ANATOMY OF CONFLICTS

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1. Introduction

Conflicts are everywhere. Decision making in the present of conflicts is one of the most challenging problems, that are faced in many areas, e.g. political and lawsuits disputes, labor-management negotiations, military operations, etc. Conflict analysis and resolution plays an important role in politics, business, government and others.

There are several formal models of conflicts (cf. e.g. Dorclan (1969), Hart (1974), Pawlak (1984), Roberts (1976)). The aim of this paper is to analyze conflict situations as formulated in Pawlak (1984) in terms of some ideas related to rough sets (cf. Pawlak (1991)) - in particular information systems. The rough set approach to conflict analysis offers new insight into this area of research and - new algorithms to solve practical problems.

2. Conflicts and Information Systems

We assume that in a conflict at least two participants, called in what follows *agents*, are in dispute over some issues. The agents may be individuals, groups, states, parties etc. The relationship of each agent to a specific issue can be clearly depicted in a form of a table, as shown in an example of the Middle East conflict, which is taken with slight modifications from Casti (cf. Casti (1988)).

Consider six agents

1 - Israel
2 - Egypt
3 - Palestinians
4 - Jordan
5 - Syria
6 - Saudi Arabia

and five issues

a - autonomous Palestinian state on the West Bank and Gaza
b - Israeli military outpost along the Jordan River
c - Israeli retains East Jerusalem
d - Israeli military outposts on the Golan Heights
e - Arab countries grant citizenship to Palestinians who choose to remain within their borders

In the table below the attitude of six nations of the Middle East region to the above issues is presented; -1 means, that the agent is against, 1 - favorable and 0 neutral toward the issue. For the sake of simplicity we will write - and + instead of -1 and 1 respectively.

U	a	b	c	d	e
1	−	+	+	+	+
2	+	0	−	−	−
3	+	−	−	−	0
4	0	−	−	0	−
5	+	−	−	−	−
6	0	+	−	0	+

Table 1

Such tables are known as *information systems* (cf. Pawlak (1991)). An information system is a table rows of which are labeled by *objects* (*agents*), columns - by *attributes* (*issues*) and entries of the table are *values of attributes* (*opinions, beliefs, views, votes, etc*), which are uniquely assigned to each agent and an attribute, i.e. each entry corresponding to row x and column a represents opinion of agent x about a.

Before we enter more specific consideration, first we give some basic formal definition and properties which will be needed in further conflicts analysis.

Information System is a pair $S = (U, A)$, where

U - is a nonempty, finite set called the *universe*; elements of U will be called (*objects*) *agents*,

A - is a nonempty, finite set of *attributes* (*issues*).

Every attribute $a \in A$ is a total function $a:U \longrightarrow V_a$, where V_a - is the set of *values* of a, called the *domain* of a; elements of V_a will be refered to as *opinions*, i.e. $a(x)$ is opinion of agent x about issue a.

In what follows, unless stated otherwise, we will assume that $V_a = \{-1, 0, 1\}$, for every a - meaning *against*, *neutral* and *favorable* respectively.

Now we are in a position to define three basic binary relations among agents: *conflict*, *neutrality* and *alliance*. To this end we have to define first the following auxiliary function:

$$\phi_a(x,y) = \begin{cases} 1, & \text{if } a(x).a(y) = 1 \text{ or } x \quad y \\ 0, & \text{if } a(x).a(y) = 0 \text{ and } x \neq y \\ -1, & \text{if } a(x).a(y) = -1 \end{cases}$$

This means that, if $\phi_a(x,y) = 1$, agents x and y have the same opinion about issue a (are allied on a); if $\phi_a(x,y) = 0$ means that at least one agent x or y has neutral approach to issue a (is neutral on a), and if

$\phi_a(x,y) = -1$, means that both agents have different opinions about issue a (are in conflict on a).

For the sake of notation we will define the following three binary relations:

$$R_a^+(x,y) \text{ iff } \phi_a(x,y) = 1$$

$$R_a^0(x,y) \text{ iff } \phi_a(x,y) = 0$$

$$R_a^-(x,y) \text{ iff } \phi_a(x,y) = -1$$

called *alliance, neutrality* and *conflict* relations respectively.

It is easily seen that the alliance relation has the following properties:

(i) $R_a^+(x,x)$

(ii) $R_a^+(x,y)$ implies $R_a^+(y,x)$

(iii) $R_a^+(x,y)$ and $R_a^+(y,z)$ implies $R_a^+(x,z)$,

thus it is an equivalence relation. Eqivalence classes of therelation R_a^+ will be called *coalitions* on a.

For conflict relation we have the following properties:

(iv) non $R_a^-(x,x)$

(v) $R_a^-(x,y)$ implies $R_a^-(y,x)$

(vi) $R_a^-(x,y)$ and $R_a^-(y,z)$ implies $R_a^+(x,z)$

(vii) $R_a^-(x,y)$ and $R_a^+(y,z)$ implies $R_a^-(x,z)$.

The relations $R_a^+(x,y)$, $R_a^0(x,y)$ and $R_a^-(x,y)$ can be extended to arbitrary subset $B \subseteq A$ of attributes in the following way. Define the function

$$\rho(x,y) = \sum_{a \in B} \phi_a(x,y) / \text{ card } A. \qquad (*)$$

Obviously $-1 \le \rho(x,y) \le 1$. In particular if $\rho(x,y) > 0$ we will say that x and y are in alliance (coalition) on B in degree $\rho(x,y)$, if $\rho(x,y) < 0$, we will say that x and y are in conflict on B in degree $\rho(x,y)$, and if $\rho(x,y) = 0$ we will say that x and y are neutral on B. Let us note that the function is symmetric, i.e. $\rho(x,y) = \rho(y,x)$ but of course partial alliance is not an equivalence relation.

The function (*) may be treated as a degree of engagement defined by Nabialek (1987a) and studied by Nabialek (1987b), Nabialek and Zakowski (1988) and Zakowski (1990). In particular it is known (Nabialek, (1987b)) that for every $x,y,z \in U$ the following inequalities are valid

$$-1 + |\rho(x,y) + \rho(y,z)| \le \rho(x,z) \le 1 - |\rho(x,y) - \rho(y,z)|.$$

Because each agent may assign various preference to issues discussed, to capture this aspect of conflict situations we introduce a function $\pi : U \times A \to \langle 0,1 \rangle$, called a *preference function*. Value of the preference function $\pi(x,a)$ expresses importance of issue a as viewed by agent x, or in other words, $\pi(x,a) > \pi(x,b)$ means that x prefers a over b, or that a is more important to x than b.

Taking into account the preference function the formula (*) can be rewritten now in the form

$$\rho^{*}(x,y) = \sum_{a \in B} \pi(x,a)\ \phi_a(x,y)/\text{card } B. \qquad (**)$$

Note that the formula (**) is asymmetric, that means that alliance (or conflict) of x to y may not be necessarily equal to that of y to x.

Formula (**) can be viewed as an overall (global) measure of relationships between agents with respect to the set of issues considered in a conflict situation. One can also easily add to the formula factor representing strength of agents, but for simplicity we will not consider this case here.

Thus an information system contains *explicit* information about the attitude of each agent to issues being consider in the debate, and *implicit* information, derived from the *explicit* one - about relationship between agents, i.e. alliance, neutrality or conflict. The *explicit* information can be seen as an "attribute" view on conflicts, and the *implicit* information - as a "relational" one (cf. Hard (1974)). We advocate here to combine both of them and use in the analysis of various aspects of conflict situations.

3. Properties of Information Systems

Analysis of information systems provides many useful clues to better understand conflict being consider. The most important problem seems to be the question of dependencies between issues. Dependencies can be pre assumed in advance (like in relational databases), or computed from the data available in the information system. We will assume the second approach in this paper, i.e, having an information system we will search for dependencies between issues. This enable us to find in a sense the "most crucial" issues and eliminate from the debate some issues without changing relative "positions" of debating parties.

In order to present these ideas in more details we need the notion of a reduct and dependency of attributes. To this end we have first to define some auxiliary notions, which will be given next (cf. Pawlak (1991)).

Suppose we are give an information system $S = (U,A)$. With every subset of attributes $B \subseteq A$, we associate a binary relation $IND(B)$, called an *indiscernibility relation* and defined thus:

$$IND(B) = \{(x,y) \in U^2: \text{ for every } a \in B,\ a(x) = a(y)\}.$$

Obviously $IND(B)$ is an equivalence relation and

$$IND(B) = \bigcap_{a \in B} IND(a)$$

By $U/IND(B)$ (in short U/B) we will denote the family of all equivalence classes of the relation $IND(B)$.

We will say that attribute $a \in B$ is *superfluous* in B, if $IND(B) = IND(B - \{a\})$; otherwise the attribute a is *indispensable* in B.

If all attributes $a \in B$ are indispensable in B, then B will be called *orthogonal.*

Subset $B' \subseteq B$ is a *reduct* of B, iff B' is orthogonal and $IND(B) = IND(B')$

The set of all indispensable attributes in B will be called the *core* of B, and will be denoted $CORE(B)$.

The following theorem establishes important relationship between the core and reducts.

Proposition 1

$$CORE(B) = \bigcap_{R \in RED(B)} R$$

where $RED(B)$ the family of all reducts of B ∎

To compute easily reducts and the core we will use discernibility matrix (cf. Skowron et all. (1991)), which is defined next.

Let $S = (U, A)$ be an information system with $U = \{x_1, x_2, \ldots, x_n\}$, and let $B \subseteq A$. By an *discernibility matrix of B* in S, denoted $M_S(B)$, or $M(B)$ if S is understood - we will mean n x n matrix defined thus:

$$(c_{ij}) = \{a \in B: a(x_i) \neq a(x_j)\} \text{ for } i,j = 1,2, \ldots , n.$$

Thus entry c_{ij} is the set of all attributes which discern objects x_i and x_j.

Remark

Let us note that the discernibility matrix $M(B)$ assigns to each pair of agents x and y a subset of attributes $\delta(x,y) \subseteq B$, with the following properties:

i) $\delta(x,x) = \emptyset$

ii) $\delta(x,y) = \delta(y,x)$

iii) $\delta(x,z) \subseteq \delta(x,y) \cup \delta(y,z)$

One can easily see that these properties resemble properties of semi-distance, and therefore the function δ may be regarded as *qualitative semi-metric* and $\delta(x,y)$ - *qualitative semi-distance.* Thus the discernibility matrix can be seen as a *semi-distance (qualitative)* matrix.

Let us also note that for every $x,y,z \in U$ we have

iv) $card\ \delta(x,x) = 0$

v) $card\ \delta(x,y) = card\ \delta(y,x)$

vi) $card\ \delta(x,z) \leq card\ \delta(x,y) + card\ \delta(y,z)$

The intuitive meaning of the above defined matrices is the following: usually we say that some elements differs on, say, there attributes (features), i.e. we specify that $card\ \delta(x,y) = 3$ - but we may be also more specific and say that the difference between two elements of interest is in size, weight, price etc., i.e. $\delta(x,y) = \{size, weight, price\}$. ■

The core can be defined now as the set of all single element entries of the discernibility matrix $M(B)$, i.e.

$$CORE(B) = \{a \in B\colon c_{ij} = (a), \text{ for some } i,j\}$$

It can be easily seen that $B' \subseteq B$ is the reduct of B, if B' is the minimal (with respect to inclusion) subset of B such that

$$B' \cap c \neq \emptyset \text{ for any nonempty entry } c\ (c \neq \emptyset) \text{ in } M(B).$$

In other words reduct is the minimal subset of attributes that discerns all objects discernible by the whole set of attributes.

Every discernibility matrix $M(B)$ defines uniquely a *discernibility (boolean) function* $f(B)$ defined as follows.

Let us assign to each attribute $a \in B$ a binary boolean variable $\bar{a}$, and let $\sum \delta(x,y)$ denote boolean sum of all boolean variables assigned to the set of attributes $\delta(x,y)$, provided $\delta(x,y) \neq \emptyset$. Then the discernibility function can be defined by the formula

$$f(B) = \prod_{(x,y) \in U^2} \sum \delta(x,y)$$

The following theorem establishes an important relationship between the minimal disjunctive normal form of the function $f(B)$ and the set of all reducts of B.

Proposition 2 (Skowron et all. (1991))

All constituents in the minimal disjunctive normal form of the function $f(B)$ are all reducts of B. ∎

To further simplification of information system we can drop some values of attributes which are unnecessary to discern objects in the system. To this end we can apply similar procedure as to eliminate superfluous attributes, which is defined next.

We will say that the value of attribute $a \in B$, is *superfluous* for x, if $[x]_{IND(B)} = [x]_{IND(B-\{a\})}$; otherwise the value of attribute a is *indispensable* for x.

If for every attribute $a \in B$ the value of a is indispensable for x, then B will be called *orthogonal* for x.

Subset $B' \subseteq B$ is a *reduct* of B for x, iff B' is orthogonal for x and $[x]_{IND(B)} = [x]_{IND(B')}$.

The set of all indispensable values of attributes in B for x will be called the *core* of B for x, and will be denoted $CORE^x(B)$.

The counterpart of Proposition 1 holds also in this case.

Proposition 1'

$$CORE^x(B) = \bigcap_{R \in RED^x(B)} R$$

where $RED^x(B)$ the is family of all reducts of B for x. ∎

In order to compute the core and reducts for x we can also use the discernibility matrix as defined before and the discernibility function, which must be slightly modified now, as shown below:

$$f^x(B) = \prod_{y \in U} \sum \delta(x,y).$$

Next important definition concerns dependency of attributes.

Intuitively speaking set of attributes $B \subseteq A$ depends on set of attributes $C \subseteq A$ ($C \Rightarrow B$), if values of attributes in B are uniquely determined by values of attributes in C, i.e. if there exists a function which assigns to each set of values of C set values of B. Formally

$$C \Rightarrow B \text{ iff } IND(C) \subseteq IND(B).$$

Below the property which establishes relation between reducts and dependency is given

Proposition 3

Let $S = (U,A)$ be an information system and let $B \subseteq A$. If B' is a reduct of B, then $B' \Rightarrow B - B'$. ∎

The following property is a direct consequence of the definition of dependency.

Proposition 4

$C \Rightarrow B$, implies $C \Rightarrow B'$, for every $B' \subseteq B$. ∎

From Proposition 4 it follows that $C \Rightarrow B$ implies $C \Rightarrow \{a\}$ for every $a \in B$. Dependencies of the form $C \Rightarrow \{a\}$ will be called *elementary.* Obviously $C \Rightarrow C'$ for every $C' \subseteq C$, in particular for every $a \in C$ we have $C \Rightarrow \{a\}$.

Propositions 3 and 4 enables us to find all dependencies between attributes.

Proposition 5

If B' is a reduct of B, then neither $a \Rightarrow b$ nor $b \Rightarrow a$ holds, for every $a,b \in B'$, i.e. all attributes in the reduct are pairwise independent. ∎

The concept of a reduct is very important to conflict analysis for two reasons. First reduct is the set of all orthogonal (pairwise independent) issues hence it determines uniquely the structure of a conflict situation, preserving minimal distances between participants, or in other words reduct preserves differences (dissimilarities) between participants. This means that instead of all issues only some of them can be discussed, without affecting essentially "balance" between participants. Secondly reducts determine all dependencies between attributes (issues), thus the dependent issues need not to be discussed because outcome of these discussions, according to Proposition 4, must be determined by results obtained debating issues belonging to the reduct.

Note that in general many reducts exist, thus some sort of optimization in choosing essential issues to the debate is possible.

The core constitutes the set of all issues which can not be avoided in a dispute and must be consider under any circumstances. In a sense core issues are the most important ones, which can not be eliminated from the debate without affecting essentially structure of the conflict situation. Thus if the core is the empty set, that means that the discussed problem is defined rather vaguely. In contrast, if there is only one reduct of issues being debated, it means that the problem is, in a certain sense, well defined.

Note also that using reduced set of issues instead of the whole one can change value of the function ρ for some x,y. That means that reduction of issues do not preserve in general relationship between participants. Hence there is a kind of trade-off between the "attribute" and the "relational" approach to conflict analysis.

3. Example

Consider conflict represented in Table 1. For example agents 1 and 6 are allied with respect to issues b and e, are in conflict with respect to issue c and are neutral regarding issues a and d.

The corresponding discernibility matrix is as follows:

	1	2	3	4	5	6
1						
2	a,b,c,d,e					
3	a,b,c,d,e	b,e				
4	a,b,c,d,e	a,b,d	a,d,e			
5	a,b,c,d,e	b	e	a,d		
6	a,c,d	a,b,d,e	a,b,d,e	b,e	a,b,d,e	

Table 2

After simplification (using the absorption law) we get the following discernibility function and its minimal disjunctive normal form.

$$eb(a+d) = abe+bde.$$

Thus the core is the set $\{e,b\}$ and there are two reducts of the set of attributes $\{a,b,e\}$ and $\{b,d,e\}$.

Corresponding reduced information systems are shown Tables 3 and 4.

U	a	b	e
1	−	+	+
2	+	0	−
3	+	−	0
4	0	−	−
5	+	−	−
6	0	+	+

Table 3

U	b	d	e
1	+	+	+
2	0	−	−
3	−	−	0
4	−	0	−
5	−	−	−
6	0	0	+

Table 4

For the above tables we get the following discernibility matrices

	1	2	3	4	5	6
1						
2	a,b,e					
3	a,b,e	b				
4	a,b,e	a,b	a			
5	a,b,e	b	e			
6	a	a,b,c	a,b,e	b,e	a,b,e	

Table 5

	1	2	3	4	5	6
1						
2	b,d,e					
3	b,d,e	b				
4	b,d,e	b,d	d,e			
5	b,d,e	b	e	d		
6	d	$b,d,e,$	b,d,e	b,e	b,d,e	

Table 6

For Table 5 we get

$$f^1(\{a,b,e\}) = d$$

$$f^2(\{a,b,e\}) = b$$

$$f^3(\{a,b,e\}) = e$$

$$f^4(\{a,b,e\}) = a(b+e) = ab+ae$$

$$f^5(\{a,b,e\}) = abe$$

$$f^6(\{a,b,e\}) = a(b+e) = ab+ae$$

and for Table 6 we have

$$f^1(\{a,b,e\}) = a$$

$$f^2(\{a,b,e\}) = b$$

$$f^3(\{a,b,e\}) = eb$$

$$f^4(\{a,b,e\}) = d(b+e) = db+de$$

$$f^5(\{a,b,e\}) = bde$$

$$f^6(\{a,b,e\}) = d(b+e) = db+de$$

Consequently we obtain

U	a	b	e
1	–	x	x
2	x	0	x
3	x	x	0
4	0	–	x
4'	0	x	–
5	+	–	–
6	0	+	x
6'	0	x	+

Table 7

U	b	d	e
1	x	+	x
2	0	x	x
3	–	x	0
4	–	0	x
4'	x	0	–
5	–	–	–
6	+	0	x
6'	x	0	+

Table 8

By Proposition 3 we get the following dependencies:

$\{a,b,e\} \Rightarrow \{c,d\}$ and $\{b,d,e\} \Rightarrow \{a,c\}$,

and consequently by Proposition 4 we have

$$\{a,b,e\} \Rightarrow \{c\}$$

$$\{a,b,e\} \Rightarrow \{d\}$$

$$\{b,d,e\} \Rightarrow \{a\}$$

$\{b,d,e\} \Rightarrow \{c\}$.

By using formula (*) we obtain the degree of conflict between all pairs of participant. Taking into account all issues a,b,c,d and e we obtain results given in Table 9 below.

	1	2	3	4	5	6
1						
2	-0.8					
3	-0.8	0.6				
4	-0.6	0.4	0.4			
5	-1.0	0.8	0.8	0.6		
6	0.2	0.0	0.0	0.2	0.2	

Table 9

Analogous results for reducts $\{a,b,e\}$ and $\{b,d,e\}$ are as follows.

	1	2	3	4	5	6
1						
2	-2/3					
3	-2/3	1/3				
4	-2/3	1/3	-1/3			
5	-1.0	1/3	2/3	2/3		
6	2/3	-1/3	-1/3	-2/3		

Table 10

	1	2	3	4	5	6
1						
2	-2/3					
3	-2/3	1/3				
4	-2/3	1/3	1/3			
5	-1.0	2/3	2/3	2/3		
6	2/3	-1/3	-1/3	-2/3	-2/3	

Table 11

Assume now the preference function as in Table 11.

U	a	b	c	d	e
1	1.0	0.8	1.0	0.8	0.6
2	0.9	0.8	0.9	1.0	0.9
3	1.0	1.0	1.0	0.9	0.5
4	1.0	0.8	0.9	0.7	1.0
5	1.0	1.0	1.0	1.0	0.8
6	0.6	0.6	0.9	0.9	0.9

Table 12

Employing now formula (**) we obtain Table 13, which is counterpart of Table 9.

	1	2	3	4	5	6
1		−0.74	−0.78	−0.54	−0.96	0.12
2	−0.68		0.58	0.38	0.76	0.00
3	−0.74	0.52		0.34	−0.96	−0.12
4	−0.48	0.36	0.40		0.56	−0.12
5	−0.84	0.74	−0.88	0.54		−0.12
6	0.12	0.00	0.00	−0.18	−0.16	

Table 13

4. Conclusions

The proposed approach to conflict analysis offers deeper look at the structure of relationships between participants as well as - issues discussed. Further research will concentrate on conflict resolution in the proposed framework.

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