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Communication Logic

by

Zdzisław PAWLAK

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Summary. In this note it will be attempted to suggest a certain logical framework to identify and describe some phenomena occurring in a communication process. Our basic aim is description of phenomena in computing systems, however, some other applications seem also possible.

1. Introduction. In this note we consider a communication system consisting of agents able to send and receive information about other agents. For example, such a communication system may consist of n processors connected by a communication network, used to distribute information about states of each processor. To describe agents we may use, for example, the unary predicates, P, Q, R etc, like $P(x)$ – "agent x is engaged" or "agent x is out of order", etc. In general case we may use n-ary predicates to describe relations among agents.

Compound formulas are formed using logical connectives $\vee$, $\wedge$, $\rightarrow$, $\leftrightarrow$, $\sim$ and quantifiers $\bigwedge$ (for every) and $\bigvee$ (there exists). Moreover, we have two operations $\uparrow_y^x$, $\downarrow_x^y$ on formulas. The intuitive meaning of these operations is the following: $\uparrow_x^y \varphi$ (φ is a formula) – "x said to y, that φ" and $\downarrow_x^y \varphi$ – "x received from y, that φ". In other words, $\uparrow_x^y \varphi$ is to mean that "x sent to y message φ" and $\downarrow_y^x$ means that y "knows" from y, after receiving the message φ, that φ.

In this note it will be shown that the two operations enable us to define a wide class of states of the communication system. It seems that the proposed approach may be more adequate to describe the variety of states in a communication system than that offered by Kripke structures (see [1]). The presented model of communication may be considered as a generalization of ideas given in [2].

2. Basic definitions. By a communication system we mean a system $S = (U, X, Y, Z, L)$ where $X, Y, Z \subseteq U$ are sets of senders, receivers and objects, respectively, and L is a communication logic. For the sake of simplicity we assume that $X = Y = Z = U$. In order to define the communication logic we need a formal language, defined in a usual way. Firstly, let us assume two constants T (for truth) and F (for falsity), and some primitive predicates P, Q, R, etc. Compound formulas are formed employing the propositional connectives and quantifiers as shown in the previous section. Moreover, two operations on formulas are assumed $\uparrow_x^y$, and $\downarrow_y^x$ – as explained previously.

3. Axioms for communication system. It seems that the communication system should satisfy the following axioms:

A) Axiom of communication (understanding)

$$\uparrow_x^y \varphi \leftrightarrow \downarrow_y^x \varphi .$$

The intuitive meaning of this axiom reads: "x said to y, that φ implies that x knows from y, that φ", and conversely, "y knows from x, that φ, implies that x said to y, that φ".

B) Axiom of truthfulness

$$\uparrow_x^y \varphi \rightarrow \varphi .$$

The meaning of this axiom is obvious.

C) Axiom of transitivity

$$\uparrow_y^u \uparrow_x^y \varphi \rightarrow \uparrow_x^u \varphi .$$

This axiom means: "y said to u, that x said to y, that φ, implies that x said to y, that φ".

D) Axiom of compression

$$\uparrow_x^y \uparrow_x^y \varphi \rightarrow \uparrow_x^y \varphi ,$$

which is to mean: "x said to y, that x said to y, that φ, implies x said to y, that φ".

E) Axiom of acknowledgement

$$\uparrow_y^x \downarrow_y^x \uparrow_x^y \varphi \rightarrow \downarrow_x^y \downarrow_y^x \varphi .$$

This axiom means the following: "y said to x, that y knows from x, that y said to y, that φ, implies that x knows from y, that y knows from x, that φ".

This set of axioms represents an exemplary collection of conditions which should be obeyed by most communication systems. The axiom of transitivity seems to be of basic importance to any communication system.

4. States of communication system. The introduced language can be easily used to define the various kinds of states in a communication system. Some examples of such states are listed below:

$$\bigwedge_{y\in Y} \uparrow_x^y \varphi$$

$$\bigwedge_{y\in Y} \downarrow_x^y \varphi$$

(x knows from every agent of Y, that φ)

$$\bigwedge_{x\in X} \uparrow_x^y \varphi$$

(every agent of X said to y that φ)

$$\bigwedge_{x\in X} \downarrow_x^y \varphi$$

(every agent of X, knowns from y, that φ)

$$\bigvee_{y\in Y} \downarrow_x^y \varphi$$

(x knows from some agent of Y, that φ)

$$\bigvee_{x\in X} \downarrow_x^y \varphi$$

(some agents of X know from y, that φ)

$$\bigvee_{y\in Y} \uparrow_x^y \varphi$$

(x said to some agents of Y, that φ)

$$\bigvee_{x\in X} \uparrow_x^y \varphi$$

(some agents of X said to y, that φ),

Also, the following states are of interest:

$$\bigwedge_{y\in Y} \bigvee_{x\in X} \uparrow_x^y \varphi$$

$$\bigwedge_{y\in Y} \bigvee_{x\in X} \downarrow_x^y \varphi$$

$$\bigvee_{y\in Y} \bigwedge_{x\in X} \downarrow_x^y \varphi$$

$$\bigvee_{y\in Y} \bigwedge_{x\in X} \uparrow_x^y \varphi, \text{ etc.}$$

The meaning of these formulas is obvious.

Interesting are also the following states:

$$\bigwedge_{x,y\in U} \downarrow_x^y \varphi$$

and

$$\bigwedge_{x,y\in U} \uparrow_x^y \varphi.$$

The first formula may be interpreted as a "common knowledge" (belief) of all agents, and the second one-may be called a "gossip" in the communication system. The following states seem to be also of a certain importance:

$\uparrow_x^y T - x$ said to y the truth

$\downarrow_x^y T - x$ knows from y the truth

$\uparrow_x^y F - x$ is telling lies to y

$\downarrow_x^y F - x$ knows that y lies (to him), etc.

5. Acquisition of knowledge. We assume that, at the very beginning, every agent x has a certain initial knowledge $\omega(x) \in L$. Further, we assume that every agent acquires knowledge from other agents throughout the communication process and/or logical inference. The inference mechanism may be described by any kind of logics assumed by an agent. The acquisition of knowledge by an agent can be described by the following formulas:

$$[\uparrow_x^y \varphi]\,\omega(z) = \begin{cases} \omega'(z) = \omega(z), & \text{if } z \neq u \\ \omega'(u) = \omega(u) \wedge \varphi \wedge \uparrow_x^y \varphi, & \text{if } z = u. \end{cases}$$

Similar formula can be written for the operator $\downarrow_x^y \varphi$.

Thus, the expression $\uparrow_x^y \varphi$ can be considered as an operator, which increases the knowledge of agent y (similarly $-$ $\downarrow_x^y \varphi$).

Our main objective is to investigate whether some states of a communication system are possible or not. The problem is reduced to the question whether some formulas of the communication logics are true or not. In order to investigate this problem the formal definition of semantics of the introduced language is needed, which will be the objective of the next article.

DEPARTMENT OF COMPLEX CONTROL SYSTEMS, POLISH ACADEMY OF SCIENCES, BAŁTYCKA 5, 44-100 GLIWICE
(ZAKŁAD SYSTEMÓW KOMPLEKSOWYCH PAN)
DEPARTMENT OF COMPUTER SCIENCE, NORTH CAROLINA UNIVERSITY, CHARLOTTE NC 28220, USA

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[2] Z. Pawlak, *About the meaning of personal pronouns*, Rev. Roum. Ling., **18** (1973), 261–262.

З. Павляк, **Логика обмена информацией**

В настоящей работе предлагается некоторой формализм для описания определенных явлений, имеющих место в процесах обмена информацией между агентами, которые в состоянии передавать и принимать информацию о других агентах (в особенности о самих себе). Основная цель этой статьи заключается в описании явлений, происходящих в вычислительных сетях, однако и другое применение предлагаемой модели тоже кажутся возможными.