

12. Appendix A.

The mean value theorem has the following form [88]:

if function $\phi(x)$ is continuous within interval $\langle a, b \rangle$ and function $\psi(x)$ is integrable within interval $\langle a, b \rangle$ and $\psi(x) \geq 0$ for $a \leq x \leq b$ then

$$\int_a^b \phi(x) \cdot \psi(x) dx = \phi(c) \cdot \int_a^b \psi(x) dx, \quad (A.1)$$

where $a < c < b$.

Applying this theorem to the integral in equation (3.33) gives:

$$\int_{-s_0/2}^{s_0/2} \exp\left[-\frac{\alpha \cdot (x \cdot e^{\alpha \cdot t} - u)^2}{2 \cdot D \cdot (e^{2 \cdot \alpha \cdot t} - 1)}\right] \cdot f(u) du = \exp\left[-\frac{\alpha \cdot (x \cdot e^{\alpha \cdot t} - \zeta)^2}{2 \cdot D \cdot (e^{2 \cdot \alpha \cdot t} - 1)}\right] \cdot \int_{-s_0/2}^{s_0/2} f(u) du, \quad (A.2)$$

where $-s_0/2 < \zeta < s_0/2$. When $t \rightarrow \infty$ in expression (A.2) one receives:

$$\lim_{t \rightarrow \infty} \left\{ \exp\left[-\frac{\alpha \cdot (x \cdot e^{\alpha \cdot t} - \zeta)^2}{2 \cdot D \cdot (e^{2 \cdot \alpha \cdot t} - 1)}\right] \cdot \int_{-s_0/2}^{s_0/2} f(u) du \right\} = \exp\left[-\frac{\alpha \cdot x^2}{2 \cdot D}\right] \cdot \int_{-s_0/2}^{s_0/2} f(u) du. \quad (A.3)$$

Similarly, applying theorem (A.1) to the integral in equation (3.38) gives:

$$\begin{aligned} & \int_0^{R_0} \exp\left(-\frac{\alpha}{4 \cdot D} \cdot \frac{r^2 \cdot e^{\alpha t} + u^2}{e^{\alpha t} - 1}\right) \cdot I_0\left(\frac{\alpha}{2 \cdot D} \cdot \frac{u \cdot r \cdot e^{\alpha t/2}}{e^{\alpha t} - 1}\right) \cdot f(u) \cdot u du = \\ & = \exp\left(-\frac{\alpha}{4 \cdot D} \cdot \frac{r^2 \cdot e^{\alpha t} + \zeta^2}{e^{\alpha t} - 1}\right) \cdot I_0\left(\frac{\alpha}{2 \cdot D} \cdot \frac{\zeta \cdot r \cdot e^{\alpha t/2}}{e^{\alpha t} - 1}\right) \cdot \int_0^{R_0} f(u) \cdot u du, \end{aligned} \quad (A.4)$$

where $0 < \zeta < R_0$. Finally, when $t \rightarrow \infty$ in expression (A.4) one receives:

$$\begin{aligned} & \lim_{t \rightarrow \infty} \left[\exp\left(-\frac{\alpha}{4 \cdot D} \cdot \frac{r^2 \cdot e^{\alpha t} + \zeta^2}{e^{\alpha t} - 1}\right) \cdot I_0\left(\frac{\alpha}{2 \cdot D} \cdot \frac{\zeta \cdot r \cdot e^{\alpha t/2}}{e^{\alpha t} - 1}\right) \cdot \int_0^{R_0} f(u) \cdot u du \right] = \\ & = \exp\left(-\frac{\alpha \cdot r^2}{4 \cdot D}\right) \cdot 1 \cdot \int_0^{R_0} f(u) \cdot u du. \end{aligned} \quad (A.5)$$