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Decision Analysis Using Rough Sets

ZDZISLAW PAWLAK* and ROMAN SŁOWINSKI†

*Warsaw University of Technology, †Technical University of Poznan, *†Polish Academy of Sciences

We show that the rough set theory is a useful tool for analysis of decision situations, in particular multi-criteria sorting problems. It deals with vagueness in the representation of a decision situation, caused by granularity of the representation. The rough set approach produces a set of decision rules involving a reduced number of most important criteria. It does not correct vagueness manifested in the representation; instead, the rules produced are categorized into deterministic and non-deterministic. The set of decision rules explains the decision situation and may be used to support new decisions. An example illustrates the rough set analysis of a multi-criteria sorting problem.

Key words: decision analysis, rough set theory, vagueness, multiple criteria, sorting problem

INTRODUCTORY REMARKS ABOUT DECISION ANALYSIS AND ROUGH SETS

Decision analysis is one of the most natural acts of human beings. It has attracted scientists for a long time who offered various mathematical tools to deal with it. Mathematical decision analysis intends to bring to light those elements of a decision situation which are not evident to the agents (decision makers, experts) involved and may influence their attitude towards the situation. More precisely, the elements revealed by mathematical decision analysis either explain the situation or prescribe, or simply privilege, some behaviour in order to increase the coherence between the evolution of the decision process on the one hand and the goals and value systems of the agents, on the other hand (*cf.* Roy, 1993).

One of the factors hindering revelation of the above mentioned elements is vagueness inherent to the representation of a decision situation. Vagueness may be caused by granularity of the representation. Due to the granularity, the facts describing a situation are either expressed precisely by means of 'granules' of the representation or only approximately.

A formal framework for discovering facts from a representation of a decision situation has been given by Pawlak (1982) and called *rough set theory*. The rough set theory assumes the representation of objects of interest in a table form called an *information system*. Rows of this table correspond to *objects* (actions, alternatives, candidates, patients, etc.) and columns correspond to *attributes*. To each pair (object, attribute) there is assigned a value called *descriptor*. Each row of the table contains descriptors representing information about the corresponding object of a given decision situation. If the set of attributes is partitioned into two subsets, *condition attributes* (criteria, tests, symptoms, etc.) and *decision attributes* (decisions, classifications, taxonomies, etc.), the information system is called a *decision table*.

Let us suppose a decision situation represented by a decision table where a finite set of objects is described by several condition attributes and a single decision attribute. The observation that objects may be indiscernible in terms of descriptors is a starting point of the rough set philosophy. Indiscernibility of objects by means of condition attributes generally prevents their precise assignment to a set following from a partition generated by the decision attribute. In this case, the only sets which can be characterized precisely in terms of the classes of indiscernible objects are lower and upper approximations of the given set. A *rough set* is just a set characterized by its lower and upper approximations. Using a lower and an upper approximation of a set (or family of sets) one can define an accuracy and a quality of approximation. These are numbers from interval [0, 1] which define how exactly one can describe the examined set of objects using available information.

As in decision problems the concept of *criterion* is often used instead of *condition attribute*; it

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Correspondence: Prof. Dr Roman Słowiński, Institute of Computing Science, Technical University of Poznan, 60-965 Poznan, Poland.

should be noticed that the latter is more general than the former because the domain (scale) of a criterion has to be ordered according to decreasing or increasing preference while the domain of a condition attribute does not have to be ordered. Similarly, the domain of a decision attribute may be ordered or not. The ordering property has to be taken into account in considerations concerning a special dependency among attributes – concordance between criteria or agents (*cf.* Boryczka, 1989). Apart from this specific case, the ordering property has no influence on rough set analysis, so there will be no distinction between criteria and condition attributes unless specified.

Depending on the kind of a decision situation, the rough set approach can bring into light different elements of this situation. In this paper, we shall limit our considerations to a *multi-criteria sorting problem*. It consists of the assignment of each object from a set to an appropriate pre-defined category (for instance: acceptance, rejection or request for additional information). The objects are described by several condition attributes and the categories are distinguished by a single decision attribute. Rough set analysis of this decision problem gives the following results:

- evaluation of importance of particular criteria,
- construction of reduced subsets of independent criteria having the same ability to approximate the decision as the whole set,
- intersection of the reduced subsets giving a core of criteria which cannot be eliminated without disturbing the ability to approximate the decision,
- elimination of redundant criteria from the decision table,
- generation of the sorting rules (deterministic or not) from the reduced decision table; they explain a decision policy and may be used for sorting new coming objects.

The aim of this paper is to show the usefulness of the rough set approach to decision analysis, in particular, the analysis of a multi-criteria sorting problem. In the next section, we recall basic concepts of the rough set theory. Then, we apply the rough set methodology to an example sorting problem. The final section groups the conclusions.

BASIC CONCEPTS OF ROUGH SET THEORY

'Common-sense' and mathematical foundations of the rough set theory

A central premise of the rough set philosophy is that *knowledge* consists of the ability to *classify*. Rational behaviour of any agent in the outer realm is based on its ability to classify real or abstract objects, for example, sensory signals. In order to classify, one has to perceive some differences between objects, thus forming classes of objects which are not noticeably different. These *indiscernibility classes* can be viewed as basic *building blocks* (*concepts*) used to build up knowledge about a real or abstract world. For example, if objects are classified according to colour and shape, then the indiscernibility classes are: red triangles, black squares, red circles, etc.; in other words, the two *attributes* make a partition in the set of objects. Hence, knowledge can be defined as a family of partitions of a fixed set (universe) U or, what is the same from a mathematical point of view, as a family of equivalence relations R on U. This view of knowledge is semantic in nature where granularity of knowledge (indiscernibility of some objects) is of primary importance and can be used to define the key concepts of the rough set theory: *approximation*, *dependency* and *reduction*. Let us explain them informally.

Suppose we are given a finite set of objects (universe) U and a finite family of equivalence relations R on U (knowledge about U). Thus, formally, knowledge R about U can be considered as a relational system $K = \langle U, R \rangle$. The main problem we are interested in is the following. Given subset $X \subseteq U$ and knowledge $K = \langle U, R \rangle$, express properties of X in terms of available knowledge. Because set theoretical intersection of equivalence relations is also an equivalence relation, the resulting family of equivalence classes (partition) can be viewed as a family of elementary sets (atoms, granules) of knowledge $K = \langle U, R \rangle$. Hence, our problem boils down to define the subset X in terms of elementary sets, i.e. to represent it as a union of atoms.

Seemingly, this is not always possible. Therefore, the concept of the *lower* and the *upper approximation* has been introduced.

The lower approximation of X in $K = \langle U, R \rangle$ is the union of all elementary sets which are included in X, whereas the upper approximation of X in $K = \langle U, R \rangle$ is the union of all elementary sets which have non empty intersection with X. These approximations correspond, respectively, to a maximal set including objects which *surely* belong to X and a minimal set of objects which *possibly* belong to X. The difference between the lower and the upper approximations is a *boundary set* consisting of all objects which cannot be classified with certainty to X or to its complement. The cardinality of the boundary set describes how exactly we can describe X in terms of available knowledge.

Discovering *dependencies* in $K = \langle U, R \rangle$ consists of finding out relationships between partitions, i.e. equivalence relations belonging to family R. In other words, dependency says how some concepts of knowledge $K = \langle U, R \rangle$ can be expressed by other concepts of knowledge $K = \langle U, R \rangle$.

Let us observe that for knowledge $K = \langle U, R \rangle$ two different families of equivalence relations, R and R' (a subfamily of R), may give the same family of elementary sets. Then, it is important to know whether it is possible to *reduce* R' while preserving the family of elementary sets, i.e. without losing a part of the knowledge.

The rough set concept has led to many theoretical results in:

- topology and abstract algebra,
- logics,
- Boolean algebra,
- probability and evidence theory,
- fuzzy sets, and others.

According to a recent bibliography of rough set theory (Ziarko, 1993), these results are presented in several hundred papers. A part of them has been summarized in Pawlak (1991).

The practical utility of the rough set concept has also been proven by many applications (*cf.* Słowiński, 1992).

Let us pass now to more formal presentation of the concepts used in the following part of the paper.

Information system

For algorithmic reasons, knowledge will be represented in the form of an information system.

By an *information system* we understand the 4-tuple $S = \langle U, Q, V, \rho \rangle$, where U is a finite set of *objects*, Q is a finite set of *attributes*, $V = \bigcup_{q \in Q} V_q$ and V_q is a *domain* of the attribute q, and $\rho : U \times Q \rightarrow V$ is a total function such that $\rho(x, q) \in V_q$ for every $q \in Q$, $x \in U$, called an *information function* (*cf.* Pawlak, 1991).

Let $S = \langle U, Q, V, \rho \rangle$ be an information system and let $P \subseteq Q$ and $x, y \in U$. We say that x and y are *indiscernible* by the set of attributes P in S iff $\rho(x, q) = \rho(y, q)$ for every $q \in P$. Thus every $P \subseteq Q$ generates a binary relation on U which will be called an *indiscernibility relation*, denoted by IND(P). Obviously, IND(P) is an equivalence relation for any P. Equivalence classes of IND(P) are called *P-elementary sets* in S. The family of all equivalence classes of relation IND(P) on U is denoted by $U \mid \text{IND}(P)$ or, in short, $U \mid P$.

$\text{Des}_P(X)$ denotes a *description* of the P-elementary set $X \in U \mid P$ in terms of values of attributes from P, i.e.

$$\text{Des}_P(X) = \{(q, v) : f(x, q) = v, \forall x \in X, \forall q \in P\}.$$

Approximation of sets

Let $P \subseteq Q$ and $Y \subseteq U$. The *P-lower approximation* of Y, denoted by $\underline{P}Y$, and the *P-upper approximation* of Y, denoted by $\bar{P}Y$, are defined as:

$$\underline{P}Y = \bigcup\{X \in U \mid P: X \subseteq Y\},$$
$$\bar{P}Y = \bigcup\{X \in U \mid P: X \cap Y \neq \varnothing\}.$$

The P-boundary (doubtful region) of set Y is defined as

$$\mathrm{Bn}_P(Y) = \bar{P}Y - \underline{P}Y.$$

Set $\underline{P}Y$ is the set of all elements of U which can be certainly classified as elements of Y, employing the set of attributes P. Set $\bar{P}Y$ is the set of elements of U which can be possibly classified as elements of Y, using the set of attributes P. The set $\mathrm{Bn}_P(Y)$ is the set of elements which cannot be certainly classified to Y using the set of attributes P.

With every set $Y \subseteq U$, we can associate an *accuracy of approximation* of set Y by P in S, or in short, accuracy of Y, defined as:

$$\alpha_P(Y) = \frac{\mathrm{card}(\underline{P}Y)}{\mathrm{card}(\bar{P}Y)}.$$

Approximation of a partition of U

Let S be an information system, $P \subseteq Q$, and let $\mathscr{Y} = \{Y_1, Y_2, \ldots, Y_n\}$ be a *partition* of U. The origin of this partition is independent of attributes from P; it can follow from solving a sorting problem by an expert. Subsets Y_i, $i = 1, \ldots, n$ are *categories* of partition $\mathscr{Y}$. By P-lower (P-upper) approximation of $\mathscr{Y}$ in S we mean sets $\underline{P}\mathscr{Y} = \{\underline{P}Y_1, \underline{P}Y_2, \ldots, \underline{P}Y_n\}$ and $\bar{P}\mathscr{Y} = \{\bar{P}Y_1, \bar{P}Y_2, \ldots, \bar{P}Y_n\}$, respectively. The coefficient

$$\gamma_P(\mathscr{Y}) = \frac{\sum_{i=1}^{n} \mathrm{card}(\underline{P}Y_i)}{\mathrm{card}(U)}$$

is called the *quality of approximation of partition* $\mathscr{Y}$ by set of attributes P, or in short, *quality of sorting*. It expresses the ratio of all P-correctly sorted objects to all objects in the system.

Reduction and dependency of attributes

We say that the set of attributes $R \subseteq Q$ *depends* on the set of attributes $P \subseteq Q$ in S (denoted $P \rightarrow R$) if $\mathrm{IND}(P) \subseteq \mathrm{IND}(R)$. Discovering dependencies between attributes is of primary importance in the rough set approach to knowledge analysis.

Another important issue is that of attribute reduction, in such a way that the reduced set of attributes provides the same quality of sorting as the original set of attributes. The minimal subset $R \subseteq P \subseteq Q$ such that $\gamma_P(\mathscr{Y}) = \gamma_R(\mathscr{Y})$ is called the *$\mathscr{Y}$-reduct* of P (or, simply, *reduct* if there is no ambiguity in the understanding of $\mathscr{Y}$) and denoted by $\mathrm{RED}_{\mathscr{Y}}(P)$. Let us note that an information system may have more than one $\mathscr{Y}$-reduct. Intersection of all $\mathscr{Y}$-reducts is called the *$\mathscr{Y}$-core* of P, i.e. $\mathrm{CORE}_{\mathscr{Y}}(P) = \bigcap \mathrm{RED}_{\mathscr{Y}}(P)$. The core is a collection of the most significant attributes in the system.

Decision tables

An information system can be seen as *decision table* assuming that $Q = C \cup D$ and $C \cap D = \varnothing$, where C are called *condition attributes*, and D *decision attributes*. The decision table $S = \langle U, C \cup D, V, \rho \rangle$ is *deterministic* iff $C \rightarrow D$; otherwise it is *non-deterministic*. The deterministic decision table uniquely describes the decisions to be made when some conditions are satisfied. In the case of a non-deterministic table, decisions are not uniquely determined by the conditions. Instead, a subset of decisions is defined which could be taken under circumstances determined by conditions.

From the decision table a set of *decision rules* can be derived. Let $U \mid \mathrm{IND}(C)$ be a family of all C-elementary sets called *condition classes*, denoted by X_i ($i = 1,\ldots,k$). Let, moreover, $U \mid \mathrm{IND}(D)$ be the family of all D-elementary sets called *decision classes*, denoted by Y_j ($j = 1,\ldots,n$).

$\mathrm{Des}_C(X_i) \Rightarrow \mathrm{Des}_D(Y_j)$ is called a (C,D)-decision rule. The rules can be also expressed as logical statements 'if . . . then . . .' relating descriptions of condition and decision classes. The set of decision rules for each decision class Y_j ($j = 1,\ldots,n$) is denoted by $\{r_{ij}\}$. Precisely,

$$\{r_{ij}\} = \{\mathrm{Des}_C(X_i) \Rightarrow \mathrm{Des}_D(Y_j): X_i \cap Y_j \neq \varnothing, i = 1,\ldots,k\}.$$

Rule r_{ij} is *deterministic* iff $X_i \subseteq Y_j$, and r_{ij} is *non-deterministic*, otherwise. Non-deterministic rules are consequences of an approximate description of decision classes (categories) in terms of condition classes (blocks of objects indiscernible by condition attributes). It means that using the available knowledge, one is unable to decide whether some objects (from the boundary region) belong to a given category or not. In other words, some objects are indiscernible in view of one's knowledge (partition) and create granules of the knowledge representation, but from the viewpoint of another knowledge they can belong to different classes – hence, description of the latter partition in terms of the granules is non-deterministic (ambiguous).

Procedures for derivation of decision rules from decision tables were presented by Boryczka and Słowiński (1988), Słowiński and Stefanowski (1992), Grzymala-Busse (1992), Skowron and Grzymala-Busse (1993), Skowron (1993) and by Ziarko *et al.* (1993).

MULTI-CRITERIA SORTING PROBLEM

The analysis of a multi-criteria sorting problem consists of discovering decision rules, taking into account the agent's (decision maker's, expert's) preferences. It is often the case that the preferences are expressed by the agent through *examples* of sorting decisions. A set of examples constitutes a decision table. In inductive learning, such a set is called a training sample. Decision rules are derived from the examples and then applied to new coming objects.

To illustrate the rough set analysis of a multi-criteria sorting problem, let us consider a simple case of selection of candicates to a school (*cf.* Moscarola, 1978).

The candidates to the school have submitted their application packages with secondary school certificate, curriculum vitae and opinion from a previous school, for consideration by an admission committee. Based on these documents, the candidates were described using seven criteria (condition attributes). The list of these criteria together with corresponding scales, ordered from the best to the worst value, is given below:

c_1 – score in mathematics, $\{5,4,3\}$
c_2 – score in physics, $\{5,4,3\}$
c_3 – score in English, $\{5,4,3\}$
c_4 – mean score in other subjects, $\{5,4,3\}$
c_5 – type of secondary school, $\{1,2,3\}$
c_6 – motivation, $\{1,2,3\}$
c_7 – opinion from previous school, $\{1,2,3\}$.

Fifteen candidates having rather different application packages have been sorted by the committee after due consideration. They create the set of examples.

The decision attribute d makes a dichotomic partition $\mathscr{Y}$ of the candidates: $d = \mathrm{A}$ means admission and $d = \mathrm{R}$ means rejection. The decision table with 15 candidates is shown in Table 1. It is clear that $C = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7\}$ and $D = \{d\}$.

Let Y_A be the set of candidates admitted and Y_R the set of candidates rejected by the committee, $Y_\mathrm{A} = \{x_1, x_4, x_5, x_7, x_8, x_{10}, x_{11}, x_{12}, x_{15}\}$, $Y_\mathrm{R} = \{x_2, x_3, x_6, x_9, x_{13}, x_{14}\}$, $\mathscr{Y} = \{Y_\mathrm{A}, Y_\mathrm{R}\}$. Sets Y_A and Y_R are D-definable sets in the decision table. There are 13 C-elementary sets: couples of indiscernible

Table 1. Decision table composed of sorting examples

Candidate \ Criterion	c_1	c_2	c_3	c_4	c_5	c_6	c_7	Decision d
x_1	4	4	4	4	2	2	1	A
x_2	3	3	4	3	2	1	1	R
x_3	3	4	3	3	1	2	2	R
x_4	5	3	5	4	2	1	2	A
x_5	4	4	5	4	2	2	1	A
x_6	3	4	3	3	2	1	3	R
x_7	4	4	5	4	2	2	2	A
x_8	4	4	4	4	2	2	2	A
x_9	4	4	4	4	2	2	2	R
x_{10}	5	3	5	4	2	1	2	A
x_{11}	5	4	4	4	1	1	2	A
x_{12}	5	3	4	4	2	2	2	A
x_{13}	4	3	3	3	3	2	2	R
x_{14}	3	3	4	3	2	3	3	R
x_{15}	4	5	5	4	2	1	1	A

candidates $\{x_4, x_{10}\}$, $\{x_8, x_9\}$ and 11 discernible candidates. The C-lower and the C-upper approximations of sets Y_A and Y_R are equal, respectively, to:

$$\underline{C}Y_A = \{x_1, x_4, x_5, x_7, x_{10}, x_{11}, x_{12}, x_{15}\}$$
$$\bar{C}Y_A = \{x_1, x_4, x_5, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{15}\}$$
$$\mathrm{Bn}_C(Y_A) = \{x_8, x_9\}$$
$$\underline{C}Y_R = \{x_2, x_3, x_6, x_{13}, x_{14}\}$$
$$\bar{C}Y_R = \{x_2, x_3, x_6, x_8, x_9, x_{13}, x_{14}\}$$
$$\mathrm{Bn}_C(Y_R) = \{x_8, x_9\}.$$

The accuracy of approximation of sets Y_A and Y_R by C is equal to 0.8 and 0.71, respectively, and the quality of approximation of the decision by C is equal to 0.87.

Let us observe that the C-doubtful region of the decision is composed of two candidates: x_8 and x_9. Indeed, they have the same value according to criteria from C but the committee has admitted x_8 and rejected x_9. This means that the decision is inconsistent with evaluation of the candidates by criteria from C. So, apparently, the committee took into account additional information from the application packages of the candidates or from an interview with them. This conclusion suggests to the committee, either adoption of an additional discriminatory criterion or, if its explicit definition would be too difficult, creation of a third category of candidates: those who should be invited to an interview.

The next step of the rough set analysis of the decision table is construction of minimal subsets of independent criteria ensuring the same quality of sorting as the whole set C, i.e. the reducts of C. In our case, there are three such reducts:

$$\mathrm{RED}^1_{\mathscr{Y}}(C) = \{c_2, c_3, c_6, c_7\}$$
$$\mathrm{RED}^2_{\mathscr{Y}}(C) = \{c_1, c_3, c_7\}$$
$$\mathrm{RED}^3_{\mathscr{Y}}(C) = \{c_2, c_3, c_5, c_7\}.$$

It can be said that the committee took the 15 sorting decisions taking into account the criteria from one of the reducts and discarded all the remaining criteria. Let us notice that criterion c_4 has no influence at all on the decision because it is not represented in any reduct.

It is interesting to see the intersection of all reducts, i.e. the core of criteria:

$$\mathrm{CORE}_{\mathscr{Y}}(C) = \mathrm{RED}^1_{\mathscr{Y}}(C) \cap \mathrm{RED}^2_{\mathscr{Y}}(C) \cap \mathrm{RED}^3_{\mathscr{Y}}(C) = \{c_3, c_7\}.$$

The core is the most essential part of set C, i.e. it cannot be eliminated without disturbing the ability of approximating the decision.

In a real case, all the reducts and the core should be submitted for consideration by the committee in view of getting its opinion about what reduct should be used to generate decision rules from the reduced decision table.

Let us suppose that the committee has chosen reduct $\mathrm{RED}^2_{\mathscr{Y}}(C)$ composed of c_1, c_3, c_7, i.e. scores in mathematics and English, and opinion from previous school. This choice could be explained in such a way that the score in mathematics (c_1) seems to the committee more important than the score in physics (c_2) plus type of secondary school (c_5) or motivation (c_6).

Now, the decision table can be reduced to criteria represented in $\mathrm{RED}^2_{\mathscr{Y}}(C)$. The decision rules generated from the reduced decision table have the following form:

rule #1: if $c_1 = 5$			then $d = \mathrm{A}$
rule #2: if	$c_3 = 5$		then $d = \mathrm{A}$
rule #3: if $c_1 = 4$		and $c_7 = 1$	then $d = \mathrm{A}$
rule #4: if $c_1 = 4$	and $c_3 = 4$	and $c_7 = 2$	then $d = \mathrm{A}$ or R
rule #5: if $c_1 = 3$			then $d = \mathrm{R}$
rule #6: if	$c_3 = 3$		then $d = \mathrm{R}$

Five rules are deterministic and one is non-deterministic. The non-deterministic rule #4 follows from the indiscernibility of candidates x_8 and x_9 which belong to different categories of decision. It defines a profile of candidates which should create the third category of decision, e.g. those candidates who should be invited to an interview.

The rules represent clearly the following policy of the selection committee:

Admit all candidates having score 5 in mathematics or in English. Admit also those who have score 4 in mathematics and in English but very good opinion from a previous school. In the case of score 4 in mathematics and in English but only a moderate opinion from a previous school, invite the candidate to an interview. Candidates having score 3 in mathematics or in English are to be rejected.

The considered sample of 15 candidates can be considered as a training sample used to reveal the selection policy of the committee. This policy could be applied next to support sorting decisions concerning other candidates.

The set of sorting rules can be viewed as a *global preference model*, alternative to a functional or a relational model classically used in multi-criteria decision making. It explains the preferential attitude of an agent through important facts in terms of significant criteria only. Interpretation of the rules is also more straightforward than that of a function or relation. The use of sorting rules to decision support is being investigated by Słowiński (1993).

Let us conclude this section by a remark that, using the rough set approach, we are able to derive implicit facts from explicit and unquestionable facts (knowledge) about a decision situation. The implicit facts are mathematical 'consequences' of the explicit facts, i.e. approximations of partitions, dependencies and reduction. Their interpretation is simple and straightforward. They are robust because they are obtained without additional assumptions. On the one hand they can explain a decision situation and, on the other hand, they can be used to prescribe a solution.

CONCLUDING REMARKS

The aim of this paper was to show that the rough set theory is a useful tool for analysis of decision situations, in particular multi-criteria sorting problems. This class of decision situations has a very large practical representation.

The main advantages of the rough set approach can be summarized in the following points:

- it analyses only facts hidden in the representation of a decision situation,
- it does not need any additional information like probability in statistics or grade of membership in fuzzy set theory,
- it does not correct vagueness manifested in the representation of a decision situation; instead, the rules produced are categorized as deterministic and non-deterministic,
- it gives reducts of independent criteria having the same ability of approximating the decision as the whole set,
- it explains a decision situation and can be used to prescribe a solution,
- it is conceptually simple and needs simple algorithms.

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