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On Rough Functions

by

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Summary. In this note we define rough fundtions, i.e. functions arguments which are not known exactly, but with a certain approximation only. The definition of rough (approximate) continuity of function is defined and some elementary properties of rough continuity are stated.

1. Introduction. This note contains a modified version of ideas presented in [1], where the notion of a rough function has been introduced. The idea of a rough function is based on the concept of a rough set (see [2]) and rough relation (see [3]).

Intuitively speaking, rough function is a function whose arguments and values are not known exactly, but with a certain approximation only, determined by an "indiscernibility" relation, which expresses the accuracy of our observations, measurement or description. In other words, we assume that if we are unable to measure (observe) the values and arguments of a function $f: X \rightarrow Y$, we "see" another function $g: P(X) \rightarrow P(Y)$, where $P(X)$ and $P(Y)$ denote power sets of X and Y, respectively.

This kind of approximation seems to be of some value in pattern recognition, object identification and some other fields.

2. Rough Relations. We recall, after [3] the basic notions needed in this paper.

A pair $A = (U, R)$, where U is a certain nonempty set and $R \subseteq U \times U$ is an equivalence relation (called the indiscernibility relation) will be called an approximation space.

Let $B = (Y, Q)$ and $C = (Z, P)$ be two approximation spaces. By a product of B and C (denoted $B \times C$) we shall mean the pair $A = (X, R)$ where

$X = Y \times Z$ and $R \subseteq (Y \times Z)$ is defined as follows: $R((y_1, z_1), (y_2, z_2))$ iff $Q(y_1, y_2)$ and $P(z_1, z_2)$.

Obviously $A = (X, R)$ is also an approximation space. Equivalence classes of the relation R will be called elementary relations in A. Finite union of elementary relations in A, are called composed relations in A.

Let $B = (Y, Q)$ and $C = (Z, P)$ be two approximation spaces, $A = (X, R)$ be a product space of B and C, and let $S \subseteq Y \times Z$ be a binary relation. By R-lower (R-upper) approximation of S in A, denoted $\underline{R}S$ ($\bar{R}S$) we mean relations defined as follows:

$$\underline{R}S = \{(x, y) \in Y \times Z : [(x, y)] \subseteq S\},$$

$$\bar{R}S = \{(x, y) \in Y \times Z : [(x, y)]_R \cap S \neq \emptyset\},$$

where $[(x, y)]_R$ denotes equivalence class of the relation R containing the pair (x, y). More about rough relations can be found in [3].

3. Rough function. Suppose that we are given a function $f: Y \to Z$. Let $B = (Y, Q)$, $C = (Z, P)$ be two approximation spaces and let $A = (X, R)$ be a product space of B and C. Obviously

$$\bar{R}f = \{(x, y) \in Y \times X : [(x, y)]_R \cap f \neq \emptyset\}.$$

In this paper we shall consider functions for which $\underline{R}f = \emptyset$.

The R-representation of f, denoted Rf is defined thus: $Rf = \bar{R}f/R$, where $\bar{R}f/R$ denotes the quotient relation.

If two functions $f: Y \to Z$ and $g: Y \to Z$, $\bar{R}f = \bar{R}g$ we say that f and g are R-indiscernible. Of course, R-indiscernible functions have the same R-representation.

Let $A = B \times C$, where $A = (X, R)$, $B = (Y, Q)$ and $C = (Z, P)$. We say that a function $f: Y \to Z$ is R-continuous in $A = (X, R)$, in the point $y_0 \in Y$, if $y_0 \in \bar{Q}Y'$ implies $f(y_0) \in \bar{P}f(Y')$, for every $Y' \subseteq Y$.

Fact 1. A function $f: Y \to Z$ is R-continuous in $A = (X, R)$ iff f is R-continuous for every $y \in Y$.

Fact 2. If $f: Y \to Z$ is R-continuous in $A = B \times C$, where $A = (X, R)$, $B = (Y, Q)$ and $C = (Z, P)$, then $f(\bar{Q}Y') \subseteq \bar{R}f(Y')$, for every $Y' \subseteq Y$.

If in an approximation space $A = (X, R)$ every equivalence class of R contains exactly one element, then A will be called a selective approximation space.

Fact 3. Let $A = (X, R)$ be a selective approximation space, and $A = B \times C$, where $B = (Y, Q)$ and $C = (Z, P)$. Every function $f: Y \to Z$ is R-continuous in A, $Rf = f$ and $f(\bar{Q}Y') = \bar{P}f(Y')$, for every $Y' \subseteq Y$.

The intuitive meaning of R-continuity is the following: a function $f: Y \to Z$ is R-continuous in A, if f does not changes its values "too fast" in comparison to measurement accuracy, expressed here by the indiscernibility relation R.

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[3] Z. Pawlak, *On rough relations*, Bull. Pol. Ac.: Tech., **34** (1986) 587–590.

З. Павляк, **Приближенные фунхции**

В работе даются определения приближенной функции, т.е. функции, величины аргументов которой и величины функции не известны точно, а с некоторым приближением, установленным соотношениями неразличимости.

Данное определение опирается на концепцию приближенных множеств. В настоящей работе дается дефиниция приближенной непрерывности функции, приводятся также несколькие элементарные свойства такого типа функций.

Предлагаемый подход может быть существенным для некоторых проблем искусственного интеллекта, а в особенности распознания форм и идентификации объектов и др.