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SPECIAL ISSUE ON

ROUGH SETS — STATE OF THE ART AND PERSPECTIVES

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The recent publication of a monograph on the theory and the handbook on applications, new developments and comparison with related methodologies* underline a dynamic growth of the rough sets methodology.

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Guest Editors

Motto: "Apart from the known and the unknown, what else is there?" (Harold Pinter in The Homecoming)

"The central problem of our age is how to act decisively in the absence of certainty" (Bertrand Russell)

ROUGH SETS
Present State and the Future

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Abstract. The paper presents the basic philosophical assumptions underlying the rough sets theory, gives its fundamental concepts and discusses briefly possible areas of applications. Finaly further problems are shortly oulined.

1. PHILOSOPHY

The rough set philosophy bears on the idea of *classification*. Any living organism or robot (an *agent*), in order to behave rationally in the outer realm, must have the ability to classify real or abstract objects (for example the sensory signals). In order to classify one has to postpone some differences between objects, thus forming classes of objects which are not noticeably different. These *indiscernibility classes* can be viewed as *basic building blocks*

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* Z. Pawlak, Rough Sets − Theoretical Aspects of Reasoning about Data Kluwer Academic Publishers, Dordrecht 1991;
Intelligent Decision Support − Handbook of Applications and Advances of the Rough Sets Theory, R. Słowiński (ed.), Kluwer Academic Publishers, Dordrecht 1992.

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(concepts) used to build up a *knowledge* about reality. For example if objects are classified according to color, then the class of all objects classified as red form the concept of *redness*. Thus our assumption is that any agent is equiped with mechanisms of variuos classification patterns and elementary concepts associated with those classifications form his *basic knowledge* about the world and himshelf. Thus knowledge in the presented approach can be understood as an ability to classify. Hence, formally knowledge can be defined as a familily of partitions of a fixed universe or, what is the same from mathematical point of view, as a family of equivalence relations. The presented view of knowledge is of semantic nature, where granularity of knowledge (indiscernibility of some objects) is of primary importance — in oposit to widely spread syntactic definition of knowledge in which formal aspects of knowledge are assumed as a starting point of the definition (cf. [3]). This can be also worded that in the rough set theory data are set before the language.

The most important issue addressed in the rough sets theory is the idea of imprecise knowledge. In this approach knowledge is imprecise if it contains imprecise concepts. But what are the imprecise concepts? The answer is straightforward. A concept which can be expressed (defined) in terms of the assumed classification patterns is *crisp* or *precise*, otherwise the concept is *imprecise* or *vague*. It turns out that the imprecise concepts can be however defined aproximatelly in the available knowledge by employing two precise concepts called their *lower* and *upper approximation*; the lower approximation of a concept consists of all objects which *surely* belong to the concept whereas the upper approximation of the concept consists of all objects which *possibly* belong to the concept in question. Difference between the lower and the upper approximation is a *boundary region* of the concept, and it consists of all objects which cannot be classified with certainty to the concept or its complement employing available knowledge. This view on vagueness can be attributed to Frege, who writes:

The concept must have a sharp boundary. To the concept without a sharp boundary there would correspond an area that has not a sharp boundary-line all around (cf. [7]).

The idea of approximations is the basic tool in the rough set philosophy.

2. THE THEORY

The concept of the rough set has inspired variety of research of both theoretical and practical nature. Logical research on approximate reasoning

seems to be more feasible and number of papers have been published in this area. The basic idea here is that conclusions are drown with some approximation only and are not exact as in the case of „classical" logic. Rough sets approach contributed already to this area of research, but the ultimate aim needs more research (cf. e.g. [6,8,32]). More references to this area of research can be found in [27].

Besides, investigations having direct practical use, like efficient algorithms, complexity of basic algorithms, comparison to other theories (e.g. like fuzzy sets, theory of evidence, statistics and others) are of great importance and are by now rather in the early state of development (cf. [4, 5, 6, 8, 12, 16, 28, 29, 39, 46, 48]).

In order to present the above ideas formally we need a suitable method of representing classifications. To this end we will use the concept of an *information systems*, known also as an *attribute-value systems* or an *knowledge representation systems*.

Information system is a finite table with rows labelled by *objects*, columns are labelled by *attributes*, moreover with each attribute a finite set of its *values*, called *domain* of the attribute, is associated. To each object and an attribute a value of the attribute is associated. For example if the object were an *apple* and the attribute — *color*, then the corresponding entry in the table could be *red*.

Simple example of such table, which characterizes six stores in terms of some factors is shown below (cf. [15]).

Store	E	Q	S	R	L	P
1	high	good	yes	yes	no	500
2	high	good	no	yes	no	−100
3	med.	good	yes	yes	no	200
4	low	avg.	yes	yes	yes	70
5	low	good	yes	yes	yes	100
6	high	avg.	no	no	yes	−20

Objects in the table are stores numbered from one to six and attributes are the following factors:

E — *empowerment of sales personnel*
Q — *perceived quality of merchandise*
S — *segment customer base*
R — *good refund policy*
L — *high traffic location*
P — *store profit or loss (in millions of US dollars)*

Attribute E has the values *high, medium* and *low*; attribute S has values *good* and *average*; attributes R, L and P have attribute values *yes* and *no*, whereas values of attribute P are *integers*.

It is easily seen that each attribute in the table defines a partition of objects, i.e. an equivalence relation, such that two objects belong to the same equivalence class if they have the same attribute values. Thus attributes in the information system represent various classification patterns and the whole table can be regarded as a simple way of notation for families of classifications, or what is the same — families of equivalence relations.

Formally an information system is a pair $S = (U, A)$, where U is a non-empty finite set of *objects* called the *universe* and A is a finite set of *attributes*. With every attribute a set of its values, called the *domain* of a, and denoted V_a, is associated. Every attribute $a \in A$, is a function a: $U \to V_a$, which to each object $x \in U$ uniquely associates an attribute value from V_a. Objects can be anything we can think of, for example states, processes, moments of time, physical or abstract entities etc.

Every subset of attributes $B \subseteq A$ defines uniquely an equivalence relation

$$IND(B) = \{(x, y) \in U^2 : a(x) = a(y) \text{ for every } a \in B\}.$$

As usually $U/IND(B)$ denotes the family of all equivalence classes of the equivalence relation $IND(B)$, i.e. the classification corresponding to $IND(B)$.

The *lower approximation* of $X \subseteq U$ by B is the union of equivalence classes of $IND(B)$ which are included in X, or formally

$$\underline{B}X = \bigcup \{Y \in U/IND(B): Y \subseteq X\}$$

The *upper approximation* of $X \subseteq U$ by B is the union of all equivalence classes of $IND(B)$ which have not-empty intersection with X, i.e.

$$\bar{B}X = \bigcup \{Y \in U/IND(B): Y \cap X \neq \phi\}$$

The boundary-line region is of course defined as $BN_B(X) = \bar{B}X - \underline{B}X$ and will be called the *B-boundary* of X.

Set $\underline{B}X$ consists of all elements of U which can be with *certainty* classified as elements of X employing knowledge B; Set $\bar{B}X$ is the set of all elements of U which can be *possibly* classified as elements of X using set of attributes B; set $BN_B(X)$ is the set of all elements which cannot be classified either to X or to $-X$ by means of attributes from B.

Now we are able to give the definition of the rough set.

A set $X \subseteq U$ is *rough with respect to B*, if $\bar{B}X \neq \underline{B}X$, otherwise the set X is *exact* (with respect to B).

Thus a set is rough if it does not have sharp defined boundary, i.e. it can not be uniquely defined employing available knowledge.

For practical applications we need numerical characterization of vagueness, which will be defined as follows:

$$\alpha_B(X) = \frac{\text{card } \underline{B}X}{\text{card } \bar{B}X}$$

where $X \neq \phi$, called the *accuracy measure*.

The accuracy measure $\alpha_B(X)$ is intended to capture the degree of completeness of our knowledge about the set (concept) X.

Obviously $0 \leq \alpha_B(X) \leq 1$, for every B and $X \subseteq U$; if $\alpha_B(X) = 1$ the R-boundary region of X is empty and the set X is definable in knowledge B; if $\alpha_B(X) < 1$ the set X has some non-empty B-boundary region and consequently is undefinable in knowledge B.

The idea of approximation of sets is the basic tool in the rough set approach and is used to approximate description of some concepts (subsets of the universe) by means of attributes. For example, we might be interested whether there are factors characteristic for stores having high (above 100 Millions dollars) profit, and if not — to find the lower and the upper characteristic of these stores. The reader is advised to answer this question using the above given definitions.

Starting from the concept of classification we can also define a variety of other notions fundamental to rough sets philosophy and applications — needed to discover various relations between attributes, and objects. The most important ones are the *dependency of attributes (cause-effect relations)*, *redundancy of attributes* and *decision rule generation*.

For example we may be interested whether the factor P (store profit or loss) depends, exactly or approximately, on the remaining five factors, i.e. whether values of factor P are determined by values of factors E, Q, S, R and L (dependency of attributes). If so, then the question arises if all the factors really influence the factor P (redundancy of attributes), and if not, which are the ones which matters. The most important problem is to find a set of decision rules (exact or approximate) which determine the stories performance.

All these problems can be easily defined and investigated within the rough set theory, however we will drop these considerations here. More details can be found in [10, 27, 51].

3. APPLICATIONS OF ROUGH SETS

The rough sets theory has proved to be very useful in practice. Many real life applications in medicine, pharmacology, industry, engineering, control, social sciences, earth sciences and other have been successfully implemented. Some of them are listed in the references [1, 9, 13, 14, 17, 18, 19, 22, 23, 25, 33, 34, 41, 42]. Besides, the book edited by professor Roman Słowiński [36] can be used as a reference book on applications of the rough sets theory.

By now rough sets have been mainly used to data analysis. Data are very often imprecise. For example in medicine body temperature, blood pressure etc. have usually not exact numerical values but are rather expressed as qualitatively values, like *normal, above normal or below normal* etc.

Rough set theory is mainly used to vague data analysis. Main problems which can be solved using rough set theory in data analysis are *data reduction*, (elimination of superfluous data), *discovering of data dependencies, data significance, decision (control) algorithms generation from data, approximate classification of data, discovering similarities or differences in data, discovering patterns in data* and the like (cf. [12, 14, 15, 18, 19, 22, 25, 33, 34, 35, 37, 38, 41, 42].

Machine learning is another important area where rough sets can be use. There is a variety of approaches to machine learning, however by now no commonly accepted theoretical foundations have been developed. It seems that the rough set approach, can be used as a theoretical basis for some problems in machine learning. Some ideas concerning the application of rough sets in this area can be found in [2, 11, 30, 31, 44, 46, 47, 49].

Rough sets approach offers alternative methods to *switching circuits synthesis and minimization, fault diagnosis* and other (cf. [20, 21]).

Image processing is also a promising field of the rough sets theory applications. Using basic concepts of the rough sets theory one can easily develop many basic algorithms for image processing and character recognition like, for example *thinning algorithms.*

Some methodological reflections seems to be in order.

Applications of rough sets can be divided into several groups having some common methodological features — which are listed below:

1. **Data analysis.** Main problems which can be solved using rough set theory in data analysis are: *data reduction, discovering of data dependencies, data significance.* This can be viewed as a counterpart of statistical data analysis.

2. **Approximate classification.** In this area rough sets can be used to *decision (control) algorithms generation from data, discovering similarities or*

differences in data, discovering patterns in data. This area can be regarded as a counterpart of cluster analysis.

3. **Switching circuits.** Rough sets approach offers alternative methods to *switching circuits synthesis, and minimization, fault diagnosis* and others. This is closely connected with boolean reasoning methods.

4. **Image processing.** Using basic concepts of the rough sets theory one can easily develop many basic algorithms for image processing like, for example *thinning* and *countor finding algorithms.*

5. **Machine learning.** Machine learning is usually meant as sort of inductive inference, in which a sample is used to draw conclusions about the whole universe. This is known in the AI literature as *learning from examples.* Rough sets metodology seems to be very well suited for this kind of study.

4. PROBLEMS

There is a wide spectrum of problems inspired by the rough sets philosophy. Some of them are listed below. Evidently rough sets view can contribute to the long lasting philosophical discussions on vagueness, uncertainty, imprecision and indiscernibility. Besides, various theoretical questions in set theory, topology and logic, which have arisen within the context of rough sets, are also of interest. Also more practical questions need appropriate attention. In particular problems related to incomplete, and distributed knowledge seem of primary importance, for not very much has been done in these areas. The developed algorithms based on the rough sets approach are very well suited to parallel processing, especially when appropriate hardware could be developed. Finally computing machine based on the rough sets concept, in which decision rules would play the role of elementary instructions is worthy consideration. Decision support systems would gain momentum having such tools. Rough controlers seems to have also bright future.

REFERENCES

[1] Arciszewski T., Ziarko W., Adaptive Expert system for Preliminary Engineering Design, [in:] Proceedings of the sixth International Workshop on Expert systems and their Applications, Paris 1986, 695—712.

[2] Arciszewski T., Ziarko W., Mustafa M., A Methodology of Design Knowledge Acquisition for Use in Learning Expert Systems, *International Journal of Man-machine Studies* (1987), 27, 23—32.

[3] Brachman R. J., Levesque H. J. (eds.), Readings in Knowledge Representation, Morgan Kaufmann Publishers, Inc., 1986.

[4] Dubois D., Prade H., Rough Fuzzy Sets and Fuzzy Rough Sets, Internal Conference on Fuzzy Sets in Informatics, Moscow, September 20—23 (1988) and *International Journal of General Systems* (1990), 17, 191—173.

[5] Dubois D., Prade H., Putting Rough Sets and Fuzzy Sets Together, [in:] Intelligent Decision Support Systems. Handbook of Applications and Advances of the Rough Sets Theory, R. Słowiński (ed.), Kluwer Academic Publishers, 1992, 203—232.

[6] Farinas del Cerro L., Prade H., Rough Sets, Twofold Fuzzy Sets and Modal Logic. Fuzzines and Indiscernibility and Partial Information, [in:] The Mathematics of Fuzzy Systems, A. D. Nola (ed.), Verlag TUG Rheinland, 1986.

[7] Frege G., Grundgesetze der Arithmetik, (1903), [in:] Selections from the Philosophical Writings of Gotlob Frege, Geach and Black (eds.), Blackweil, Oxford 1970.

[8] Gao J. M., Nakamura A., Algebraic Analysis of Fuzzy Indiscernibility, Proceedings of Third International Conference: Information Processing and Management of Uncertainty in Knowledge-Based Systems, Paris—France, July 2—6 (1990), 260—262.

[9] Greenburg A. G., Commentary on the paper by Pawlak etc., *Computing Reviews* (1987), 27, 413—433.

[10] Grzymała-Busse J., Knowledge Acquisition under Uncertainty — a Rough Set Approach, *Journal of Intelligent and Robotic Systems* (1988), 1, 3—36.

[11] Grzymała-Busse J., LERS- A System for Learning from Examples Based on Rough Sets, [in:] Intelligent Decision Support Systems. Handbook of Applications and Advances of the Rough Sets Theory, R. Słowiński (ed.), Kluwer Academic Publishers, 1992, 3—18.

[12] Gupta D., Rough Sets and Information Systems, Proceedings of the Eleventh International Conference on Research and Development in Information Retrieval, Baltimore, USA, 1988.

[13] Hadijimicheal M., Wasilewska A., Rough Sets-based Study of Voter Preference in 1988 USA Presidential Election, [in:] Intelligent Decision Support Systems. Handbook of Applications and Advances of the Rough Sets Theory, R. Słowiński (ed.), Kluwer Academic Publishers, 1992, 137—152.

[14] Kandulski M., Marciniec J., Tukałło K., Surgical Wound Infection-Conductive Factors and their Mutual Dependencies, [in:] Intelligent Decision Support Systems. Handbook of Applications and Advances of the Rough Sets Theory, R. Słowiński (ed.), Kluwer Academic Publishers, 1992, 95—110.

[15] Kick R. C., Koczkodaj W. W., Bussines Data Analysis by Rough Set Theory, Technological University of Tennessee, Manuscript, 1990.

[16] Krusińska E., Słowiński R., Stefanowski J., Discriminant Versus Rough Sets Approach to Vague Data Analysis, *Journal of Applied Statistics and Data Analysis* (1992), 8, 43—56.

[17] Krysiński J., Rough Set Approach to Analysis of Relationship between Structure and Activity of Quaternary Imidazolium Compounds, *Arzenmittel-Forschung Drug Research* (1990), 40, 795—799.

[18] Krysiński J., Grob-Mengen-Theorie in der Analysis der Structure-Wirkungs-Beziehungen von quartearen Piridiniumverbindungen, *Pharmazie* (1991), 46, 878—881.

[19] Krysiński J., Analysis of Structure-Activity Relationships of Quaternary Ammonium Compounds, [in:] Intelligent Decision Support Systems. Handbook of Applications and Advances of the Rough Sets Theory, R. Słowiński (ed.), Kluwer Academic Publishers, 1992, 119—136.

[20] Luba T., Rybnik J., Rough Sets in Logic Synthesis, [in:] Intelligent Decision Support Systems. Handbook of Applications and Advances of the Rough Sets Theory, R. Słowiński (ed.), Kluwer Academic Publishers, 1992, 181—202.

[21] Modrzejewski M., On Searching for Test Experiments Using Rough Sets Theory, *Bull. PAS. ser Tech.* (1992), 40, 179—194.

[22] Mrózek A., Rough Set Dependency Analysis Among Attributes in Computer Implementation of Expert Inference Models, *Int. Journal of Man-Machine Studies* (1989), 30, 457—473.

[23] Mrózek A., Rough Sets in Computer Implementation of Rule-Based Control of Industrial Processes, [in:] Intelligent Decision Support Systems. Handbook of Applications and Advances of the Rough Sets Theory, R. Słowiński (ed.), Kluwer Academic Publishers, 1992, 19—32.

[24] Nakamura A., Fuzzy Rough Sets, *Note on Multiple- valued Logic in Japan* (1988), 9.

[25] Nowicki R., Słowiński R., Stefanowski J., Analysis of Diagnostic Symptoms in Vibroacustic Diagnostics by means of the Rough Sets Theory, [in:] Intelligent Decision Support Systems. Handbook of Applications and Advances of the Rough Sets Theory, R. Słowiński (ed.), Kluwer Academic Publishers, 1992, 33—48.

[26] Pawlak Z., Rough Sets, *International Journal of Computer and Information Sciences* (1982), 11, 341—356.

[27] Pawlak Z., Rough Sets. Theoretical Aspects of Reasoning about Data, Kluwer Academic Publishers, 1991.

[28] Pawlak Z., Wong S. K. M., Ziarko W., Rough Sets: Probabilistic Versus Deterministic Approach, *International Journal of Man-Machine Studies* (1988), 29, 81—85.

[29] Pawlak Z., Skowron A., Rough Membership Functions, [in:] Fuzzy Logic for Management of Uncertainty, L. A. Zadeh, J. Kacprzyk (eds.), John Wiley and Sons, New York (to appear).

[30] Pettorossi A., Ras Z., Zemankova M., On Learning with Imperfect Teachers, Proceedings of the 2nd ACM SIGART International Symposium on Methodologies for Intelligent Systems, North Holland, Amsterdam 1987, 256—263.

[31] Ras Z., Zemankova M., Learning in Knowledge Based Systems. A probabilistic Approach, Proceedings of the 1986 CISS, Princeton, NJ., 1986, 844—847.

[32] Rauszer C., An Equivalence between Indiscernibility Relations and in Information Systems and a Fragment of Intuitionistic Logic, Lecture Notes in Computer Sciences, Springer Verlag, 208, (1984), 298—317.

[33] Reinhard A., Stawski B., Weber T., Wybraniec-Skardowska U., An Application of Rough Set Theory in the Control of Water Conditions on a Polder, [in:] Intelligent Decision Support Systems. Handbook of Applications and Advances of the Rough Sets Theory, R. Słowiński (ed.), Kluwer Academic Publishers, 1992, 153—164.

[34] Słowiński K., Słowiński R., Stefanowski J., Rough Sets Approach to Analysis of Data from Peritoneal Lavage in Acute Pancreatitis, *Medical Informatics* (1989), 13, 143—159.

[35] Słowiński K., Słowiński R., Sensitivity Analysis of Rough Classification, *Int. Journal of Man-Machine Studies* (1990), 32, 693—705.705.

[36] Słowiński R. (ed.), Intelligent Decision Support Systems. Handbook of Applications and Advances of the Rough Sets Theory, Kluwer Academic Publishers, 1992.

[37] Słowiński K., Rough Classification of HSV Patients, [in:] Intelligent Decision Support Systems. Handbook of Applications and Advances of the Rough Sets Theory, R. Słowiński (ed.), Kluwer Academic Publishers, 1992, 77—94.

[38] Słowiński R., Stefanowski J., ROUGHDAS and ROUGHCLASS Software Implementations of the Rough Sets Approach, [in:] Intelligent Decision Support Systems. Handbook of Applications and Advances of the Rough Sets Theory, R. Słowiński (ed.), Kluwer Academic Publishers, 1992, 445—456.

[39] Skowron A., Grzymala-Busse J., From Rough Sets Theory to Evidence Theory, [in:] Advances in Dempster-Shafer Theory, M. Fedrizzi, J. Kacprzyk and R. R. Yager (eds.), 1992 (to appear).

[40] Szladow A., Ziarko W., Knowledge based process Control Using Rough Sets, [in:] Intelligent Decision Support Systems. Handbook of Applications and Advances of the Rough Sets Theory, R. Słowiński (ed.), Kluwer Academic Publishers, 1992, 49—60.

[41] Tanaka H., Ishibuchi H., Shigenega T., Fuzzy Inference System based on Rough Sets and its Application to Medical Diagnosis, [in:] Intelligent Decision Support Systems. Handbook of Applications and Advances of the Rough Sets Theory, R. Słowiński (ed.), Kluwer Academic Publishers, 1992, 111—118.

[42] Teghem J., Charlet J. M., Use of "Rough Sets" Method to Draw Premonitory Factors for Earthquakes by Emphasizing Gas Geochemistry: The case of a Low Seismic Activity Context, in Belgium, [in:] Intelligent Decision Support Systems. Handbook of Applications and Advances of the Rough Sets Theory, R. Słowiński (ed.), Kluwer Academic Publishers, 1992, 165—180.

[43] Wasilewska A., Syntactic Decision procedures in Information Systems, *International Journal of Man-machine Studies* (1989), 50, 273—285.

[44] Wong S. K. M., Wong J. H., An Inductive Learning system-ILS, Proceedings of the 2nd ACM SIGART International Symposium on Methodologies for Intelligent Systems, Nort Holland, Amsterdam 1987, 370—378.

[45] Wong S. K. M., Ziarko W., INFER — an Adaptive Decision Support System Based on the Probabilistic Approximate Classification, The 6th international Workshop on Expert systems and their Applications, Avignon, France, 713—726. 1987, 1.

[46] Wong S. K. M., Ziarko W., Ye R. L., Comparison of Rough Set and Statistical Methods in Inductive Learning, *International Journal of Man-Machine Studies* (1986), 24, 53—72.

[47] Wong S. K. M., Ziarko W., Ye R. K., On Learning and Evaluation of Decision Rules in Context of Rough Sets, Proceedings of the first ACM SIGART International Symposium on Methodologies for Intelligent Systems, Knoxville, Tenn., 1986, 308—324.

[48] Wygralak W., Rough Sets and Fuzzy Sets — Some Remarks on Interrelations, *Fuzzy Sets and Systems* (1989), 29, 241—243.

[49] Yasdi R., Learning Classification Rules from Database in the Context of Knowledge-Acquisition and Representation, [in:] Intelligent Decision Support Systems. Handbook of Applications and Advances of the Rough Sets Theory, R. Słowiński (ed.), Kluwer Academic Publishers, 1992, 419—444.

[50] Ziarko W., On Reduction of Knowledge Representation, Proceedings of the Second International Symposium on Methodologies for Intelligent Systems (Colloquia Program), Charlotte, ORNL, 1987, 99—113.

[51] Ziarko W., The Discovery, Analysis and Representation of Data Dependencies in Databases, [in:] Knowledge Discovery in Databasis, AAAI Press, 1991.

[52] Ziarko W., Acquisition of Control Algorithms from Operation Data, [in:] Intelligent Decision Support Systems. Handbook of Applications and Advances of the Rough Sets Theory, R. Słowiński (ed.), Kluwer Academic Publishers, 1992, 61—76.

A CATEGORY FOR ROUGH SETS*

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Abstract. A category ROUGH for rough sets has been proposed, a subcategory ξ-ROUGH of ROUGH considered and a relationship of ROUGH with other categories studied. Some properties of ROUGH have been proved.

1. INTRODUCTION

The concept of "rough sets" has been proposed by Pawlak [7]. So far, considerable research has been done involving various aspects of this new concept. We present here a category that may capture the notion.

A rough set is often looked upon as the pair of the upper and lower approximations of a set in an approximation space. We think such a representation captures only the extensional facet of the concept and fails to express the notion of "rough equality". The category proposed in this paper, on the other hand, captures both the intention and the extension of rough sets. Morphisms defined appear, quite naturally, to be extension-preserving. In the first section, we have described the category and derived some of its properties. Further studies would lead to other revelations. In the next section, a subcategory ξ-ROUGH of ROUGH has been considered. In this subcategory, morphisms are so designed that they preserve structures in the objects more rigidly. In the last section, it has been shown that although the category ROUGH is not a topos, it occupies a position in between two categories which are topoi and mutually equivalent. For most of the categorial concepts, we have followed Goldblatt [1] and Mitchell [4].