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COMPUTERS

Organization of the Address-free Digital Computer for Calculating Simple Arithmetical Expressions

by

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In the paper [1] are given some simple relations between the organization of digital computers and elementary notions of computable functions, and the organization of the machine resulting from these dependences. The present paper gives an example of an organization of the machine, differing from the former [1].

Parenthesis and parenthesis-free notation

Let L_1 be a formalized language with the following primitive symbols:

a) $p, q, r, \ldots,$ variable,

b) $\Delta_1, \Delta_2, \ldots, \Delta_n$ symbols of dyadic operators,

c) (,) parentheses.

The formula Φ in L_1 will be defined by recursion:

i. If α and β are variables, then $(\alpha\Delta_i\beta)$ is the formula in L_1

ii. If α or β are variables or formulae in L_1, then $(\alpha\Delta_i\beta)$ is also a formula in L_1.

Let L_2 be a formalized language, containing primitive symbols:

a) $p, q, r, \ldots$ — variables,

b) $\Delta_1, \Delta_2, \ldots, \Delta_n$ — symbols of dyadic operators,

c) B — letter denoting blank space,

The formula in L_2 is defined below:

i. If α and β are variables or the letter B, then $\alpha\beta\Delta_i$ is the expression in L_2.

ii. If $\alpha_1, \alpha_2, \ldots, \alpha_n$ are expressions in L_2, then the sequence $\alpha_1\alpha_2\ldots\alpha_n$ is the expression in L_2.

iii. If α is the expression in L_2, then $B\alpha$ is the formula in L_2.

Example. $BBB+$, $BpB-$, $Bpq+Br-st\cdot$ are formulae in L_2. The expression in L_2 containing only three primitive symbols is called the elementary expression.

Formula $Ba_1 a_2 \dots a_n$ where $a_1, a_2, \dots, a_n$ are elementary expressions may also be written in the form of a column:

$$\begin{array}{l} a_n \\ \vdots \\ a_2 \\ a_1 \\ B \end{array}$$

Example.

$$\begin{array}{l} BB+ \\ B \end{array} \qquad \begin{array}{l} pB- \\ B \end{array} \qquad \begin{array}{l} st\cdot \\ Br- \\ pq+ \\ B \end{array}$$

It results from the properties of the addressing functions [1] that:

i. To every formula Φ in L_1 corresponds exactly one formula Ψ in L_2, Ψ will be called the well-formed formula.

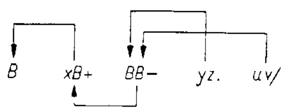
ii. The algorithm transforming each formula Φ into formula Ψ can be given.

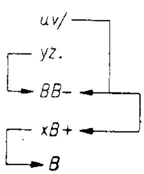
iii. If Ψ is a well formed formula in L_2, and $\Psi = Ba_1 a_2, \dots, a_n$, the result of the computation of the elementary expression a_i is to be put in place of letter B, the nearest on the left side of expression a_i.

Example. To the formula $(x+((y\cdot z)-(u/v))$ in L_1 corresponds in L_2 the formula $BxB+BB-yz\cdot uv/$ or

$$\begin{array}{l} uv/ \\ yz. \\ BB- \\ xB+ \\ B \end{array}$$

The course of the computation is stated below. Computation is started from right to left, or from top to bottom. The arrows indicate, where the result of computation of each elementary expression is to be placed.





If, in the elementary expression a_{i+k} of the formulae written in the form of a column two letters B appear, the letter placed on the right hand side is then considered as the nearest to the expression a_i.

Organization of the machine

An example of the machine operating on the principle described above is shown in Fig. 1.

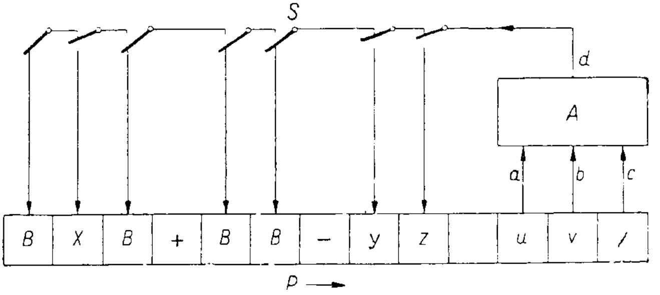


Fig. 1

The machine consists of the *memory registers P, arithmometer A* and the *switching circuit S*. The memory registers, may for purposes of simplification be assumed to be a paper tape. The tape is divided into squares. The value of the variable, the dyadic operator Δ_i or the letter B can be written in each of the squares. The arithmometer A performs — on the numbers set at the inputs a and b — the operation set at the input c and gives the result at the output d.

Each memory square is examined by the control unit (not shown on the drawing). If the letter B occurs in the examined square, the control units sets the switch corresponding to the given square in bottom position; if the square under examination contains the number or a symbol of an operation, the switch is set in top position. It is easy to observe that this kind of switching system transmits the result of the computation to the nearest blank space. After each elementary expression has been computed, the tape moves three squares on to the right and the cycle starts anew.

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[1] Z. Pawlak, *The organization of digital computers and computable functions*, Bull. Acad. Polon. Sci., Sér. sci. techn., 8 (1960), 41.