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On Discernibility of Objects in Knowledge Representation Systems

by

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Summary. In this paper we analyse how subsets of a given set of attributes contribute to discernment of sets of objects.

1. Introduction. In the knowledge representation systems we describe objects by means of attributes. The question arises how some subsets of attributes contribute to the discernment of sets of objects.

The problem mentioned above is nalysed by the rough sets approach (see Pawlak [2], Pawlak [3] and Orłowska, Pawlak [4]).

2. Knowledge representation system. Indiscernibility. We recall after Pawlak [1], the notion of a knowledge representation system, which is the starting point of our considerations.

By a knowledge representation system we mean a system

$$S = (U, A, V, \varrho)$$

where

U—is a set of *objects*,

A—is a set of *attributes*,

$V = \bigcup_{a \in A} V_a$—is a set of *values* of attributes,

$\varrho: U \times A \to V$—is an *information function.*

Set $V_a, a \in A$ will be referred to as *domain* of the attribute a.

Function $\varrho_x: A \to V$ such that $\varrho_x(a) = \varrho(x, a)$ for every $a \in A$, $x \in U$ will be called an *information* about x in S.

Let B be a nonempty subset of attributes A. We say that objects $x, y \in U$ are B-indiscernible in S, $x \tilde{B} y$, iff

$$\varrho_x(a) = \varrho_y(a) \text{ for every } a \in B$$

Obviously B is an equivalence relation for any $B \subseteq A$.

Equivalence classes of relation B are called *B-elementary sets* in A. A-elementary sets are called simply *elementary sets* in S.

B-elementary set containing object $x \in U$, will be denoted by $[x]_{\tilde{B}}^{S}$, or $[x]_{\tilde{B}}$ when S is understood.

Subset $x \in U$ will be called a B-definable set in S if X is a union of some B-elementary sets in S, an empty set is B-definable for every $B \subseteq A$.

3. Approximation of sets in knowledge representation system. Let $S = (U, A, V, \varrho)$ be a knowledge representation system, let $X \subseteq U$ and let $B \subseteq A$ $(B \neq \emptyset)$.

A *lower B-approximation* of X in S ($\underline{B}_S(X)$ or $\underline{B}(X)$ when S is understood) we define as follows:

$$\underline{B}(X) = \{x \in U : [x]_{\tilde{B}} \subseteq X\}$$

An *upper B-approximation* of X in S ($\bar{B}_S(X)$ or $\bar{B}(X)$ when S is understood) we mean set

$$\bar{B}(X) = \{x \in U : [x]_{\tilde{B}} \cap X \neq \emptyset\}$$

Set $\underline{B}(X)$ will be called a *positive region* of set X with respect to B in S, and will be denoted by $\mathrm{pos}_B(X)$;

Set $U - \bar{B}(X)$ will be called a *negative region* of set X with respect to B in S, and will be denoted by $\mathrm{neg}_B(X)$;

Set $\mathrm{Fr}_B(X) = \bar{B}(X) - \underline{B}(X)$ will be called a *borderline region* (or a *boundary*) of set X with respect to B in S.

4. Sets decidable by sets of attributes. Given subset $X \subseteq U$ of objects in system $S = (U, A, V, \varrho)$. We might be interested how subset $B \subseteq A$ of attributes affects the accuracy of approximation of set X in S.

To answer this question we introduce set

$$X_B = \mathrm{Fr}_{A-B}(X) - \mathrm{Fr}_A(X)$$

called a set *decidable* with respect to X by set of attributes B in S.

Set X_B is simply a set of objects in S membership out of which set X is dependent upon a set of attributes B; in other words set X_B says how the boundary region of set X changes when removing the set of attributes B from system S.

We can split set X_B into two sets X_B^+ and X_B^- $(X_B = X_B^+ \cup X_B^-)$ such that

$$X_A^+ = \underline{A}(X) - \underline{A-B}(X) = \underline{A}(X) \cap \mathrm{Fr}_{A-B}(X)$$
$$X_B^- = \overline{A-B}(X) - \bar{A}(X) = (U - \bar{A}(X)) \cap \mathrm{Fr}_{A-B}(X)$$

called *positively* and *negatively decidable* with respect to X in S by B, respectively.

In other words

$$X_B^+ \subseteq \mathrm{pos}_A(X) \text{ and } X_B^+ \subseteq \mathrm{Fr}_{A-B}(X)$$

and

$$X_B^- \subseteq \mathrm{neg}_A(X) \text{ and } X_B^- \subseteq \mathrm{Fr}_{A-B}(X)$$

From the above properties one can easily see what is the role of subset B of attributes in defining set X in system S.

5. Sets split by attributes. If we remove set B of attributes from set of attributes A in system $S = (U, A, V, \varrho)$ some A-elementary sets glue together forming $(A-B)$-elementary sets in S, (in other words set B splits some $(A-B)$-elementary sets into A-elementary sets).

Union of $(A-B)$-elementary sets obtained by "glue together" some A-elementary sets when removing subset of attributes B from set A in system S, is defined as follows:

$$\mathrm{Sp}_S(B) = \{x \in U : [x]_{\widetilde{A-B}} \in U/_{\widetilde{A-B}} - U/_{\tilde{A}}\}$$

One can show the following property

$$\text{If } X \cap \mathrm{Sp}_S(B) = \emptyset \text{ then } X_B = \emptyset$$

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З. Павляк, **О различимости объектов в системах представлении знаний**

В настоящей работе рассматривается, каким образом подмножества данного множества аттрибутов воздействуют на резличимость множества объектов.