R. Swiniarski, Data Mining Methods for Knowledge Discovery, Kluwer, Norwell, MA, 1998.
[29] S.D. Comer, An algebraic approach to the approximation of information, Fundamenta Informaticae 14 (4) (1991) 495–502.
[30] A. Czyżewski, Automatic identification of sound source position employing neural networks and rough sets, Pattern Recognition Letters 24 (6) (2003) 921–933.
[31] A. Czyżewski, R. Królikowski, Neuro-rough control of masking thresholds for audio signal enhancement, Neurocomputing 36 (2001) 5–27.
[32] A. Czyżewski, M. Szczerba, B. Kostek, Musical phrase representation and recognition by means of neural networks and rough sets. In: Peters and Skowron [225], pp. 254–278.
[33] S. Demri, E. Orłowska (Eds.), Incomplete Information: Structure, Inference, Complexity, Monographs in Theoretical Computer Sience, Springer-Verlag, Heidelberg, 2002.
[34] S. Demri, U. Sattler, Automata-theoretic decision procedures for information logics, Fundamenta Informaticae 53 (1) (2002) 1–22.
[35] S. Demri, J. Stepaniuk, Computational complexity of multimodal logics based on rough sets, Fundamenta Informaticae 44 (4) (2000) 373–396.
[36] J. Deogun, V.V. Raghavan, A. Sarkar, H. Sever, Data mining: trends in research and development. In: Rough Sets and Data Mining – Analysis of Imperfect Data, pp. 9–46.
[37] P. Doherty, W. Łukaszewicz, A. Skowron, A. Szałas, Approximation transducers and trees: a technique for combining rough and crisp knowledge. In: Knowledge Engineering: A Rough Set Approach [38], pp. 189–218.
[38] P. Doherty, W. Łukaszewicz, A. Skowron, A. Szałas, Knowledge Engineering: A Rough Set Approach, Studies in Fizziness and Soft Computing, vol. 202, Springer, Heidelberg, 2006.
[39] D. Dubois, H. Prade, Rough fuzzy sets and fuzzy rough sets, Fuzzy Sets and Systems 23 (1987) 3–18.
[40] D. Dubois, H. Prade, Rough fuzzy sets and fuzzy rough sets, International Journal of General Systems 17 (1990) 191–209.
[41] D. Dubois, H. Prade, Foreword. In: Rough Sets: Theoretical Aspects of Reasoning about Data [206].
[42] V. Dubois, M. Quafafou, Concept learning with approximation: rough version spaces. In: Alpigini et al. [1], pp. 239–246.
[43] R. Duda, P. Hart, R. Stork, Pattern Classification, John Wiley & Sons, New York, NY, 2002.
[44] B. Dunin-Kęplicz, A. Jankowski, A. Skowron, M. Szczuka (Eds.), Monitoring, Security, and Rescue Tasks in Multiagent Systems (MSRAS'2004), Advances in Soft Computing, Springer, Heidelberg, 2005.
[45] I. Düntsch, A logic for rough sets, Theoretical Computer Science 179 (1997) 427–436.
[46] I. Düntsch, G. Gediga, Uncertainty measures of rough set prediction, Artificial Intelligence 106 (1) (1998) 77–107.
[47] I. Düntsch, G. Gediga, Rough set data analysis, Encyclopedia of Computer Science and Technology, vol. 43, Marcel Dekker, 2000, pp. 281–301.
[48] I. Düntsch, G. Gediga, Rough Set Data Analysis: A Road to Non-invasive Knowledge Discovery, Methodos Publishers, Bangor, UK, 2000.
[49] I. Düntsch, E. Orlowska, H. Wang, Algebras of approximating regions, Fundamenta Informaticae 46 (1–2) (2001) 71–82.
[50] T.-F. Fan, C.-J. Liau, Y. Yao, On modal and fuzzy decision logics based on rough set theory, Fundamenta Informaticae 52 (4) (2002) 323–344.
[51] K. Farion, W. Michalowski, R. Słowiński, S. Wilk, S. Rubin, Rough set methodology in clinical practice: Controlled hospital trial of the MET system. In: Tsumoto et al. [337], pp. 805–814.
[52] H. Filip, Nominal and verbal semantic structure: analogies and interactions, Language Sciences 23 (2000) 453–501.
[53] K. Fine, Vagueness, truth and logic, Synthese 30 (1975) 265–300.
[54] P. Forrest, Sets as mereological tropes, Metaphysical 3 (2002) 5–10.
[55] G. Frege, Grundgesetzen der Arithmetik, 2, Verlag von Hermann Pohle, Jena, 1903.
[56] J.H. Friedman, T. Hastie, R. Tibshirani, The Elements of Statistical Learning: Data Mining, Inference, and Prediction, Springer-Verlag, Heidelberg, 2001.
[57] D.M. Gabbay, C.J. Hogger, J.A. Robinson (Eds.), Handbook of Logic in Artificial Intelligence and Logic Programming, Nonmonotonic Reasoning and Uncertain Reasoning, vol. 3, Calderon Press, Oxford, 1994.
[58] H. Garcia-Molina, J. Ullman, J. Widom, Database Systems: The Complete Book, Prentice Hall, Upper Saddle River, New Jersey, 2002.
[59] G. Gediga, I. Düntsch, Rough approximation quality revisited, Artificial Intelligence 132 (2001) 219–234.
[60] G. Gediga, I. Düntsch, Maximum consistency of incomplete data via non-invasive imputation, Artificial Intelligence Review 19 (2003) 93–107.
[61] G. Gediga, I. Düntsch, On model evaluation, indices of importance, and interaction values in rough set analysis. In: Pal et al. [194], pp. 251–276.
[62] A. Gomolińska, A comparative study of some generalized rough approximations, Fundamenta Informaticae 51 (1–2) (2002) 103–119.

[63] A. Gomolińska, A graded meaning of formulas in approximation spaces, Fundamenta Informaticae 60 (1–4) (2004) 159–172.
[64] A. Gomolińska, Rough validity, confidence, and coverage of rules in approximation spaces. In: Peters and Skowron [226], pp. 57–81.
[65] G. Góra, A.G. Wojna, RIONA: A new classification system combining rule induction and instance-based learning, Fundamenta Informaticae 51 (4) (2002) 369–390.
[66] S. Greco, M. Inuiguchi, R. Słowiński, A new proposal for fuzzy rough approximations and gradual decision rule representation. In: Peters et al. [228], pp. 319–342.
[67] S. Greco, M. Inuiguchi, R. Słowiński, Fuzzy rough sets and multiple-premise gradual decision rules, International Journal of Approximate Reasoning 41 (2) (2006) 179–211.
[68] S. Greco, B. Matarazzo, R. Słowiński, Dealing with missing data in rough set analysis of multi-attribute and multi-criteria decision problems, in: S. Zanakis, G. Doukidis, C. Zopounidis (Eds.), Decision Making: Recent Developments and Worldwide Applications, Kluwer Academic Publishers, Boston, MA, 2000, pp. 295–316.
[69] S. Greco, B. Matarazzo, R. Słowiński, Rough set theory for multicriteria decision analysis, European Journal of Operational Research 129 (1) (2001) 1–47.
[70] S. Greco, B. Matarazzo, R. Słowiński, Data mining tasks and methods: classification: multicriteria classification, in: W. Kloesgen, J. Żytkow (Eds.), Handbook of KDD, Oxford University Press, Oxford, 2002, pp. 318–328.
[71] S. Greco, B. Matarazzo, R. Słowiński, Dominance-based rough set approach to knowledge discovery (I) – general perspective, (ii) – extensions and applications. In: Zhong and Liu [374], pp. 513–552, 553–612.
[72] S. Greco, Z. Pawlak, R. Słowiński, Can Bayesian confirmation measures be useful for rough set decision rules? Engineering Applications of Artificial Intelligence 17 (4) (2004) 345–361.
[73] S. Greco, R. Słowiński, J. Stefanowski, M. Zurawski, Incremental versus non-incremental rule induction for multicriteria classification. In: Peters et al. [228], pp. 54–62.
[74] J.W. Grzymała-Busse, Managing Uncertainty in Expert Systems, Kluwer Academic Publishers, Norwell, MA, 1990.
[75] J.W. Grzymała-Busse, LERS – A system for learning from examples based on rough sets. In: Słowiński [305], pp. 3–18.
[76] J.W. Grzymała-Busse, Selected algorithms of machine learning from examples, Fundamenta Informaticae 18 (1993) 193–207.
[77] J.W. Grzymała-Busse, Classification of unseen examples under uncertainty, Fundamenta Informaticae 30 (3–4) (1997) 255–267.
[78] J.W. Grzymała-Busse, A new version of the rule induction system LERS, Fundamenta Informaticae 31 (1) (1997) 27–39.
[79] J.W. Grzymała-Busse, Three strategies to rule induction from data with numerical attributes. In: Peters et al. [228], pp. 54–62.
[80] J.W. Grzymała-Busse, LERS – A data mining system. In: Maimon and Rokach [138], pp. 1347–1351.
[81] J.W. Grzymała-Busse, Rule induction. In: Maimon and Rokach [138], pp. 277–294.
[82] J.W. Grzymała-Busse, W.J. Grzymała-Busse, Handling missing attribute values. In: Maimon and Rokach [138], pp. 37–57.
[83] J.W. Grzymała-Busse, W.J. Grzymała-Busse, L.K. Goodwin, Coping with missing attribute values based on closest fit in preterm birth data: a rough set approach, Computational Intelligence: An International Journal 17 (3) (2001) 425–434.
[84] J.W. Grzymaa-Busse, Z.S. Hippe, Data mining methods supporting diagnosis of melanoma, In: 18th IEEE Symposium on Computer-Based Medical Systems (CBMS 2005), 23–24 June 2005, Dublin, Ireland, IEEE Computer Society, 2005, pp. 371–373.
[85] J.W. Grzymała-Busse, W. Ziarko, Data mining and rough set theory, Communications of the ACM 43 (2000) 108–109.
[86] S. Han, J. Wang, Reduct and attribute order, Journal of Computer Science and Technology 19 (4) (2004) 429–449.
[87] C.G. Hempel, Vagueness and logic, Philosophy of Science 6 (1939) 163–180.
[88] S. Hirano, M. Inuiguchi, S. Tsumoto (Eds.). Proceedings of International Workshop on Rough Set Theory and Granular Computing (RSTGC'2001), Matsue, Shimane, Japan, May 20–22, 2001, Bulletin of the International Rough Set Society, vol. 5(1–2). International Rough Set Society, Matsue, Shimane, 2001.
[89] S. Hirano, S. Tsumoto, Rough representation of a region of interest in medical images, International Journal of Approximate Reasoning 40 (1–2) (2005) 23–34.
[90] X. Hu, N. Cercone, Learning in relational databases: a rough set approach, Computational Intelligence: An International Journal 11 (2) (1995) 323–338.
[91] X. Hu, N. Cercone, Data mining via discretization, generalization and rough set feature selection, Knowledge and Information Systems: An International Journal 1 (1) (1999) 33–60.
[92] X. Hu, N. Cercone, Discovering maximal generalized decision rules through horizontal and vertical data reduction, Computational Intelligence: An International Journal 17 (4) (2001) 685–702.
[93] X. Hu, N. Cercone, N. Shan, A rough set approach to compute all maximal generalized rules, Journal of Computing and Information 1 (1) (1995) 1078–1089.
[94] X. Hu, T.Y. Lin, J. Han, A new rough set model based on database systems, Journal of Fundamental Informatics 59 (2–3) (2004) 135–152.
[95] T.R. Hvidsten, B. Wilczyński, A. Kryshtafovych, J. Tiuryn, J. Komorowski, K. Fidelis, Discovering regulatory binding-site modules using rule-based learning, Genome Research 6 (15) (2005) 856–866.
[96] M. Inuiguchi, Generalizations of rough sets: from crisp to fuzzy cases. In: Tsumoto et al. [337], pp. 26–37 (plenary talk).
[97] M. Inuiguchi, S. Hirano, S. Tsumoto (Eds.), Rough Set Theory and Granular Computing, Studies in Fuzziness and Soft Computing, vol. 125, Springer-Verlag, Heidelberg, 2003.
[98] T. Iwiński, Rough analysis of lattices, Working papers, vol. 23. University of Carlos III, Madrid, 1991.
[99] J. Järvinen, Representation of information systems and dependence spaces, and some basic algorithms. Licentiate's thesis. Ph.D. thesis, University of Turku, Department of Mathematics, Turku, Finland, 1997.
[100] J. Järvinen, On the structure of rough approximations, Fundamenta Informaticae 53 (2) (2002) 135–153.
[101] T. Jech, Set Theory, second ed., Springer Verlag, New York, 1997.

[102] J. Jelonek, J. Stefanowski, Feature subset selection for classification of histological images, Artificial Intelligence in Medicine 9 (3) (1997) 227–239.
[103] R. Jensen, Q. Shen, Semantics-preserving dimensionality reduction: rough and fuzzy-rough approaches, IEEE Transactions on Knowledge and Data Engineering 16 (2) (2004) 1457–1471.
[104] R. Jensen, Q. Shen, A. Tuso, Finding rough set reducts with SAT. In: Ślęzak et al. [300], pp. 194–203.
[105] R. Keefe, Theories of Vagueness. Cambridge Studies in Philosophy, Cambridge, UK, 2000.
[106] R. Keefe, P. Smith, Vagueness: A Reader, MIT Press, Massachusetts, MA, 1997.
[107] D. Kim, Data classification based on tolerant rough set, Pattern Recognition 34 (8) (2001) 1613–1624.
[108] D. Kim, S.Y. Bang, A handwritten numeral character classification using tolerant rough set, IEEE Transactions on Pattern Analysis and Machine Intelligence 22 (9) (2000) 923–937.
[109] W. Kloesgen, J. Żytkow (Eds.), Handbook of Knowledge Discovery and Data Mining, Oxford University Press, Oxford, 2002.
[110] J. Komorowski, Z. Pawlak, L. Polkowski, A. Skowron, Rough sets: a tutorial. In: Pal and Skowron [195], pp. 3–98.
[111] B. Kostek, Soft computing-based recognition of musical sounds. In: Polkowski and Skowron [245], pp. 193–213.
[112] B. Kostek, Soft Computing in Acoustics, Applications of Neural Networks, Fuzzy Logic and Rough Sets to Physical Acoustics, Studies in Fuzziness and Soft Computing, vol. 31, Physica-Verlag, Heidelberg, 1999.
[113] B. Kostek, Perception-Based Data Processing in Acoustics: Applications to Music Information Retrieval and Psychophysiology of Hearing, Studies in Computational Intelligence, vol. 3, Springer, Heidelberg, 2005.
[114] B. Kostek, A. Czyżewski, Processing of musical metadata employing Pawlak's flow graphs. In: Peters and Skowron [225], pp. 279–298.
[115] B. Kostek, P. Szczuko, P. Żwan, P. Dalka, Processing of musical data employing rough sets and artificial neural networks. In: Peters and Skowron [226], pp. 112–133.
[116] M. Kryszkiewicz, Maintenance of reducts in the varable precision rough set model. In: Rough Sets and Data Mining – Analysis of Imperfect Data. pp. 355–372.
[117] M. Kryszkiewicz, Properties of incomplete information systems in the framework of rough sets. In: Polkowski and Skowron [244], pp. 422–450.
[118] M. Kryszkiewicz, Rough set approach to incomplete information systems, Information Sciences 112 (1–4) (1998) 39–49.
[119] M. Kryszkiewicz, Rules in incomplete information systems, Information Sciences 113 (3–4) (1999) 271–292.
[120] M. Kryszkiewicz, K. Cichoń, Towards scalable algorithms for discovering rough set reducts. In: Peters et al. [228], pp. 120–143.
[121] A. Lægreid, T.R. Hvidsten, H. Midelfart, J. Komorowski, A.K. Sandvik, Discovering regulatory binding-site modules using rule-based learning, Genome Researche 5 (13) (2003) 965–979.
[122] R. Latkowski, On decomposition for incomplete data, Fundamenta Informaticae 54 (1) (2003) 1–16.
[123] R. Latkowski, Flexible indiscernibility relations for missing attribute values, Fundamenta Informaticae 67 (1–3) (2005) 131–147.
[124] A.O.V. Le Blanc, Lesniewski's Computative Protothetic. Report (Ph.D. thesis), University of Manchester, Manchester, UK, 2003.
[125] G.W. Leibniz, Discourse on metaphysics. In: Ariew and Garber [4], pp. 35–68.
[126] S. Leśniewski, Grungzüge eines neuen Systems der Grundlagen der Mathematik, Fundamenta Mathematicae 14 (1929) 1–81.
[127] Y. Li, S.C.-K. Shiu, S.K. Pal, J.N.-K. Liu, A rough set-based case-based reasoner for text categorization, International Journal of Approximate Reasoning 41 (2) (2006) 229–255.
[128] T.Y. Lin, Neighborhood systems and approximation in database and knowledge base systems, in: M.L. Emrich, M.S. Phifer, M. Hadzikadic, Z.W. Ras (Eds.), Proceedings of the Fourth International Symposium on Methodologies of Intelligent Systems (Poster Session), October 12–15, 1989, Oak Ridge National Laboratory, Charlotte, NC, 1989, pp. 75–86.
[129] T.Y. Lin (Ed.)Journal of the Intelligent Automation and Soft Computing vol. 2 (2) (1996) (Special issue).
[130] T.Y. Lin, A.M. Wildberger (Eds.), Soft Computing: Rough Sets, Fuzzy Logic, Neural Networks, Uncertainty Management, Knowledge Discovery, Simulation Councils, Inc., San Diego, CA, USA, 1995.
[131] T.Y. Lin, Y.Y. Yao, L.A. Zadeh (Eds.), Rough Sets, Granular Computing and Data Mining, Studies in Fuzziness and Soft Computing, Physica-Verlag, Heidelberg, 2001.
[132] P. Lingras, Fuzzy – rough and rough – fuzzy serial combinations in neuro-computing, Neurocomputing 36 (1–4) (2001) 29–44.
[133] P. Lingras, Unsupervised rough set classification using gas, Journal of Intelligent Information Systems 16 (3) (2001) 215–228.
[134] P. Lingras, C. Davies, Application of rough genetic algorithms, Computational Intelligence: An International Journal 17 (3) (2001) 435–445.
[135] P. Lingras, C. West, Interval set clustering of Web users with rough *K*-means, Journal of Intelligent Information Systems 23 (1) (2004) 5–16.
[136] C. Liu, N. Zhong, Rough problem settings for ilp dealing with imperfect data, Computational Intelligence: An International Journal 17 (3) (2001) 446–459.
[137] J. Łukasiewicz, Die logischen Grundlagen der Wahrscheinlichkeitsrechnung, 1913, in: L. Borkowski (Ed.), Jan Łukasiewicz – Selected Works, North Holland Publishing Company, Polish Scientific Publishers, Amsterdam, London, Warsaw, 1970, pp. 16–63.
[138] O. Maimon, L. Rokach (Eds.), The Data Mining and Knowledge Discovery Handbook, Springer, Heidelberg, 2005.
[139] J. Małuszyński, A. Vitória, Toward rough datalog. In: Pal et al. [194], pp. 297–332.
[140] S. Marcus, The paradox of the heap of grains, in respect to roughness, fuzziness and negligibility. In: Polkowski and Skowron [243], pp. 19–23.
[141] V.W. Marek, H. Rasiowa, Approximating sets with equivalence relations, Theoretical Computer Science 48 (3) (1986) 145–152.
[142] V.W. Marek, M. Truszczyński, Contributions to the theory of rough sets, Fundamenta Informaticae 39 (4) (1999) 389–409.

[143] E. Menasalvas, A. Wasilewska, Data mining as generalization: a formal model, in: T.Y. Lin, S. Ohsuga, C.J. Liau, X. Hu (Eds.), Foundations and Novel Approaches in Data Mining, Computational Intelligence, Springer, Heidelberg, 2006, pp. 99–126.
[144] H. Midelfart, Supervised learning in the gene ontology. Part I: rough set framework. Part II: a bottom-up algorithm. In: Peters and Skowron [227], pp. 69–97, 98–124.
[145] H. Midelfart, J. Komorowski, K. Nørsett, F. Yadetie, A.K. Sandvik, A. Lægreid, Learning rough set classifiers from gene expression and clinical data, Fundamenta Informaticae 2 (53) (2004) 155–183.
[146] J.S. Mill, Ratiocinative and Inductive, Being a Connected View of the Principles of Evidence, and the Methods of Scientific Investigation, Parker, Son, and Bourn, West Strand London, 1862.
[147] T.M. Mitchel, Machine Learning, Computer Science, McGraw-Hill, Boston, MA, 1999.
[148] P. Mitra, S. Mitra, S.K. Pal, Modular rough fuzzy mlp: Evolutionary design. In: Skowron et al. [280], pp. 128–136.
[149] P. Mitra, S.K. Pal, M.A. Siddiqi, Non-convex clustering using expectation maximization algorithm with rough set initialization, Pattern Recognition Letters 24 (6) (2003) 863–873.
[150] S. Mitra, Computational intelligence in bioinformatics. In: Peters and Skowron [226], pp. 134–152.
[151] S. Mitra, T. Acharya, Data mining. Multimedia, Soft Computing, and Bioinformatics, John Wiley & Sons, New York, NY, 2003.
[152] S. Miyamoto, Application of rough sets to information retrieval, Journal of the American Society for Information Science 49 (3) (1998) 195–220.
[153] S. Miyamoto, Generalizations of multisets and rough approximations, International Journal of Intelligent Systems 19 (7) (2004) 639–652.
[154] M.J. Moshkov, Time complexity of decision trees. In: Peters and Skowron [226], pp. 244–459.
[155] M.J. Moshkov, M. Piliszczuk, On partial tests and partial reducts for decision tables. In: Ślęzak et al. [300], pp. 149–155.
[156] A. Mrózek, Rough sets in computer implementation of rule-based control of industrial processes. In: Słowiński [305], pp. 19–31.
[157] T. Munakata, Rough control: a perspective. In: Rough Sets and Data Mining – Analysis of Imperfect Data, pp. 77–88.
[158] M. Muraszkiewicz, H. Rybiński, Towards a parallel rough sets computer. In: Ziarko [376], pp. 434–443.
[159] A. Nakamura, Fuzzy quantifiers and rough quantifiers, in: P.P. Wang (Ed.), Advances in Fuzzy Theory and Technology II, Duke University Press, Durham, NC, 1994, pp. 111–131.
[160] A. Nakamura, On a logic of information for reasoning about knowledge. In: Ziarko [376], pp. 186–195.
[161] A. Nakamura, A rough logic based on incomplete information and its application, International Journal of Approximate Reasoning 15 (4) (1996) 367–378.
[162] H.S. Nguyen, On the decision table with maximal number of reducts, Electronic Notes in Theoretical Computer Science 82 (4) (2003).
[163] H.S. Nguyen, Approximate boolean reasoning approach to rough sets and data mining. In: Ślęzak et al. [301], pp. 12–22 (plenary talk).
[164] H.S. Nguyen, S.H. Nguyen, Rough sets and association rule generation, Fundamenta Informaticae 40 (4) (1999) 383–405.
[165] H.S. Nguyen, D. Ślęzak. Approximate reducts and association rules – correspondence and complexity results. In: Skowron et al. [280], pp. 137–145.
[166] S.H. Nguyen, Regularity analysis and its applications in data mining. In: Polkowski et al. [241], pp. 289–378.
[167] S.H. Nguyen, J. Bazan, A. Skowron, H.S. Nguyen, Layered learning for concept synthesis. In: Peters and Skowron [225], pp. 187–208.
[168] S.H. Nguyen, H.S. Nguyen, Some efficient algorithms for rough set methods. In: Sixth International Conference on Information Processing and Management of Uncertainty on Knowledge Based Systems IPMU'1996, Granada, Spain, 1996, vol. III, pp. 1451–1456.
[169] T.T. Nguyen, Eliciting domain knowledge in handwritten digit recognition. In: Pal et al. [189], pp. 762–767.
[170] T.T. Nguyen, A. Skowron, Rough set approach to domain knowledge approximation. In: Wang et al. [350], pp. 221–228.
[171] T. Nishino, M. Nagamachi, H. Tanaka, Variable precision Bayesian rough set model and its application to human evaluation data. In: Ślęzak et al. [300], pp. 294–303.
[172] K.G. Norsett, A. Lægreid, H. Midelfart, F. Yadetie, S.E. Erlandsen, S. Falkmer, J.E. Gronbech, H.L. Waldum, J. Komorowski, A.K. Sandvik, Gene expression based classification of gastric carcinoma, Cancer Letters 2 (210) (2004) 227–237.
[173] M. Novotný, Z. Pawlak, Algebraic theory of independence in information systems, Fundamenta Informaticae 14 (4) (1991) 454–476.
[174] M. Novotný, Z. Pawlak, Algebraic theory of independence in information systems, Fundamenta Informaticae 14 (1991) 454–476.
[175] M. Novotný, Z. Pawlak, On a problem concerning dependence space, Fundamenta Informaticae 16 (1992) 275–287.
[176] C.-S. Ong, J.-J. Huang, G.-H. Tzeng, Using rough set theory for detecting the interaction terms in a generalized logit model. In: Tsumoto et al. [337], pp. 624–629.
[177] E. Orłowska, Semantics of vague concepts, in: G. Dorn, P. Weingartner (Eds.), Foundation of Logic and Linguistics, Plenum Press, New York, 1984, pp. 465–482.
[178] E. Orłowska, Rough concept logic. In: Skowron [272], pp. 177–186.
[179] E. Orłowska, Reasoning about vague concepts, Bulletin of the Polish Academy of Sciences, Mathematics 35 (1987) 643–652.
[180] E. Orłowska, Logic for reasoning about knowledge, Zeitschrift für Mathematische Logik und Grundlagen der Mathematik 35 (1989) 559–572.
[181] E. Orłowska, Kripke semantics for knowledge representation logics, Studia Logica 49 (2) (1990) 255–272.
[182] E. Orowska (Ed.), Incomplete Information: Rough Set Analysis, Studies in Fuzziness and Soft Computing, vol. 13, Springer-Verlag/Physica-Verlag, Heidelberg, 1997.

[183] E. Orłowska, Z. Pawlak, Expressive power of knowledge representation system. Technical Report, Institute of Computer Science, Polish Academy of Sciences 432.

[184] E. Orłowska, Z. Pawlak, Representation of non–deterministic information, Theoretical Computer Science 29 (1984) 27–39.

[185] P. Pagliani, From concept lattices to approximation spaces: algebraic structures of some spaces of partial objects, Fundamenta Informaticae 18 (1993) 1–25.

[186] P. Pagliani, Rough sets and nelson algebras, Fundamenta Informaticae 27 (2–3) (1996) 205–219.

[187] P. Pagliani, Pretopologies and dynamic spaces, Fundamenta Informaticae 59 (2–3) (2004) 221–239.

[188] S.K. Pal, Soft data mining, computational theory of perceptions, and rough-fuzzy approach, Information Sciences 163 (1–3) (2004) 5–12.

[189] S.K. Pal, S. Bandoyopadhay, S. Biswas (Eds.), Proceedings of the First International Conference on Pattern Recognition and Machine Intelligence (PReMI 2005), December 18–22, 2005, Lecture Notes in Computer Science, vol. 3776, Indian Statistical Institute, Springer, Heidelberg, Kolkata, 2005.

[190] S.K. Pal, B. Dasgupta, P. Mitra, Rough self organizing map, Applied Intelligence 21 (2004) 289–299.

[191] S.K. Pal, P. Mitra, Case generation using rough sets with fuzzy representation, IEEE Transactions on Knowledge and Data Engineering 16 (3) (2004) 292–300.

[192] S.K. Pal, P. Mitra, Pattern Recognition Algorithms for Data Mining, CRC Press, Boca Raton, Florida, 2004.

[193] S.K. Pal, W. Pedrycz, A. Skowron, R. Swiniarski (Eds.), Rough-neuro computing, Neurocomputing 36 (2001) (Special volume).

[194] S.K. Pal, L. Polkowski, A. Skowron (Eds.), Rough-Neural Computing: Techniques for Computing with Words, Cognitive Technologies, Springer-Verlag, Heidelberg, 2004.

[195] S.K. Pal, A. Skowron (Eds.), Rough Fuzzy Hybridization: A New Trend in Decision-Making, Springer-Verlag, Singapore, 1999.

[196] K. Pancerz, Z. Suraj, Discovering concurrent models from data tables with the ROSECON system, Fundamenta Informaticae 60 (1–4) (2004) 251–268.

[197] G. Paun, L. Polkowski, A. Skowron, Rough set approximation of languages, Fundamenta Informaticae 32 (1997) 149–162.

[198] Z. Pawlak, Rough real functions and rough controllers. In: Rough Sets and Data Mining – Analysis of Imperfect Data, pp. 139–147.

[199] Z. Pawlak, Classification of Objects by Means of Attributes, Reports, vol. 429. Institute of Computer Science, Polish Academy of Sciences, Warsaw, Poland, 1981.

[200] Z. Pawlak, Information systems – theoretical foundations, Information Systems 6 (1981) 205–218.

[201] Z. Pawlak, Rough Relations, Reports, vol. 435. Institute of Computer Science, Polish Academy of Sciences, Warsaw, Poland, 1981.

[202] Z. Pawlak, Rough sets, International Journal of Computer and Information Sciences 11 (1982) 341–356.

[203] Z. Pawlak, Rough classification, International Journal of Man-Machine Studies 20 (5) (1984) 469–483.

[204] Z. Pawlak, Rough logic, Bulletin of the Polish Academy of Sciences, Technical Sciences 35 (5–6) (1987) 253–258.

[205] Z. Pawlak, Decision logic, Bulletin of the EATCS 44 (1991) 201–225.

[206] Z. Pawlak, Rough Sets: Theoretical Aspects of Reasoning about Data, System Theory, Knowledge Engineering and Problem Solving, vol. 9, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1991.

[207] Z. Pawlak, Concurrent versus sequential – the rough sets perspective, Bulletin of the EATCS 48 (1992) 178–190.

[208] Z. Pawlak, Decision rules, Bayes' rule and rough sets. In: Skowron et al. [280], pp. 1–9.

[209] Z. Pawlak, A treatise on rough sets. In: Peters and Skowron [227], pp. 1–17.

[210] Z. Pawlak, A. Skowron, Rough sets: Some extensions, Information Sciences, in press, doi:10.1016/j.ins.2006.06.006.

[211] Z. Pawlak, L. Polkowski, A. Skowron, Rough sets and rough logic: a KDD perspective. In: Polkowski et al. [241], pp. 583–646.

[212] Z. Pawlak, A. Skowron, A rough set approach for decision rules generation, in: Thirteenth International Joint Conference on Artificial Intelligence IJCAI'1993, Morgan Kaufmann, Chambéry, France, 1993, pp. 114–119.

[213] Z. Pawlak, A. Skowron, Rough membership functions, in: R. Yager, M. Fedrizzi, J. Kacprzyk (Eds.), Advances in the Dempster–Shafer Theory of Evidence, John Wiley & Sons, New York, NY, 1994, pp. 251–271.

[214] Z. Pawlak, A. Skowron, Rough sets and boolean reasoning, Information Sciences, in press, doi:10.1016/j.ins.2006.06.007.

[215] Z. Pawlak, K. Słowiński, R. Słowiński, Rough classification of patients after highly selective vagotomy for duodenal ulcer, International Journal of Man-Machine Studies 24 (5) (1986) 413–433.

[216] Z. Pawlak, S.K.M. Wong, W. Ziarko, Rough sets: probabilistic versus deterministic approach, in: B. Gaines, J. Boose (Eds.), Machine Learning and Uncertain Reasoning, vol. 3, Academic Press, London, 1990, pp. 227–242.

[217] W. Pedrycz, L. Han, J.F. Peters, S. Ramanna, R. Zhai, Calibration of software quality: fuzzy neural and rough neural computing approaches, Neurocomputing 36 (1–4) (2001) 149–170.

[218] J. Peters, A. Skowron (Eds.), A rough set approach to reasoning about dataInternational Journal of Intelligent Systems vol. 16 (1) (2001) (Special issue).

[219] J.F. Peters, Rough ethology: Towards a biologically-inspired study of collective behavior in intelligent systems with approximation spaces. In: Peters and Skowron [226], pp. 153–174.

[220] J.F. Peters, L. Han, S. Ramanna, Rough neural computing in signal analysis, Computational Intelligence: An International Journal 17 (3) (2001) 493–513.

[221] J.F. Peters, C. Henry, Reinforcement learning with approximation spaces, Fundamenta Informaticae 71 (2006) 1–27.

[222] J.F. Peters, S. Ramanna, Towards a software change classification system: A rough set approach, Software Quality Journal 11 (2) (2003) 121–147.

[223] J.F. Peters, S. Ramanna, Approximation space for software models. In: Peters et al. [228], pp. 338–355.

[224] J.F. Peters, S. Ramanna, M.S. Szczuka, Towards a line-crawling robot obstacle classification system: a rough set approach. In: Wang et al. [350], pp. 303–307.

[225] J.F. Peters, A. Skowron (Eds.), Transactions on Rough Sets I: Journal Subline, Lecture Notes in Computer Science, vol. 3100, Springer, Heidelberg, 2004.
[226] J.F. Peters, A. Skowron (Eds.), Transactions on Rough Sets III: Journal Subline, Lecture Notes in Computer Science, vol. 3400, Springer, Heidelberg, 2005.
[227] J.F. Peters, A. Skowron (Eds.), Transactions on Rough Sets IV: Journal Subline, Lecture Notes in Computer Science, vol. 3700, Springer, Heidelberg, 2005.
[228] J.F. Peters, A. Skowron, D. Dubois, J.W. Grzymała-Busse, M. Inuiguchi, L. Polkowski (Eds.), Transactions on Rough Sets II. Rough sets and fuzzy sets: Journal Subline, Lecture Notes in Computer Science, vol. 3135, Springer, Heidelberg, 2004.
[229] J.F. Peters, A. Skowron, Z. Suraj, An application of rough set methods in control design, Fundamenta Informaticae 43 (1–4) (2000) 269–290.
[230] J.F. Peters, A. Skowron, P. Synak, S. Ramanna, Rough sets and information granulation, in: O.K.T. Bilgic, D. Baets (Eds.), Tenth International Fuzzy Systems Association World Congress (IFSA 2003), Istanbul, Turkey, June 30–July 2, 2003, Lecture Notes in Artificial Intelligence, vol. 2715, Springer-Verlag, Heidelberg, 2003, pp. 370–377.
[231] J.F. Peters, Z. Suraj, S. Shan, S. Ramanna, W. Pedrycz, N.J. Pizzi, Classification of meteorological volumetric radar data using rough set methods, Pattern Recognition Letters 24 (6) (2003) 911–920.
[232] J.F. Peters, M.S. Szczuka, Rough neurocomputing: A survey of basic models of neurocomputation. In: Alpigini et al. [1], pp. 308–315.
[233] J.F. Peters, K. Ziaei, S. Ramanna, Approximate time rough control: Concepts and application to satellite attitude control. In: Polkowski and Skowron [243], pp. 491–498.
[234] R. Pindur, R. Susmaga, J. Stefanowski, Hyperplane aggregation of dominance decision rules, Fundamenta Informaticae 61 (2) (2004) 117–137.
[235] L. Polkowski, On convergence of rough sets. In: Słowiński [305], pp. 305–311.
[236] L. Polkowski, On fractal dimension in information systems. toward exact sets in infinite information systems, Fundamenta Informaticae 50 (3–4) (2002) 305–314.
[237] L. Polkowski, Rough Sets: Mathematical Foundations, Advances in Soft Computing, Physica-Verlag, Heidelberg, 2002.
[238] L. Polkowski, Rough mereology: A rough set paradigm for unifying rough set theory and fuzzy set theory, Fundamenta Informaticae 54 (2003) 67–88.
[239] L. Polkowski, A note on 3-valued rough logic accepting decision rules, Fundamenta Informaticae 61 (1) (2004) 37–45.
[240] L. Polkowski, Toward rough set foundations. mereological approach. In: Tsumoto et al. [337], pp. 8–25. (plenary talk).
[241] L. Polkowski, T.Y. Lin, S. Tsumoto (Eds.), Rough Set Methods and Applications: New Developments in Knowledge Discovery in Information Systems, Studies in Fuzziness and Soft Computing, vol. 56, Springer-Verlag/Physica-Verlag, Heidelberg, 2000.
[242] L. Polkowski, A. Skowron, Rough mereology: A new paradigm for approximate reasoning, International Journal of Approximate Reasoning 15 (4) (1996) 333–365.
[243] L. Polkowski, A. Skowron (Eds.), First International Conference on Rough Sets and Soft Computing RSCTC'1998, Lecture Notes in Artificial Intelligence, vol. 1424, Springer-Verlag, Warsaw, Poland, 1998.
[244] L. Polkowski, A. Skowron (Eds.), Rough Sets in Knowledge Discovery 1: Methodology and Applications, Studies in Fuzziness and Soft Computing, vol. 18, Physica-Verlag, Heidelberg, 1998.
[245] L. Polkowski, A. Skowron (Eds.), Rough Sets in Knowledge Discovery 2: Applications, Case Studies and Software Systems, Studies in Fuzziness and Soft Computing, vol. 19, Physica-Verlag, Heidelberg, 1998.
[246] L. Polkowski, A. Skowron, Rough mereology in information systems. a case study: Qualitative spatial reasoning. In: Polkowski et al. [241], pp. 89–135.
[247] L. Polkowski, A. Skowron, Rough mereological calculi of granules: a rough set approach to computation, Computational Intelligence: An International Journal 17 (3) (2001) 472–492.
[248] J. Pomykała, J.A. Pomykała, The stone algebra of rough sets, Bulletin of the Polish Academy of Sciences, Mathematics 36 (1988) 495–508.
[249] G.-F. Qiu, W.-X. Zhang, W.-Z. Wu, Characterizations of attributes in generalized approximation representation spaces. In: Ślęzak et al. [300], pp. 84–93.
[250] M. Quafafou, M. Boussouf, Generalized rough sets based feature selection, Intelligent Data Analysis 4 (1) (2000) 3–17.
[251] A. Radzikowska, E.E. Kerre, A comparative study of fuzzy rough sets, Fuzzy Sets and Systems 126 (2) (2002) 137–155.
[252] A. Radzikowska, E.E. Kerre, Fuzzy rough sets based on residuated lattices. In: Peters et al. [228], pp. 278–296.
[253] Z.W. Ras, Reducts-driven query answering for distributed autonomous knowledge systems, International Journal of Intelligent Systems 17 (2) (2002) 113–124.
[254] Z.W. Ras, A. Dardzinska, Collaborative query processing in DKS controlled by reducts. In: Alpigini et al. [1], pp. 189–196.
[255] H. Rasiowa, Axiomatization and completeness of uncountably valued approximation logic, Studia Logica 53 (1) (1994) 137–160.
[256] H. Rasiowa, A. Skowron, Approximation logic, in: W. Bibel, K.P. Jantke (Eds.), Mathematical Methods of Specification and Synthesis of Software Systems, Mathematical Research, vol. 31, Akademie Verlag, Berlin, 1985, pp. 123–139.
[257] H. Rasiowa, A. Skowron, Rough concept logic. In: Skowron [272], pp. 288–297.
[258] C. Rauszer, An equivalence between indiscernibility relations in information systems and a fragment of intuitionistic logic. In: Skowron [272], pp. 298–317.
[259] C. Rauszer, An equivalence between theory of functional dependence and a fragment of intuitionistic logic, Bulletin of the Polish Academy of Sciences, Mathematics 33 (1985) 571–579.
[260] C. Rauszer, Logic for information systems, Fundamenta Informaticae 16 (1992) 371–382.

[261] C. Rauszer, Knowledge representation systems for groups of agents, in: J. Wroński (Ed.), Philosophical Logic in Poland, Kluwer, Dordrecht, Netherlands, 1994, pp. 217–238.
[262] S. Read, Thinking about Logic: An Introduction to the Philosophy of Logic, Oxford University Press, Oxford, New York, 1994.
[263] J. Rissanen, Modeling by shortes data description, Automatica 14 (1978) 465–471.
[264] J. Rissanen, Minimum-description-length principle, in: S. Kotz, N. Johnson (Eds.), Encyclopedia of Statistical Sciences, John Wiley & Sons, New York, NY, 1985, pp. 523–527.
[265] A. Roy, S.K. Pal, Fuzzy discretization of feature space for a rough set classifier, Pattern Recognition Letters 24 (6) (2003) 895–902.
[266] B. Russell, The Principles of Mathematics, George Allen & Unwin Ltd., London, Great Britain, 1903 (2nd Edition in 1937).
[267] B. Russell, Vagueness, The Australian Journal of Psychology and Philosophy 1 (1923) 84–92.
[268] B. Russell, An Inquiry into Meaning and Truth, George Allen & Unwin Ltd. and W.W. Norton,, London and New York, 1940.
[269] H. Sever, V.V. Raghavan, T.D. Johnsten, The status of research on rough sets for knowledge discovery in databases, in: S. Sivasundaram (Ed.), Proceedings of the Second Internationall Conference on Nonlinear Problems in Aviation and Aerospace (ICNPAA'1998), April 29–May 1, 1998, Daytona Beach, FL, vol. 2, Embry-Riddle Aeronautical University, Daytona Beach, FL, 1998, pp. 673–680.
[270] N. Shan, W. Ziarko, An incremental learning algorithm for constructing decision rules, in: W. Ziarko (Ed.), Rough Sets, Fuzzy Sets and Knowledge Discovery, Springer Verlag, Berlin, 1994, pp. 326–334.
[271] P. Simons, A Study in Ontology, Oxford University Press, Oxford, UK, 1987.
[272] A. Skowron (Ed.), Proceedings of the 5th Symposium on Computation Theory, Zaborów, Poland, 1984, Lecture Notes in Computer Science, vol. 208, Springer-Verlag, Berlin, 1985.
[273] A. Skowron, Boolean reasoning for decision rules generation, in: J. Komorowski, Z.W. Raś (Eds.), Proceedings of ISMIS'1993, Trondheim, Norway, June 15–18, 1993, Lecture Notes in Artificial Intelligence, vol. 689, Springer-Verlag, 1993, pp. 295–305.
[274] A. Skowron, Extracting laws from decision tables, Computational Intelligence: An International Journal 11 (1995) 371–388.
[275] A. Skowron, Rough sets in KDD – plenary talk, in: Z. Shi, B. Faltings, M. Musen (Eds.), 16th World Computer Congress (IFIP'2000): Proceedings of Conference on Intelligent Information Processing (IIP'2000), Publishing House of Electronic Industry, Beijing, 2000, pp. 1–14.
[276] A. Skowron, Rough sets and boolean reasoning, in: W. Pedrycz (Ed.), Granular Computing: an Emerging Paradigm, Studies in Fuzziness and Soft Computing, vol. 70, Springer-Verlag/Physica-Verlag, Heidelberg, 2001, pp. 95–124.
[277] A. Skowron, Approximate reasoning in distributed environments. In: Zhong and Liu [374], pp. 433–474.
[278] A. Skowron, Rough sets and vague concepts, Fundamenta Informaticae 64 (1–4) (2005) 417–431.
[279] A. Skowron, J.W. Grzymała-Busse, From rough set theory to evidence theory, in: R. Yager, M. Fedrizzi, J. Kacprzyk (Eds.), Advances in the Dempster–Shafer Theory of Evidence, John Wiley & Sons, New York, NY, 1994, pp. 193–236.
[280] A. Skowron, S. Ohsuga, N. Zhong (Eds.). Proceedings of the 7th International Workshop on Rough Sets, Fuzzy Sets, Data Mining, and Granular-Soft Computing (RSFDGrC'99), Yamaguchi, November 9–11, 1999, Lecture Notes in Artificial Intelligence, vol. 1711, Springer-Verlag, Heidelberg, 1999.
[281] A. Skowron, S.K. Pal (Eds.), Rough sets, pattern recognition and data mining, Pattern Recognition Letters vol. 24 (6) (2003) (Special volume).
[282] A. Skowron, Z. Pawlak, J. Komorowski, L. Polkowski, A rough set perspective on data and knowledge, in: W. Kloesgen, J. Żytkow (Eds.), Handbook of KDD, Oxford University Press, Oxford, 2002, pp. 134–149.
[283] A. Skowron, J. Peters, Rough sets: trends and challenges. In: Wang et al. [350], pp. 25–34 (plenary talk).
[284] A. Skowron, C. Rauszer, The discernibility matrices and functions in information systems. In: Słowiński [305], pp. 331–362.
[285] A. Skowron, J. Stepaniuk, Tolerance approximation spaces, Fundamenta Informaticae 27 (2–3) (1996) 245–253.
[286] A. Skowron, J. Stepaniuk, Information granules and rough-neural computing. In: Pal et al. [194], pp. 43–84.
[287] A. Skowron, J. Stepaniuk, Ontological framework for approximation. In: Ślęzak et al. [300], pp. 718–727.
[288] A. Skowron, J. Stepaniuk, J.F. Peters, Rough sets and infomorphisms: towards approximation of relations in distributed environments, Fundamenta Informaticae 54 (1–2) (2003) 263–277.
[289] A. Skowron, R. Swiniarski, Rough sets and higher order vagueness. In: Ślęzak et al. [300], pp. 33–42.
[290] A. Skowron, R. Swiniarski, P. Synak, Approximation spaces and information granulation. In: Peters and Skowron [226], pp. 175–189.
[291] A. Skowron, P. Synak, Complex patterns, Fundamenta Informaticae 60 (1–4) (2004) 351–366.
[292] A. Skowron, P. Synak, Reasoning in information maps, Fundamenta Informaticae 59 (2–3) (2004) 241–259.
[293] A. Skowron, M. Szczuka (Eds.), Proceedings of the Workshop on Rough Sets in Knowledge Discovery and Soft Computing at ETAPS 2003, April 12–13, 2003, Electronic Notes in Computer Science, vol. 82(4), Elsevier, Amsterdam, Netherlands, 2003. Available from: http://www.elsevier.nl/locate/entcs/volume82.html.
[294] D. Ślęzak, Approximate reducts in decision tables. In: Sixth International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems IPMU'1996. Granada, Spain, 1996, vol. III, pp. 1159–1164.
[295] D. Ślęzak, Approximate Markov boundaries and Bayesian networks. In: Inuiguchi et al. [97], pp. 109–121.
[296] D. Ślęzak, Normalized decision functions and measures for inconsistent decision tables analysis, Fundamenta Informaticae 44 (2000) 291–319.
[297] D. Ślęzak, Various approaches to reasoning with frequency-based decision reducts: A survey. In: Polkowski et al. [241], pp. 235–285.
[298] D. Ślęzak, Approximate entropy reducts, Fundamenta Informaticae 53 (2002) 365–387.
[299] D. Ślęzak, Rough sets and Bayes factor. In: Peters and Skowron [226], pp. 202–229.

[300] D. Ślęzak, G. Wang, M. Szczuka, I. Düntsch, Y. Yao (Eds.), Proceedings of the 10th International Conference on Rough Sets, Fuzzy Sets, Data Mining, and Granular Computing (RSFDGrC'2005), Regina, Canada, August 31–September 3, 2005, Part I, Lecture Notes in Artificial Intelligence, vol. 3641, Springer-Verlag, Heidelberg, 2005.
[301] D. Ślęzak, J.T. Yao, J.F. Peters, W. Ziarko, X. Hu (Eds.), Proceedings of the 10th International Conference on Rough Sets, Fuzzy Sets, Data Mining, and Granular Computing (RSFDGrC'2005), Regina, Canada, August 31–September 3, 2005, Part II, Lecture Notes in Artificial Intelligence, vol. 3642, Springer-Verlag, Heidelberg, 2005.
[302] D. Ślęzak, W. Ziarko, The investigation of the Bayesian rough set model, International Journal of Approximate Reasoning 40 (2005) 81–91.
[303] K. Słowiński, R. Słowiński, J. Stefanowski, Rough sets approach to analysis of data from peritoneal lavage in acute pancreatitis, Medical Informatics 13 (3) (1998) 143–159.
[304] K. Słowiński, J. Stefanowski, Medical information systems – problems with analysis and way of solution. In: Pal and Skowron [195], pp. 301–315.
[305] R. Słowiński (Ed.), Intelligent Decision Support – Handbook of Applications and Advances of the Rough Sets Theory, System Theory, Knowledge Engineering and Problem Solving, vol. 11, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1992.
[306] R. Słowiński, J. Stefanowski (Eds.). Special issue: Proceedings of the First International Workshop on Rough Sets: State of the Art and Perspectives, Kiekrz, Poznań, Poland, September 2–4 (1992). In: Foundations of Computing and Decision Sciences, vol. 18(3–4). 1993.
[307] R. Słowiński, J. Stefanowski, Rough set reasoning about uncertain data, Fundamenta Informaticae 27 (1996) 229–244.
[308] R. Słowiński, J. Stefanowski, S. Greco, B. Matarazzo, Rough sets processing of inconsistent information, Control and Cybernetics 29 (2000) 379–404.
[309] R. Słowiński, J. Stefanowski, D. Siwiński, Application of rule induction and rough sets to verification of magnetic resonance diagnosis, Fundamenta Informaticae 53 (2002) 345–363.
[310] R. Słowiński, D. Vanderpooten, Similarity relation as a basis for rough approximations, in: P. Wang (Ed.), Advances in Machine Intelligence and Soft Computing, vol. 4, Duke University Press, Duke, NC, 1997, pp. 17–33.
[311] B. Smith, Formal ontology, common sense and cognitive science, International Journal of Human-Computer Studies 43 (1995) 641–667.
[312] J. Stefanowski, A. Tsoukiàs, Incomplete information tables and rough classification, Computational Intelligence 17 (3) (2001) 545–566.
[313] J. Stefanowski, S. Wilk, Minimizing business credit risk by means of approach integrating decision rules and case based learning, Journal of Intelligent Systems in Accounting, Finance and Management 10 (2001) 97–114.
[314] J.G. Stell, Boolean connection algebras: A new approach to the region-connection calculus, Artificial Intelligence 122 (2000) 111–136.
[315] J. Stepaniuk, Approximation spaces, reducts and representatives. In: Polkowski and Skowron [245], pp. 109–126.
[316] J. Stepaniuk, Knowledge discovery by application of rough set models. In: Polkowski et al. [241], pp. 137–233.
[317] K. Sugihara, Y. Maeda, H. Tanaka, Interval evaluation by AHP with rough set concept. In: Skowron et al. [280], pp. 375–381.
[318] Z. Suraj, Rough set methods for the synthesis and analysis of concurrent processes. In: Polkowski et al. [241], pp. 379–488.
[319] J. Swift. Gulliver's Travels into Several Remote Nations of the World. (ananymous publisher), London, M, DCC, XXVI, 1726.
[320] R. Swiniarski, Rough sets and Bayesian methods applied to cancer detection. In: Polkowski and Skowron [243], pp. 609–616.
[321] R. Swiniarski, Rough sets and principal component analysis and their applications. data model building and classification. In: Pal and Skowron [195], pp. 275–300.
[322] R. Swiniarski, An application of rough sets and Haar wavelets to face recognition. In: Ziarko and Yao [380], pp. 561–568.
[323] R. Swiniarski, L. Hargis, A new halftoning method based on error diffusion with rough set filterin. In: Polkowski and Skowron [245], pp. 336–342.
[324] R. Swiniarski, L. Hargis, Rough sets as a front end of neural networks texture classifiers, Neurocomputing 36 (1–4) (2001) 85–103.
[325] R.W. Swiniarski, A. Skowron, Independent component analysis, principal component analysis and rough sets in face recognition. In: Peters and Skowron [225], pp. 392–404.
[326] M. Szczuka, Refining classifiers with neural networks, International Journal of Intelligent Systems 16 (2001) 39–55.
[327] M. Szczuka, P. Wojdyłło, Neuro-wavelet classifiers for EEG signals based on rough set methods, Neurocomputing 36 (2001) 103–122.
[328] H. Tanaka, Dual mathematical models based on rough approximations in data analysis. In: Wang et al. [350], pp. 52–59.
[329] H. Tanaka, H. Lee, Interval regression analysis with polynomials and its similarity to rough sets concept, Fundamenta Informaticae 37 (1–2) (1999) 71–87.
[330] A. Tarski, Logic, Semantics, Metamathematics, Oxford University Press, Oxford, UK, 1983 [translated by J.H. Woodger].
[331] T. Terano, T. Nishida, A. Namatame, S. Tsumoto, Y. Ohsawa, T. Washio (Eds.), New Frontiers in Artificial Intelligence, Joint JSAI'2001 Workshop Post-Proceedings, Lecture Notes in Artificial Intelligence, vol. 2253, Springer-Verlag, Heidelberg, 2001.
[332] S. Tsumoto, Automated induction of medical expert system rules from clinical databases based on rough set theory, Information Sciences 112 (1998) 67–84.
[333] S. Tsumoto, Empirical induction on medical system expert rules based on rough set theory. In: Polkowski and Skowron [243], pp. 307–323.
[334] S. Tsumoto, Mining diagnostic rules from clinical databases using rough sets and medical diagnostic model, Information Sciences 162 (2) (2004) 65–80.
[335] S. Tsumoto, S. Hirano, Automated discovery of chronological patterns in long time-series medical datasets, International Journal of Intelligent Systems 20 (6) (2005) 737–757.

[336] S. Tsumoto, S. Kobayashi, T. Yokomori, H. Tanaka, A. Nakamura (Eds.). Proceedings of the The Fourth International Workshop on Rough Sets, Fuzzy Sets and Machine Discovery, November 6-8, University of Tokyo, Japan. The University of Tokyo, Tokyo, 1996.

[337] S. Tsumoto, R. Słowiński, J. Komorowski, J. Grzymała-Busse (Eds.), Proceedings of the 4th International Conference on Rough Sets and Current Trends in Computing (RSCTC'2004), Uppsala, Sweden, June 1–5, 2004, Lecture Notes in Artificial Intelligence, vol. 3066, Springer-Verlag, Heidelberg, 2004.

[338] S. Tsumoto, H. Tanaka, PRIMEROSE: probabilistic rule induction method based on rough sets and resampling methods, Computational Intelligence: An International Journal 11 (1995) 389–405.

[339] S. Tsumoto, W. Ziarko, The application of rough sets-based data mining technique to differential diagnosis of meningoenchepahlitis, in: Z.W. Raś, M. Michalewicz (Eds.), Ninth International Symposium on Methodologies for Intelligent Systems ISMIS'1996, Lecture Notes in Artificial Intelligence, vol. 1079, Springer-Verlag, Zakopane, Poland, 1996, pp. 438–447.

[340] D. Vakarelov, A modal logic for similarity relations in Pawlak knowledge representation systems, Fundamenta Informaticae 15 (1) (1991) 61–79.

[341] D. Vakarelov, Modal logics for knowledge representation systems, Theoretical Computer Science 90 (2) (1991) 433–456.

[342] D. Vakarelov, A duality between Pawlak's knowledge representation systems and bi-consequence systems, Studia Logica 55 (1) (1995) 205–228.

[343] D. Vakarelov, A modal characterization of indiscernibility and similarity relations in Pawlak's information systems. In: Ślęzak et al. [300], pp. 12–22 (plenary talk).

[344] J.J. Valdés, A.J. Barton, Relevant attribute discovery in high dimensional data based on rough sets and unsupervised classification: Application to leukemia gene expression. In: Ślęzak et al. [301], pp. 362–371.

[345] A.C. Varzi, Change, temporal parts, and the argument from vagueness, Dialectica 59 (4) (2005) 485–498.

[346] A. Vitória, A framework for reasoning with rough sets. Licentiate Thesis, Linköping University 2004. In: Peters and Skowron [227], pp. 178–276.

[347] P. Vopenka, Mathematics in the Alternative Set Theory, Teubner, Leipzig, 1979.

[348] A. Wakulicz-Deja, P. Paszek, Diagnose progressive encephalopathy applying the rough set theory, International Journal of Medical Informatics 46 (2) (1997) 119–127.

[349] A. Wakulicz-Deja, P. Paszek, Applying rough set theory to multi stage medical diagnosing, Fundamenta Informaticae 54 (4) (2003) 387–408.

[350] G. Wang, Q. Liu, Y. Yao, A. Skowron (Eds.), Proceedings of the 9th International Conference on Rough Sets, Fuzzy Sets, Data Mining, and Granular Computing (RSFDGrC'2003), Chongqing, China, May 26–29, 2003, Lecture Notes in Artificial Intelligence, vol. 2639, Springer-Verlag, Heidelberg, 2003.

[351] J. Wang, C. Jia, K. Zhao, Investigation on AQ11, ID3 and the principle of discernibility matrix, Journal of Computer Science and Technology 16 (1) (2001) 1–12.

[352] J. Wang, W. Ju, Reduction algorithms based on discernibility matrix: the ordered attributes method, Journal of Computer Science and Technology 16 (6) (2001) 489–504.

[353] A. Wasilewska, Topological rough algebras. In: Rough Sets and Data Mining – Analysis of Imperfect Data. pp. 411–425.

[354] A. Wasilewska, L. Vigneron, Rough equality algebras, in: Proceedings of the Second Joint Annual Conference on Information Sciences, Wrightsville Beach, North Carolina, USA, 1995, pp. 26–30.

[355] A. Wasilewska, L. Vigneron, Rough algebras and automated deduction. In: Polkowski and Skowron [244], pp. 261–275.

[356] A. Wieczorkowska, J. Wróblewski, P. Synak, D. Ślęzak, Application of temporal descriptors to musical instrument sound recognition, Journal of Intelligent Information Systems 21 (1) (2003) 71–93.

[357] A. Wojna, Analogy based reasoning in classifier construction. In: Peters and Skowron [227], pp. 277–374.

[358] S.K.M. Wong, W. Ziarko, Comparison of the probabilistic approximate classification and the fuzzy model, Fuzzy Sets and Systems 21 (1987) 357–362.

[359] J. Wróblewski, Theoretical foundations of order-based genetic algorithms, Fundamenta Informaticae 28 (1996) 423–430.

[360] J. Wróblewski, Genetic algorithms in decomposition and classification problem. In: Polkowski and Skowron [245], pp. 471–487.

[361] J. Wróblewski, Adaptive aspects of combining approximation spaces. In: Pal et al. [194], pp. 139–156.

[362] W.-Z. Wu, J.-S. Mi, W.-X. Zhang, Generalized fuzzy rough sets, Information Sciences 151 (2003) 263–282.

[363] W.-Z. Wu, W.-X. Zhang, Constructive and axiomatic approaches of fuzzy approximation operators, Information Sciences 159 (2) (2004) 233–254.

[364] Y.Y. Yao, Generalized rough set models. In: Polkowski and Skowron [244], pp. 286–318.

[365] Y.Y. Yao, Information granulation and rough set approximation, International Journal of Intelligent Systems 16 (2001) 87–104.

[366] Y.Y. Yao, On generalizing rough set theory. In: Wang et al. [350], pp. 44–51.

[367] Y.Y. Yao, Probabilistic approaches to rough sets, Expert Systems 20 (2003) 287–297.

[368] Y.Y. Yao, P. Lingras, Interpretation of belief functions in the theory of rough sets, Information Sciences 104 (1-2) (1998) 81–106.

[369] Y.Y. Yao, S.K.M. Wong, T.Y. Lin, A review of rough set models. In: Rough Sets and Data Mining – Analysis of Imperfect Data, pp. 47–75.

[370] L.A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.

[371] W.-X. Zhang, J.-S. Mi, W.-Z. Wu, Approaches to knowledge reductions in inconsistent systems, International Journal of Intelligent Systtems 18 (9) (2003) 989–1000.

[372] Z. Zheng, G. Wang, RRIA: A rough set and rule tree based incremental knowledge acquisition algorithm, Fundamenta Informaticae 59 (2–3) (2004) 299–313.

[373] N. Zhong, J. Dong, S. Ohsuga, Meningitis data mining by cooperatively using GDT-RS and RSBR, Pattern Recognition Letters 24 (6) (2003) 887–894.
[374] N. Zhong, J. Liu (Eds.), Intelligent Technologies for Information Analysis, Springer, Heidelberg, 2004.
[375] W. Ziarko, Variable precision rough set model, Journal of Computer and System Sciences 46 (1993) 39–59.
[376] W. Ziarko (Ed.), Rough Sets, Fuzzy Sets and Knowledge Discovery: Proceedings of the Second International Workshop on Rough Sets and Knowledge Discovery (RSKD'93), Banff, Alberta, Canada, October 12–15, 1993, Workshops in Computing, Springer–Verlag & British Computer Society, London, Berlin, 1994.
[377] W. Ziarko (Ed.), Computational IntelligenceAn International Journal 11 (2) (1995), Special issue.
[378] W. Ziarko (Ed.)Fundamenta Informaticae 27 (2–3) (1996), Special issue.
[379] W. Ziarko, Probabilistic decision tables in the variable precision rough set model, Computational Intelligence 17 (2001) 593–603.
[380] W. Ziarko, Y. Yao (Eds.), Proceedings of the 2nd International Conference on Rough Sets and Current Trends in Computing (RSCTC'2000), Banff, Canada, October 16–19, 2000, Lecture Notes in Artificial Intelligence, vol. 2005, Springer-Verlag, Heidelberg, 2001.