

$$A = A_0 \cdot \sqrt{\left(\frac{\cos \alpha_x}{X}\right)^2 + \left(\frac{\cos \alpha_y}{Y}\right)^2 + \left(\frac{\cos \alpha_z}{Z}\right)^2}, \quad (2.10)$$

where X , Y and Z denote the relative changes of distances between the same particles in x , y and z directions caused by non-zero values of velocity gradients: $(\partial v_x / \partial x)$, $(\partial v_y / \partial y)$ and $(\partial v_z / \partial z)$.

This simple theory was used by Mohr et al. [17] to relate mixing performance in a single screw-extruder to screw geometry and degree of pressure flow. The distribution patterns of the "minor" tracer liquid in the cross section of extruded rods were found to be in a qualitative agreement with the theoretical predictions. Another success of the theory of Mohr is presented by Schrenk et al. [18]. They have shown that in the case of pressure driven, laminar flow through a rotating tube, measured and calculated values of the striation thickness remain in a good quantitative agreement. The importance of effects of deformation, orientation and redistribution of the intermaterial area in mixing of very viscous liquids was also confirmed in special cases by Shearer [19] and Erwin[20].

The theory of Mohr was further developed by Ranz, Ottino and Macosko [21,22,23] by including the effects of molecular diffusion and chemical reaction.

In this context it is very interesting that the Mohr's interpretation of the effect of viscosity ratio on mixing was seriously criticized already in 1972 by Murakami et al. [24]. The theory was examined experimentally by the authors using a cone rotor type mixer to generate the uniform shear. The authors concluded that the effect of viscosity ratio on the striation thickness might be negligible in the range of $0.6 < \mu_d / \mu_c < 3$. In the range of viscosity ratio $\mu_d / \mu_c > 3$ and $\mu_d / \mu_c < 0.6$ Murakami et al. [24] observed formation of uneven streaks and spots of the "minor" component, which completely changes interpretation of mixing; the relation between the shape of these spots and viscosity ratio is extremely different from that for insoluble liquids. All these observations are contrary to Mohr's predictions. Only when the viscosity ratio was equal to one, were the experimental results in good agreement with the theoretical results.

2.3. Mixing with Diffusion and Reaction.

Transport processes occurring at the molecular level such as molecular diffusion and chemical reaction were generally neglected in the early studies of mixing in very viscous liquids. The rate of molecular mass transfer was considered to be too slow and acting over

too small distances to aid to the overall transport mechanism [15, 16]. Nevertheless, Mohr [16] analyzed theoretically the case of molecular diffusion in the system with shear flow in the configuration shown in figure 2.3. He found that the time required to decrease the intensity of segregation from 1 to I_s can be estimated from the relation:

$$t_m = \sqrt[3]{3 \cdot s_0^2 \cdot \ln \left[\frac{\sqrt{2} \cdot \sin(\pi \cdot \delta_0 / s_0)}{I_s \cdot \pi \cdot \delta_0 / s_0} \right] / \pi^2 \cdot D \cdot \left(\frac{\partial v_x}{\partial y} \cdot \cos \alpha_x \right)^2}, \quad (2.11)$$

valid for large values of $(\partial v_x / \partial y) \cdot t$. A quotient δ_0 / s_0 represents the ratio of the initial thickness of the slab containing tracer to the initial thickness of the system as shown in figure 2.5.

The importance of effects of molecular diffusion in the process of mixing on the molecular scale was neglected in the early works mainly because of lack of experimental data - [25]. In 1972 Mitsubishi et al. [26] proved experimentally that the course of homogeneous chemical reaction occurring between equimolar amounts of unpremixed reactants (ethylene halohydrins and hydroxyl ions) in very viscous liquids ($\mu = 0.04 \div 3$ Pa·s) was affected by molecular diffusion. Mitsiushi et al. [26] recognized

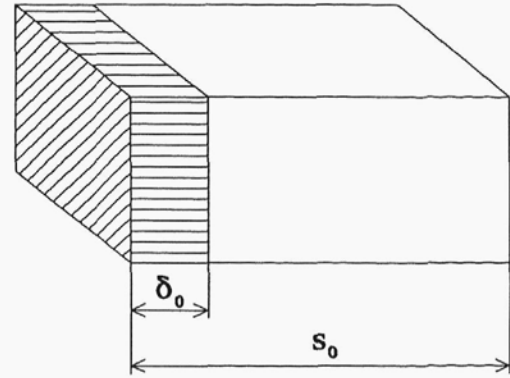


Figure 2.5. System for simultaneous deformation and diffusion.

three stages or periods in the process. In the first stage, initiated by addition of halohydrin solution to the reactor containing solution of the hydroxyl ions, the conversion rate was found to be controlled by the agitator revolution speed. In the second stage the reaction was neither controlled by convective mixing nor by the intrinsic reaction kinetics. During the last period of mixing the reaction was proceeding in a kinetic regime. The observed phenomenon clearly indicates that the segregation (unmixedness on the molecular scale) still exists in the second period of the process. The deviations from the complete mixing on the molecular scale vanish due to molecular diffusion, which proves existence and importance of the micromixing regime in mixing of very viscous liquids.

The first model of laminar mixing as consisting of simultaneous deformation, diffusion and chemical reactions was presented by Ranz [21]. Contrary to the previous descriptions of mixing in very viscous liquids formulated in Eulerian frame of reference, Ranz [21] applied a Lagrangian approach. He considered a liquid element as it passes through various regions

of the system and exchanges momentum, mass and energy with the environment. When the incompressible, very viscous and completely miscible liquids are mixed, the resulting mixture has a stratified structure as shown schematically in figure 2.6; the photographs of such structures are published for example by Macosko[27].

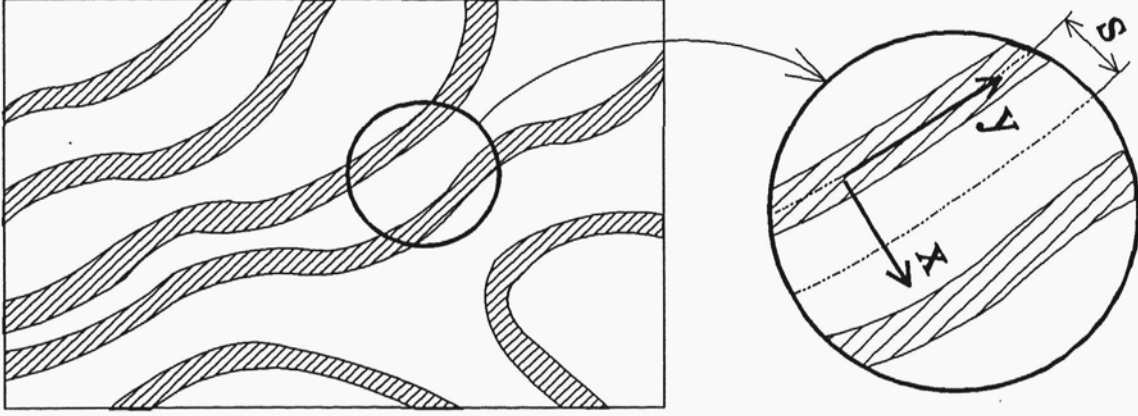


Figure 2.6. Schematic representation of a lamellar structure.

Ranz [21] attached the local system of coordinates to fluid element as shown in figure 2.6. Within this frame of reference the lamina extends or contracts depending on the orientation of its normals with the respect to the principal axes of deformation. This description is obviously valid only in the close surrounding of the coordinate origin, where the flow can be divided into pure straining motion and a rigid body rotation. Ranz [21] assumed that in Lagrangian frame of reference the flow can be univocally characterized by a function $\alpha(t)$:

$$v_x = -\alpha \cdot x, \quad v_y = \alpha \cdot y, \quad v_z = 0. \quad (2.12)$$

The rate of stretch - $\alpha(t)$ is directly related to the rate of change of striation thickness and the rate of the intermaterial area generation per unit volume:

$$\alpha = -\frac{1}{s} \cdot \frac{ds}{dt} = \frac{1}{a_v} \cdot \frac{da_v}{dt}. \quad (2.13)$$

Thus, when the history of deformation is known, one can easily calculate the final value of striation thickness or the contact surface area. For the stagnation and shear flows Ranz [21] obtained:

$$\text{- stagnation flow} \quad \alpha = \text{const} \quad s = s_0 \cdot \exp(-\alpha \cdot t), \quad (2.14a)$$

$$\text{- shear flow} \quad \alpha = G^2 \cdot t / [1 + (G \cdot t)^2] \quad s = s_0 / \sqrt{1 + (G \cdot t)^2}, \quad (2.14b)$$

where G is a rate of shear. Neglecting the variation of species concentrations at planes parallel to the intermaterial surface, the material balance in Lagrangian frame reads:

$$\frac{\partial c_i}{\partial t} - \alpha \cdot x \cdot \frac{\partial c_i}{\partial x} = D_i \cdot \frac{\partial^2 c_i}{\partial x^2} + R_i . \quad (2.15)$$

This equation allowed Ranz to incorporate molecular diffusion and homogeneous reaction to the model of mixing in the stretching laminae.

Using this model Ranz [21] was able to estimate the mixing time in a process known as reaction injection moulding (RIM). A sketch of the stratified structure obtained in a RIM mixing head is shown schematically in figure 2.7. In the process the rate of reaction is controlled by diffusion of reactant (monomer A) through a product layer (polymer) to the regions containing the monomer catalyst B. The width of the polymer layer thickens

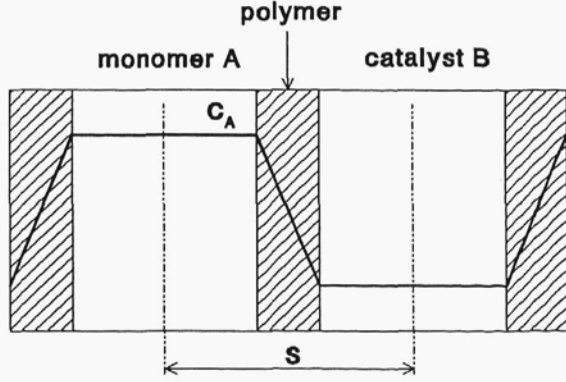


Figure 2.7. Sketch of the stratified, reacting mixture.

by reaction and thins by stretching; the average reaction rate is thus controlled by the extent of reaction zone and the rate of growth of an intermaterial area. The time required to complete the reaction was estimated as:

$$t_m \cong \frac{1}{2 \cdot \alpha} \cdot \ln \left(1 + \frac{s_0^2 \cdot \alpha \cdot \rho}{D_A \cdot c_{A0}} \right) , \quad (2.16)$$

where c_{A0} denotes solubility of the monomer in the polymer and is expressed as mass of A per unit volume of the polymer.

The analysis proposed by Ranz was further developed by Ottino et al. [22]. These authors formalized and generalized the model. First, they introduced the stretch ratio - λ , defined as:

$$\lambda \equiv \frac{|d\vec{x}|}{|d\vec{X}|} \quad (2.17)$$

with orientation given by

$$\hat{n} = \frac{d\vec{x}}{|d\vec{x}|} \quad (2.18)$$

as shown in figure 2.8. They proved that:

$$\frac{1}{\lambda} \cdot \frac{d\lambda}{dt} = \overline{\overline{D}} : \hat{n} \hat{n} , \quad (2.19)$$

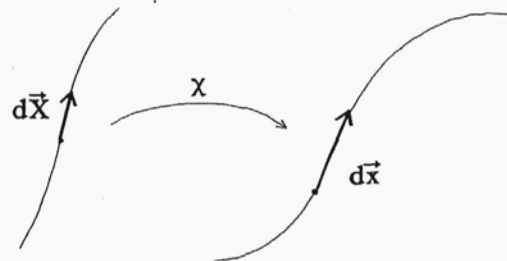


Figure 2.8. Deformation of an infinitesimal material line under the motion $\vec{x} = \chi(\vec{X}, t)$.

where $\overline{\overline{D}}$, the deformation tensor is the symmetric part of velocity gradient tensor:

$$\overline{\overline{D}} = \frac{1}{2} \cdot [(\text{Grad} \vec{v}) + (\text{Grad} \vec{v})^T] . \quad (2.20)$$

Similarly they described the deformation of an elementary material plane (see figure 2.9):

$$\eta \equiv \frac{|d\vec{a}|}{|d\vec{A}|} \quad (2.21)$$

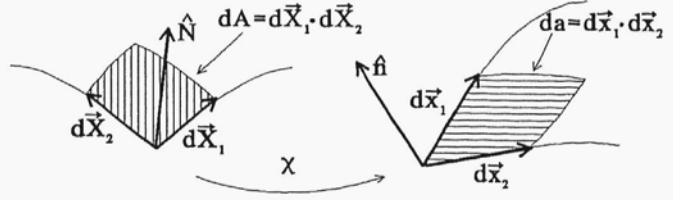


Figure 2.9. Deformation of an infinitesimal material plane under the motion $\bar{x} = \bar{\chi}(\bar{X}, t)$.

and proved that the surface area stretching can be computed from the following expression:

$$\alpha = \frac{1}{\eta} \cdot \frac{d\eta}{dt} = \text{div} \vec{v} - \bar{\bar{D}} : \hat{n} \hat{n} . \quad (2.22)$$

Due to the fact, that for incompressible fluids $\lambda \cdot \eta = 1$, one has a simple relation between the length and area stretching:

$$\frac{1}{\lambda} \cdot \frac{d\lambda}{dt} = -\frac{1}{\eta} \cdot \frac{d\eta}{dt} = -\alpha . \quad (2.23)$$

The deformation tensor $\bar{\bar{D}}$ and the viscosity μ determine the rate of viscous dissipation per unit volume:

$$\epsilon = 2 \cdot \mu \text{tr} \bar{\bar{D}}^2 . \quad (2.24)$$

Making use of the Cauchy-Schwarz inequality:

$$\bar{\bar{D}} : \hat{n} \hat{n} \leq \sqrt{\bar{\bar{D}} : \bar{\bar{D}}} \quad (2.25)$$

and using equation (2.22) with $\text{div} \vec{v} = 0$ for incompressible fluid Ottino et al. [22] determined the upper bound for the rate of intermaterial area generation as:

$$\alpha = \frac{1}{\eta} \cdot \frac{d\eta}{dt} \leq \sqrt{\frac{\epsilon}{2 \cdot \mu}} . \quad (2.26)$$

The above equation confirms an earlier remark of Spencer and Wiley [15] that velocity gradients always produce viscous dissipation but the dissipation of the energy does not always imply increase of the intermaterial area. Assuming that intermaterial area per unit volume can be defined locally as

$$a_v \equiv \lim_{V \rightarrow 0} \frac{A}{V} \quad (2.27)$$

and treated as a smooth function of time and space, the rate of generation of a_v and the rate of stretch are thus related by:

$$\frac{1}{a_v} \cdot \frac{da_v}{dt} = \frac{1}{\eta} \cdot \frac{d\eta}{dt} = -\frac{1}{\lambda} \cdot \frac{d\lambda}{dt} = -\bar{D} : \hat{n} \hat{n} \leq \sqrt{\frac{\epsilon}{2 \cdot \mu}} . \quad (2.28)$$

Following the method by Ranz [21], Ottino studied diffusion and reaction from a Lagrangian point of view. In particular, considering the contact area in the lamellar structure and the striation thickness

$$s \equiv 1/a_v \quad (2.29)$$

he obtained [22,23]:

$$\left. \frac{\partial \ln a_v}{\partial t} \right|_{\vec{x}} = - \left. \frac{\partial \ln s}{\partial t} \right|_{\vec{x}} = -\bar{D} : \hat{n} \hat{n} = \alpha(\vec{X}, t) . \quad (2.30)$$

The equation is valid in the neighborhood of the material point \vec{X} . Clearly, this formula generalizes the meaning of the rate of deformation α introduced by Ranz [21] to characterize a local stagnation flow. From definition, the principal axes of strain are coincident with Lagrangian frame axes so the flow around the origin of the framing is an elliptic stagnation symmetrical flow; in particular the relative velocity of any material points in the close proximity of \vec{X} is given by:

$$v'_i = D_{ii}(\vec{X}, t) \cdot x'_i , \quad (2.31)$$

where primes refer to quantities in the considered frame and D_{ii} denotes element of the deformation tensor. In the case of lamellar structure the total effect of the flow can be thus confined to normal velocity components, i.e. v'_x :

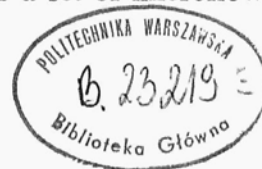
$$v'_x = - \alpha(\vec{X}, t) \cdot x' . \quad (2.32)$$

The small region ζ_x around \vec{X} where the stagnation flow can be assumed (so the velocity gradients are uniform) is called microflow element. The material balance in each microflow element takes thus a simple form:

$$\frac{\partial c_i}{\partial t} - \alpha(\vec{X}, t) \cdot x' \cdot \frac{\partial c_i}{\partial x'} = D_i \cdot \frac{\partial^2 c_i}{\partial x'^2} + R_i . \quad (2.33)$$

The structure and evolution of the stratified microflow element are shown schematically in figure 2.10.

Each region ζ_x has its own history of deformation and residence time in the reactor. The main assumption is that the local state of the mixture at any point in the system and in any time has a lamellar structure, so the whole system can be considered as a set of microflow elements



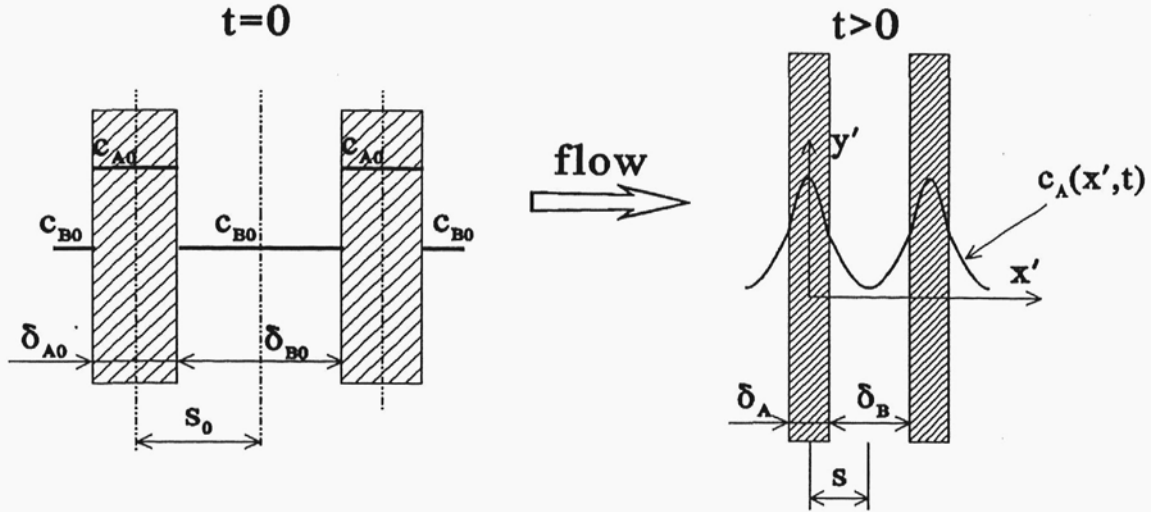


Figure 2.10. Evolution of a lamellar structure composed of two materials containing species A and B.

ζ_x . Solution of the differential balance equations (2.33) gives distribution of reactants concentrations in space (in the microflow element ζ_x) and time provided that the appropriate initial and boundary conditions are known. The problem of initial and boundary conditions in the microflow element ζ_x becomes very difficult and complicated when there is a back-mixing in the system; these problems were, however, completely disregarded and neglected by Ottino [23].

The next step in description of the system (reactor or mixer) is to apply some kind of population balance to the set of microflow elements and to calculate in this way the mean concentration in the reactor exit. The average concentration in the material volumes ζ_x can be calculated by simple integration leading to the solution in the form:

$$\langle c_i \rangle_{\zeta_x} = \langle c_i[\vec{X}, t, \alpha(\vec{X}, t)] \rangle_{\zeta_x} = \text{function}(\vec{X}, \text{time}, \text{local fluid mechanics}) . \quad (2.34)$$

Using now a residence time distribution function $E(t)$, "being the mathematical description of time history of a set of ζ_x -elements present together at the entrance flow" (and afterwards possibly separated) one can calculate the mean concentration in the reactor exit:

$$\langle c_i \rangle_{\text{exit}} = \int_0^{\infty} E(t) \cdot \langle c_i \rangle_{\zeta_x} dt . \quad (2.35)$$

A combination of the lamellar mixing model with the concept of residence time distribution is presented schematically in figure 2.11.

In the chemical reaction engineering the realization of the residence time distribution is called

macromixing, whereas micromixing is defined as local deformation, molecular diffusion and reaction. Equations (2.33)÷(2.35) thus present an attempt to link the models of macro- and micromixing. Linking of macro- and micromixing models is a serious problem of chemical reaction engineering, so it is worth discussing the idea of Ottino in more detail. To this end let us compare characteristic length and time scales for

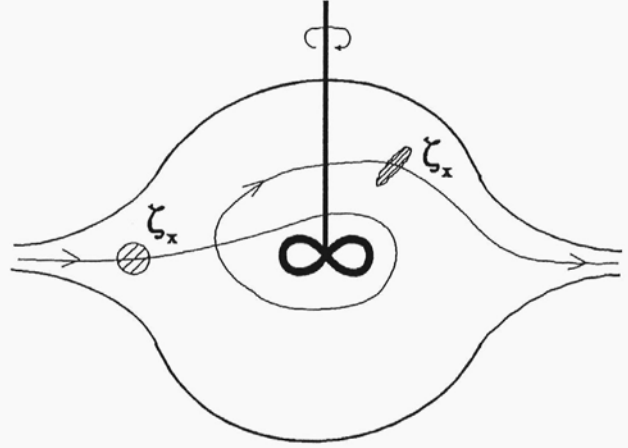


Figure 2.11. Relation between microflow and macroflow in the reactor.

macro- and micromixing. The characteristic length scale of the microflow element ζ_x is equal to the size of the region where the velocity gradients are uniform and the inertial stresses are smaller than the viscous normal stresses. Ranz [28] estimated this scale from the following relation:

$$\lambda_x = 2 \cdot \sqrt{\frac{\mu_c}{\rho_c \cdot \alpha}} . \quad (2.36)$$

Clearly, the model can be applied provided that the microflow element scale, λ_x , is much smaller than the scale of the reactor, L , and much larger than striation thickness, s .

$$L \gg \lambda_x \gg s . \quad (2.37)$$

It means that the theory can be applied starting from the moment when the scale of segregation is reduced (by convection) and the fine lamellar structure is developed.

Characteristic times for micromixing and chemical reaction resulting from analysis of equation (2.33) are as follows [29]:

$$\text{– characteristic fluid mechanical time} \quad t_F = 1/\bar{\alpha}(t) , \quad (2.38)$$

where

$$\bar{\alpha}(t) = \frac{1}{t} \cdot \int_0^t \alpha(t) dt \quad (2.39)$$

is the average value of stretching function which can be estimated either from conversion of instantaneous reaction or from the approximate relation

$$\bar{\alpha} = \frac{1}{t_{res}} \cdot \ln \left(\frac{s_0}{s_f} \right) , \quad (2.40)$$

– characteristic time of molecular diffusion $t_D = s_0^2/D_i$, (2.41)

– characteristic reaction time $t_R = c_{i0}/|R_{i0}|$. (2.42)

Equation (2.35) can be applied provided that the life time of material elements ζ_x (i.e. the time of keeping identity by the fluid elements) is longer than the residence time, t_{res} . The material element ζ_x is loosing its identity due to molecular diffusion accelerated by deformation, so in liquids this characteristic life time should be longer than t_F and shorter than t_D . A more exact estimation of the characteristic life time of material element ζ_x will be presented subsequently in this work; here the condition for using equation (2.35) can be proposed in the form:

$$t_D > \bar{t}_{res} , \quad (2.43)$$

where \bar{t}_{res} is the mean residence time in the system. In the case of large chemical reactors the above condition is hardly fulfilled, so in most cases equation (2.35) should not be used.

The characteristic times defined in equations (2.38), (2.41) and (2.42) have some very important applications. First, using the characteristic times one can transform equation (2.33) into the following dimensionless forms [29]:

$$\frac{\partial C_i}{\partial \Gamma} = \eta^2(\Gamma) \cdot \left(\frac{t_C}{t_D} \right) \cdot \frac{\partial^2 C_i}{\partial \xi^2} + \left(\frac{t_C}{t_R} \right) \cdot \frac{R_i}{|R_{i0}|} , \quad (2.44)$$

$$\frac{\partial C_i}{\partial \tau} = \left(\frac{t_C}{t_D} \right) \cdot \frac{\partial^2 C_i}{\partial \xi^2} + \frac{1}{\eta^2(\tau)} \cdot \left(\frac{t_C}{t_R} \right) \cdot \frac{R_i}{|R_{i0}|} , \quad (2.45)$$

where t_C is characteristic time which can be taken as t_D or t_F and $\eta(t)$ is the area stretch

$$\eta(t) = \exp \left[\int_0^t \alpha(t') dt' \right] . \quad (2.46)$$

Independent variables in equations (2.44) and (2.45) are defined as follows:

$$\Gamma = t/t_C , \quad \text{reduced time} , \quad (2.47)$$

$$\tau = \frac{1}{t_C} \cdot \int_0^t \eta^2(t') dt' , \quad \text{warped time} , \quad (2.48)$$

$$\xi = x/s(t) , \quad \text{space scale} . \quad (2.49)$$

Another application of the characteristic times is related to identification of the controlling mechanism. Reactions may be classified by comparing the characteristic times for mixing and reaction; the reaction is:

- instantaneous if $t_D \gg t_R$,
- fast for $t_D \approx t_R$,
- slow if $t_D \ll t_R$.

In the case of slow reactions the process does not depend on local mixing but it entirely depends on chemical kinetics; the process can be described in this case by a system of ordinary differential equations coupled by reaction term. In the limit of instantaneous reaction the reactants cannot coexist, the reaction zone is reduced to the plane and the process being controlled by molecular diffusion does not depend on chemical kinetics. The differential balance equations are reduced to the form:

$$\frac{\partial c_i}{\partial t} - \alpha(\vec{X}, t) \cdot \vec{x}' \cdot \frac{\partial c_i}{\partial x'} = D_i \cdot \frac{\partial^2 c_i}{\partial x'^2} \quad (2.50)$$

with the condition that the fluxes of reactants to the reaction plane should satisfy reaction stoichiometry. Only in the case of fast reactions the full system of partial differential equations (2.33) completed by reaction terms has to be used.

The model of lamellar structure by Ranz and Ottino [21,22,23] was widely applied for interpretation of experimental data in the subject literature.

Saito and Kamiwano [30] applied the model to estimate the mixing time in the systems with high-viscosity liquids ($\mu=0.4\div 5$ Pa·s) in a screw and helical ribbon impeller mixers. The mixing time was defined as the time required to reduce striation thickness to the desired value. Of course it means that only mechanical thinning of slabs is considered. The authors found good correlation between predicted and measured experimentally values of the mixing times. In the next paper Saito and Kamiwano [31] applied the model of lamellar structure with molecular diffusion to calculate the mixing time interpreted as the period required to reduce the differences between the maximum and the average concentration of non-reacting tracer below 4%. They found again a good agreement between theoretical predictions and experimental data for very viscous liquids ($\mu=0.35\div 1.5$ Pa·s). Finally Saito et al. [32] have applied the test reaction of reduction of I_2 to $2I^-$ by either $Na_2S_2O_3$ or Na_2HPO_4 in the presence of starch used as an indicator. The reaction was performed in the batch reactor (helical screw/draft tube mixer and helical ribbon impeller mixer). The initial amount of $Na_2S_2O_3$ or Na_2HPO_4 was 4 or 8 times larger in chemical equivalent than the amount of I_2 . The measured decoloration times agreed well with the ones predicted with the use of the model of lamellar structure with diffusion and chemical reaction.

The model of lamellar structure as proposed by Ranz and Ottino was successfully verified in

batch systems; it predicts correctly mixing times for mixing of highly viscous liquids. The influence of mixing on the course of chemical reaction was tested only in the case of very high excess of one reactant; when the reaction behaves in fact as pseudo first-order one.

The model of lamellar structures has, however, same serious limitations and creates serious problems specially when the system with back-mixing is to be described:

- In the case of a continuous reactor operating with separated feed streams **A** and **B** it is assumed in the model that the lamellar structure in the microflow element contains initially only slabs of fresh solutions **A** and **B**; in fact there are other more plausible combinations, for example **A** + bulk and **B** + bulk. Besides, the initial slab thickness ratio δ_A/δ_B is assumed to be equivalent to their initial volume ratio V_{A0}/V_{B0} ; in fact the lamellar structure should rather depend upon the local flow conditions.
- The problem of life time of material elements was not considered and thus any contact between fluid elements of different age is excluded in the model. The fluid elements cannot exchange mass with environment (the microflow elements are segregated from environment in the model) and thus cannot lose their identity. This may result in not realistic estimation of the extent of the reaction zone and consequently cause errors in predictions of conversion and selectivity of chemical reactions as shown by Baldyga and Bourne [33].
- In many situations the stratified structures are not formed. A good example is mixing of partially or completely immiscible liquids (see [34]) or mixing of completely miscible liquids highly differing in viscosities (see [28] and [35]).

Laminar mixing was also modelled without using any mixing model by a direct integration of differential balance equations for momentum and mass in Eulerian frame of reference. Success of this approach depends on the complexity of the system and on the type of numerical algorithm used, its accuracy and stability. Literature provides many examples of the numerical study of mixing processes; below are given three representative examples.

Bigg and Middleman [36] used the marker and cell method to study two-dimensional shear flow of two viscous Newtonian fluids, initially stratified within a rectangular cavity. The upper surface of the cavity was set in uniform motion to mix the cavity content. The analysis showed considerable influence of both: the viscosity ratio of the mixed liquids and Reynolds number on the flow patterns and the rate of mixing. The predictions were in good agreement with the experimental data.

Lafon and Bertrand [37] employed finite-difference method to analyze laminar mixing of Newtonian and power-law liquids in unbaffled vessel agitated with an anchor or paddle impeller for Reynolds number lower than 5. The calculated two-dimensional concentration

distributions of non-reactive, diffusive tracer were applied for comparison of mixing performance of investigated impellers.

A very complicated, transient and three-dimensional analysis of mixing with chemical reaction was recently presented by Kaminoyama and Kamiwano [38]. In this study, reaction and mixing process of highly viscous Newtonian and pseudoplastic liquid was considered in a vessel stirred with a double helical ribbon impeller. The authors correctly predicted dynamic changes of flow patterns and concentration distributions of diffusing and reacting solutes: iodine and either sodium thiosulfate or disodium phosphate. The influence of the impeller rotational speed on the decolouration time was also correctly predicted. The last example clearly shows the abilities of the modern computation technique in solving problems of laminar mixing.

2.4. Efficiency of Mixing.

Among criterions of mixer performance proposed in the literature, energetic cost of mixing is the ultimate one when a device for mixing of very viscous liquids is to be chosen. A development of a general procedure which allows evaluation and comparison of the energetic efficiency of mixing in various mixers types was always a challenging task for researchers.

Gray [39] compared efficiency of batch mixing in different mixers by comparing the mixing time values measured at the same power input and identical viscosity. The condition of identical power input into compared systems enables direct comparison of energy required for batch mixing: the energy ratio is simply equal to the mixing time ratio. The mixing time was defined as a time necessary for obtaining a visually uniform mixture throughout the mixer vessel after introduction of dyed solution into the clear content of the vessel. In practice the measurements of the mixing time in the systems to be compared are performed at different power input and different viscosity. In such a situation the mixing time should be recalculated. In the regime of laminar mixing a power input and mixing time can be expressed as follows:

$$Po = \frac{P}{\rho \cdot n^3 \cdot d^5} = \frac{const_1}{Re} = \frac{const_1 \cdot \mu}{\rho \cdot n \cdot d^2} \quad (2.51)$$

$$n \cdot t_m = const_2 \quad (2.52)$$

Thus, comparison at the power level P_1 and for the same liquid mixed (characterized by dynamic viscosity μ_1) requires the rotational stirrer speed: