WZORY — FORMULAE

Wzory redukcyjne — Reduction formulae

$e^{a+x} = e^a \cdot e^x,$

$a^x = e^{x \log_e a} = 10^{x \log_{10} a} \quad (a > 0),$

$\log(ax) = \log a + \log x,$

$\log_a x = \dfrac{\log_e x}{\log_e a} = \dfrac{\log_{10} x}{\log_{10} a} \quad (a > 0,\ a \neq 1,\ x > 0),$

$\sinh x = \frac{1}{2}(e^x - e^{-x}),$

$\cosh x = \frac{1}{2}(e^x + e^{-x}),$

$\operatorname{tgh} x = \dfrac{\sinh x}{\cosh x},$

$\operatorname{ctgh} x = \dfrac{1}{\operatorname{tgh} x} = \dfrac{\cosh x}{\sinh x},$

$\cosh^2 x - \sinh^2 x = 1,$

$$\sinh(a + x) = \sinh a \cosh x + \cosh a \sinh x = \sinh a + 2 \sinh \frac{x}{2} \cosh\left(a + \frac{x}{2}\right),$$

$$\cosh(a + x) = \cosh a \cosh x + \sinh a \sinh x = \cosh a + 2 \sinh \frac{x}{2} \sinh\left(a + \frac{x}{2}\right),$$

$$\operatorname{tgh}(a + x) = \frac{\operatorname{tgh} a + \operatorname{tgh} x}{1 + \operatorname{tgh} a \operatorname{tgh} x} = \operatorname{tgh} a + \frac{\sinh x}{\cosh a \cosh(a + x)},$$

$$\operatorname{ctgh}(a + x) = \frac{1 + \operatorname{ctgh} a \operatorname{ctgh} x}{\operatorname{ctgh} a + \operatorname{ctgh} x} = \operatorname{ctgh} a - \frac{\sinh x}{\sinh a \sinh(a + x)},$$

$\sinh 2x = 2 \sinh x \cosh x,$

$\sinh 3x = 3 \sinh x + 4 \sinh^3 x,$

$\sinh 4x = \cosh x (4 \sinh x + 8 \sinh^3 x),$

$\sinh 5x = 5 \sinh x + 20 \sinh^3 x + 16 \sinh^5 x,$

$\cosh 2x = 2 \cosh^2 x - 1,$

$\cosh 3x = 4 \cosh^3 x - 3 \cosh x,$

$\cosh 4x = 8 \cosh^4 x - 8 \cosh^2 x + 1,$

$\cosh 5x = 16 \cosh^5 x - 20 \cosh^3 x + 5 \cosh x,$

$\operatorname{ar\,sinh} x = \log_e\left(x + \sqrt{x^2 + 1}\right),$

$\operatorname{ar\,cosh} x = \log_e\left(x + \sqrt{x^2 - 1}\right) \quad (x > 1),$

$$\operatorname{ar\,tgh} x = \log_e \sqrt{\frac{1 + x}{1 - x}} = \frac{1}{2}\left(\log_e(1 + x) - \log_e(1 - x)\right) \quad (-1 < x < 1),$$

$$\operatorname{ar\,ctgh} x = \log_e \sqrt{\frac{x + 1}{x - 1}} = \frac{1}{2}\left(\log_e|x + 1| - \log_e|x - 1|\right) \quad (|x| > 1),$$

$$\operatorname{ar\,sinh} x = \operatorname{sgn} x \cdot \operatorname{ar\,cosh} \sqrt{x^2 + 1} = \operatorname{ar\,tgh} \frac{x}{\sqrt{x^2 + 1}}, \qquad \operatorname{sgn} x = \begin{cases} 1 & \text{dla (for)} \quad x > 0, \\ 0 & \text{dla (for)} \quad x = 0, \\ -1 & \text{dla (for)} \quad x < 0. \end{cases}$$

$$\operatorname{ar\,cosh} x = \operatorname{ar\,sinh} \sqrt{x^2 - 1} = \operatorname{ar\,tgh} \frac{\sqrt{x^2 - 1}}{x} \quad (x \geqslant 1),$$

$$\operatorname{ar\,tgh} x = \operatorname{ar\,sinh} \frac{x}{\sqrt{1 - x^2}} = \operatorname{sgn} x \cdot \operatorname{ar\,cosh} \frac{x}{\sqrt{1 - x^2}} = \operatorname{ar\,ctgh} \frac{1}{x} \quad (|x| < 1),$$

$$\operatorname{ar\,ctgh} x = \operatorname{ar\,sinh} \frac{1}{\sqrt{x^2 - 1}} = \operatorname{sgn} x \cdot \operatorname{ar\,cosh} \frac{x}{\sqrt{x^2 - 1}} = \operatorname{ar\,tgh} \frac{1}{x} \quad (|x| > 1),$$

$$\operatorname{ar\,sinh}(a+x) = \operatorname{ar\,sinh} a + \operatorname{ar\,sinh}\left((a+x)\sqrt{1+a^2} - a\sqrt{1+(a+x)^2}\right),$$

$$\operatorname{ar\,cosh}(a+x) = \operatorname{ar\,cosh} a + \operatorname{ar\,cosh}\left(a(a+x) - \sqrt{(a^2-1)[(a+x)^2-1]}\right) \qquad (a>1,\ a+x>1),$$

$$\operatorname{ar\,tgh}(a+x) = \operatorname{ar\,tgh} a + \operatorname{ar\,tgh}\frac{x}{1-a(a+x)} \qquad (|a|<1,\ |a+x|<1),$$

$$\operatorname{ar\,ctgh}(a+x) = \operatorname{ar\,ctgh} a - \operatorname{ar\,ctgh}\frac{a(a+x)-1}{x} \qquad (|a|>1,\ |a+x|>1),$$

$$\cos^2 x + \sin^2 x = 1,$$

$$\operatorname{tg} x = \frac{\sin x}{\cos x}, \qquad \operatorname{ctg} x = \frac{1}{\operatorname{tg} x} = \frac{\cos x}{\sin x},$$

$$\sin(a+x) = \sin a \cos x + \cos a \sin x = \sin a + 2\sin\frac{x}{2}\cos\left(a+\frac{x}{2}\right),$$

$$\cos(a+x) = \cos a \cos x - \sin a \sin x = \cos a - 2\sin\frac{x}{2}\sin\left(a+\frac{x}{2}\right),$$

$$\operatorname{tg}(a+x) = \frac{\operatorname{tg} a + \operatorname{tg} x}{1-\operatorname{tg} a \operatorname{tg} x} = \operatorname{tg} a + \frac{\sin x}{\cos a \cos(a+x)},$$

$$\operatorname{ctg}(a+x) = \frac{\operatorname{ctg} a \operatorname{ctg} x - 1}{\operatorname{ctg} a + \operatorname{ctg} x} = \operatorname{ctg} a - \frac{\sin x}{\sin a \sin(a+x)},$$

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1}{2}(1-\cos x)}, \qquad \cos\frac{x}{2} = \pm\sqrt{\frac{1}{2}(1+\cos x)},$$

$$\operatorname{tg}\frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x}, \qquad \operatorname{ctg}\frac{x}{2} = \frac{\sin x}{1-\cos x} = \frac{1+\cos x}{\sin x},$$

$$\sin 2x = 2\sin x\cos x, \qquad \cos 2x = 2\cos^2 x - 1,$$

$$\sin 3x = 3\sin x - 4\sin^3 x, \qquad \cos 3x = 4\cos^3 x - 3\cos x,$$

$$\sin 4x = \cos x\,(4\sin x - 8\sin^3 x), \qquad \cos 4x = 8\cos^4 x - 8\cos^2 x + 1,$$

$$\sin 5x = 5\sin x - 20\sin^3 x + 16\sin^5 x, \qquad \cos 5x = 16\cos^5 x - 20\cos^3 x + 5\cos x,$$

$$\operatorname{arc\,sin} x + \operatorname{arc\,cos} x = \frac{\pi}{2}, \qquad \operatorname{arc\,tg} x + \operatorname{arc\,ctg} x = \frac{\pi}{2},$$

$$\operatorname{arc\,sin} x = \begin{cases} \operatorname{arc\,cos}\sqrt{1-x^2} & \text{dla (for)} \quad 0 \leqslant x \leqslant 1, \\ -\operatorname{arc\,cos}\sqrt{1-x^2} & \text{dla (for)} \quad -1 \leqslant x \leqslant 0, \end{cases}$$

$$= \operatorname{arc\,tg}\frac{x}{\sqrt{1-x^2}}$$

$$= \begin{cases} \operatorname{arc\,ctg}\dfrac{\sqrt{1-x^2}}{x} & \text{dla (for)} \quad 0 < x \leqslant 1, \\ \operatorname{arc\,ctg}\dfrac{\sqrt{1-x^2}}{x} - \pi & \text{dla (for)} \quad -1 \leqslant x < 0, \end{cases}$$

$$\operatorname{arc\,cos} x = \begin{cases} \operatorname{arc\,sin}\sqrt{1-x^2} & \text{dla (for)} \quad 0 \leqslant x \leqslant 1, \\ \pi - \operatorname{arc\,sin}\sqrt{1-x^2} & \text{dla (for)} \quad -1 \leqslant x \leqslant 0, \end{cases}$$

$$= \begin{cases} \operatorname{arc\,tg}\dfrac{\sqrt{1-x^2}}{x} & \text{dla (for)} \quad 0 < x \leqslant 1, \\ \pi + \operatorname{arc\,tg}\dfrac{\sqrt{1-x^2}}{x} & \text{dla (for)} \quad -1 \leqslant x < 0, \end{cases}$$

$$= \operatorname{arc\,ctg}\frac{x}{\sqrt{1-x^2}} \qquad \text{dla (for)} \quad x^2 < 1,$$

Wzory redukcyjne — Reduction formulae

$$\operatorname{arc\,tg} x = \arcsin \frac{x}{\sqrt{1+x^2}}$$

$$= \begin{cases} \arccos \dfrac{1}{\sqrt{1+x^2}} & \text{dla (for)} \quad x \geqslant 0, \\ -\arccos \dfrac{1}{\sqrt{1+x^2}} & \text{dla (for)} \quad x \leqslant 0, \end{cases}$$

$$= \begin{cases} \operatorname{arc\,ctg} \dfrac{1}{x} & \text{dla (for)} \quad x > 0, \\ \operatorname{arc\,ctg} \dfrac{1}{x} - \pi & \text{dla (for)} \quad x < 0, \end{cases}$$

$$\operatorname{arc\,ctg} x = \begin{cases} \arcsin \dfrac{1}{\sqrt{1+x^2}} & \text{dla (for)} \quad x \geqslant 0, \\ \pi - \arcsin \dfrac{1}{\sqrt{1+x^2}} & \text{dla (for)} \quad x \leqslant 0, \end{cases}$$

$$= \arccos \frac{x}{\sqrt{1+x^2}}$$

$$= \begin{cases} \operatorname{arc\,tg} \dfrac{1}{x} & \text{dla (for)} \quad x > 0, \\ \pi + \operatorname{arc\,tg} \dfrac{1}{x} & \text{dla (for)} \quad x < 0. \end{cases}$$

$$\arcsin(a+x) = \arcsin a + \begin{cases} \arcsin\left((a+x)\sqrt{1-a^2} - a\sqrt{1-(a+x)^2}\right) & \underset{\text{(for)}}{\text{dla}} \; a(a+x) \geqslant 0 \; \underset{\text{(or)}}{\text{albo}} \; a^2+(a+x)^2 \leqslant 1. \\ \pi - \arcsin\left((a+x)\sqrt{1-a^2} - a\sqrt{1-(a+x)^2}\right) & \underset{\text{(for)}}{\text{dla}} \; a+x > 0,\; a < 0,\; a^2+(a+x)^2 > 1, \\ -\pi - \arcsin\left((a+x)\sqrt{1-a^2} - a\sqrt{1-(a+x)^2}\right) & \underset{\text{(for)}}{\text{dla}} \; a+x < 0,\; a > 0,\; a^2+(a+x)^2 > 1, \end{cases}$$

$$= \arcsin a + \begin{cases} \arccos\left(\sqrt{(1-a^2)[1-(a+x)^2]} + a(a+x)\right) & \text{dla (for)} \quad x > 0, \\ -\arccos\left(\sqrt{(1-a^2)[1-(a+x)^2]} + a(a+x)\right) & \text{dla (for)} \quad x < 0, \end{cases}$$

$$\arccos(a+x) = \arccos a + \begin{cases} -\arccos\left(a(a+x) + \sqrt{(1-a^2)[1-(a+x)^2]}\right) & \text{dla (for)} \quad x > 0, \\ \arccos\left(a(a+x) + \sqrt{(1-a^2)[1-(a+x)^2]}\right) & \text{dla (for)} \quad x < 0, \end{cases}$$

$$\operatorname{arc\,tg}(a+x) = \operatorname{arc\,tg} a + \begin{cases} \operatorname{arc\,tg} \dfrac{x}{1+a(a+x)} & \text{dla (for)} \quad a(a+x) > -1, \\ \pi + \operatorname{arc\,tg} \dfrac{x}{1+a(a+x)} & \text{dla (for)} \quad x > 0,\; a(a+x) < -1, \\ -\pi + \operatorname{arc\,tg} \dfrac{x}{1+a(a+x)} & \text{dla (for)} \quad x < 0,\; a(a+x) < -1, \end{cases}$$

$$\operatorname{arc\,ctg}(a+x) = \operatorname{arc\,ctg} a + \begin{cases} -\operatorname{arc\,ctg} \dfrac{1+a(a+x)}{x} & \text{dla (for)} \quad x > 0, \\ \pi - \operatorname{arc\,ctg} \dfrac{1+a(a+x)}{x} & \text{dla (for)} \quad x < 0. \end{cases}$$

Rozwinięcia w szereg potęgowy — Expansions in a power series

$$\frac{1}{a+x} = \frac{1}{a}\left[1 - \frac{x}{a} + \left(\frac{x}{a}\right)^2 - \left(\frac{x}{a}\right)^3 + \dots\right], \qquad -1 < \frac{x}{a} \leqslant 1,$$

$$\sqrt{a+x} = \sqrt{a}\left[1 + \frac{1}{2}\left(\frac{x}{a}\right) - \frac{1}{2\cdot 4}\left(\frac{x}{a}\right)^2 + \frac{1\cdot 3}{2\cdot 4\cdot 6}\left(\frac{x}{a}\right)^3 - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 8}\left(\frac{x}{a}\right)^4 + \dots\right], \qquad -1 < \frac{x}{a} \leqslant 1,$$

$$\frac{1}{\sqrt{a+x}} = \frac{1}{\sqrt{a}}\left[1 - \frac{1}{2}\left(\frac{x}{a}\right) + \frac{1\cdot 3}{2\cdot 4}\left(\frac{x}{a}\right)^2 - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\left(\frac{x}{a}\right)^3 + \dots\right], \qquad -1 < \frac{x}{a} \leqslant 1,$$

$$\sqrt[3]{a+x} = \sqrt[3]{a}\left[1 + \frac{1}{3}\left(\frac{x}{a}\right) - \frac{2}{3\cdot 6}\left(\frac{x}{a}\right)^2 + \frac{2\cdot 5}{3\cdot 6\cdot 9}\left(\frac{x}{a}\right)^3 - \frac{2\cdot 5\cdot 8}{3\cdot 6\cdot 9\cdot 12}\left(\frac{x}{a}\right)^4 + \dots\right], \qquad -1 < \frac{x}{a} \leqslant 1,$$

$$\frac{1}{\sqrt[3]{a+x}}=\frac{1}{\sqrt[3]{a}}\left[1-\frac{1}{3}\left(\frac{x}{a}\right)+\frac{1\cdot4}{3\cdot6}\left(\frac{x}{a}\right)^2-\frac{1\cdot4\cdot7}{3\cdot6\cdot9}\left(\frac{x}{a}\right)^3+\dots\right], \qquad -1<\frac{x}{a}\leqslant 1,$$

$$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots,$$

$$\log_e(a+x)=\log_e a+\frac{x}{a}-\frac{1}{2}\left(\frac{x}{a}\right)^2+\frac{1}{3}\left(\frac{x}{a}\right)^3-\frac{1}{4}\left(\frac{x}{a}\right)^4+\dots, \qquad -1<\frac{x}{a}\leqslant 1,$$

$$\log_e\frac{a+x}{a-x}=2\left[\frac{x}{a}+\frac{1}{3}\left(\frac{x}{a}\right)^3+\frac{1}{5}\left(\frac{x}{a}\right)^5+\dots\right], \qquad \left(\frac{x}{a}\right)^2<1,$$

$$\sinh x=x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}+\dots,$$

$$\operatorname{ar\,sinh} x=\begin{cases} x-\frac{1}{2}\cdot\frac{x^3}{3}+\frac{1\cdot3}{2\cdot4}\cdot\frac{x^5}{5}-\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\cdot\frac{x^7}{7}+\dots, & x^2\leqslant 1,\\ \log_e 2x+\frac{1}{2}\cdot\frac{1}{2x^2}-\frac{1\cdot3}{2\cdot4}\cdot\frac{1}{4x^4}+\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\cdot\frac{1}{6x^6}-\dots, & x\geqslant 1,\end{cases}$$

$$\cosh x=1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\dots,$$

$$\operatorname{ar\,cosh} x=\begin{cases} \sqrt{2(x-1)}\left[1-\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{x-1}{2}+\frac{1\cdot3}{2\cdot4}\cdot\frac{1}{5}\cdot\left(\frac{x-1}{2}\right)^2-\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\cdot\frac{1}{7}\left(\frac{x-1}{2}\right)^3+\dots\right], & 1\leqslant x\leqslant 3,\\ \log_e 2x-\frac{1}{2}\cdot\frac{1}{2x^2}-\frac{1\cdot3}{2\cdot4}\cdot\frac{1}{4x^4}-\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\cdot\frac{1}{6x^6}-\dots, & x>1,\end{cases}$$

$$\operatorname{tgh} x=x-\frac{1}{3}x^3+\frac{2}{15}x^5-\frac{17}{315}x^7+\frac{62}{2835}x^9-\frac{1382}{155925}x^{11}+\dots, \qquad x^2<\frac{\pi^2}{4},$$

$$\operatorname{ar\,tgh} x=\begin{cases} x+\frac{x^3}{3}+\frac{x^5}{5}+\frac{x^7}{7}+\dots, & x^2<1,\\ -\frac{1}{2}\left[\ln\frac{1-x}{2}+\frac{1-x}{2}+\frac{1}{2}\left(\frac{1-x}{2}\right)^2+\frac{1}{3}\left(\frac{1-x}{2}\right)^3+\dots\right], & x^2<1,\end{cases}$$

$$\operatorname{ctgh} x=\frac{1}{x}+\frac{x}{3}-\frac{x^3}{45}+\frac{2x^5}{945}-\frac{x^7}{4725}+\frac{2x^9}{93555}-\frac{1382x^{11}}{638512875}+\dots, \qquad x^2<\pi^2,$$

$$\operatorname{ar\,ctgh} x=\begin{cases} -\frac{1}{2}\left[\ln\frac{x-1}{2}-\frac{x-1}{2}+\frac{1}{2}\left(\frac{x-1}{2}\right)^2-\frac{1}{3}\left(\frac{x-1}{2}\right)^3+\dots\right], & 1<x\leqslant 3,\\ \frac{1}{x}+\frac{1}{3x^3}+\frac{1}{5x^5}+\frac{1}{7x^7}+\dots, & x^2>1,\end{cases}$$

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\dots,$$

$$\operatorname{arc\,sin} x=\begin{cases} x+\frac{1}{2}\cdot\frac{x^3}{3}+\frac{1\cdot3}{2\cdot4}\cdot\frac{x^5}{5}+\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\cdot\frac{x^7}{7}+\dots, & x^2<1,\\ \frac{\pi}{2}-\sqrt{2(1-x)}\left[1+\frac{1}{2}\cdot\frac{1}{3}\left(\frac{1-x}{2}\right)+\frac{1\cdot3}{2\cdot4}\cdot\frac{1}{5}\left(\frac{1-x}{2}\right)^2+\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\cdot\frac{1}{7}\left(\frac{1-x}{2}\right)^3+\right.\\ \qquad\left.+\frac{1\cdot3\cdot5\cdot7}{2\cdot4\cdot6\cdot8}\cdot\frac{1}{9}\left(\frac{1-x}{2}\right)^4+\dots\right], & x^2<1,\end{cases}$$

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\dots,$$

$$\operatorname{arc\,cos} x=\begin{cases} \frac{\pi}{2}-x-\frac{1}{2}\cdot\frac{x^3}{3}-\frac{1\cdot3}{2\cdot4}\cdot\frac{x^5}{5}-\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\cdot\frac{x^7}{7}-\dots, & x^2<1.\\ \sqrt{2(1-x)}\left[1+\frac{1}{2}\cdot\frac{1}{3}\left(\frac{1-x}{2}\right)+\frac{1\cdot3}{2\cdot4}\cdot\frac{1}{5}\left(\frac{1-x}{2}\right)^2+\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\cdot\frac{1}{7}\left(\frac{1-x}{2}\right)^3+\right.\\ \qquad\left.+\frac{1\cdot3\cdot5\cdot7}{2\cdot4\cdot6\cdot8}\cdot\frac{1}{9}\left(\frac{1-x}{2}\right)^4+\dots\right], & x^2<1,\end{cases}$$

Rozwinięcia w szereg potęgowy — Expansions in a power series

$$\operatorname{tg} x = \begin{cases} x + \dfrac{1}{3}x^3 + \dfrac{2}{15}x^5 + \dfrac{17}{315}x^7 + \dfrac{62}{2835}x^9 + \dfrac{1382}{155925}x^{11} + \dots, & x^2 < \dfrac{\pi^2}{4}, \\ \dfrac{1}{\dfrac{\pi}{2} - x} - \dfrac{1}{3}\left(\dfrac{\pi}{2} - x\right) - \dfrac{1}{45}\left(\dfrac{\pi}{2} - x\right)^3 - \dfrac{2}{945}\left(\dfrac{\pi}{2} - x\right)^5 - \dfrac{1}{4725}\left(\dfrac{\pi}{2} - x\right)^7 - & \\ \qquad - \dfrac{2}{93555}\left(\dfrac{\pi}{2} - x\right)^9 - \dfrac{1382}{638512875}\left(\dfrac{\pi}{2} - x\right)^{11} - \dots, & \left(\dfrac{\pi}{2} - x\right)^2 < \pi^2, \end{cases}$$

$$\operatorname{arc\,tg} x = \begin{cases} x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \dots, & x^2 \leqslant 1, \\ \dfrac{\pi}{2} - \dfrac{1}{x} + \dfrac{1}{3x^3} - \dfrac{1}{5x^5} + \dfrac{1}{7x^7} - \dots, & x > 1, \\ -\dfrac{\pi}{2} - \dfrac{1}{x} + \dfrac{1}{3x^3} - \dfrac{1}{5x^5} + \dfrac{1}{7x^7} - \dots, & x \leqslant -1, \end{cases}$$

$$\operatorname{ctg} x = \begin{cases} \dfrac{1}{x} - \dfrac{1}{3}x - \dfrac{1}{45}x^3 - \dfrac{2x^5}{945} - \dfrac{x^7}{4725} - \dfrac{2x^9}{93555} - \dfrac{1382x^{11}}{638512875} - \dots, & x^2 < \pi^2, \\ \dfrac{\pi}{2} - x + \dfrac{1}{3}\left(\dfrac{\pi}{2} - x\right)^3 + \dfrac{2}{15}\left(\dfrac{\pi}{2} - x\right)^5 + \dfrac{17}{315}\left(\dfrac{\pi}{2} - x\right)^7 + \dfrac{62}{2835}\left(\dfrac{\pi}{2} - x\right)^9 + & \\ \qquad + \dfrac{1382}{155925}\left(\dfrac{\pi}{2} - x\right)^{11} + \dots, & \left(\dfrac{\pi}{2} - x\right)^2 < \dfrac{\pi^2}{4}, \end{cases}$$

$$\operatorname{arc\,ctg} x = \begin{cases} \dfrac{\pi}{2} - x + \dfrac{x^3}{3} - \dfrac{x^5}{5} + \dfrac{x^7}{7} - \dfrac{x^9}{9} + \dots, & x^2 \leqslant 1, \\ \dfrac{1}{x} - \dfrac{1}{3x^3} + \dfrac{1}{5x^5} - \dfrac{1}{7x^7} + \dots, & x \geqslant 1, \\ \pi + \dfrac{1}{x} - \dfrac{1}{3x^3} + \dfrac{1}{5x^5} - \dfrac{1}{7x^7} + \dots, & x \leqslant -1. \end{cases}$$

Błędy interpolacji funkcji $f(x)$ w przedziale $0 \leqslant x_0 < x < x_0 + h$
Errors of interpolation of function $f(x)$ in the interval $0 \leqslant x_0 < x < x_0 + h$

Funkcja Function	Błąd interpolacji Error of interpolation
$\dfrac{1}{x}$	$\dfrac{h^2}{4x_0^3}$,
$\log_{10} x$	$0{,}054287\,\dfrac{h^2}{x_0^2}$
10^x	$0{,}66274 \cdot 10^{x_0+h} \cdot h^2$,
x^2	$\dfrac{1}{4}h^2$,
$\sqrt{x}$	$\dfrac{h^2}{32x_0\sqrt{x_0}}$,
x^3	$\dfrac{3}{4}(x_0 + h)\,h^2$,
$\sqrt[3]{x}$	$\dfrac{h^2}{36x_0\sqrt[3]{x_0^2}}$,
e^x	$\dfrac{1}{8}e^{x_0+h} \cdot h^2$,
e^{-x}	$\dfrac{1}{8}e^{-x_0} \cdot h^2$,
$\sinh x$	$\dfrac{1}{8}\sinh(x_0 + h) \cdot h^2$,

Błędy interpolacji funkcji $f(x)$ w przedziale $0 \leqslant x_0 < x < x_0 + h$
Errors of interpolation of function $f(x)$ in the interval $0 \leqslant x_0 < x < x_0 + h$

ar sinh x	$\begin{cases} \dfrac{x_0+h}{[(x_0+h)^2+1]^{3/2}}\cdot\dfrac{h^2}{8} & \text{dla (for)} \quad x_0 \leqslant \dfrac{\sqrt{2}}{2}-h, \\ \dfrac{x_0}{(x_0^2+1)^{3/2}}\cdot\dfrac{h^2}{8} & \text{dla (for)} \quad x_0 \geqslant \dfrac{\sqrt{2}}{2}, \\ \dfrac{1}{36}\sqrt{3}\,h^2 & \text{dla (for)} \quad \dfrac{\sqrt{2}}{2}-h \leqslant x_0 \leqslant \dfrac{\sqrt{2}}{2}, \end{cases}$
cosh x	$\frac{1}{8}\cosh(x_0+h)\cdot h^2,$
ar cosh x	$\frac{x_0}{(x_0^2-1)^{3/2}}\cdot\frac{h^2}{8},$
tgh x	$\begin{cases} \dfrac{1}{4}[\operatorname{tgh}(x_0+h)-\operatorname{tgh}^3(x_0+h)] & \text{dla (for)} \quad x_0 \leqslant \operatorname{ar\,tgh}\dfrac{\sqrt{3}}{3}-h, \\ \dfrac{1}{4}(\operatorname{tgh} x_0-\operatorname{tgh}^3 x_0) & \text{dla (for)} \quad x_0 \geqslant \operatorname{ar\,tgh}\dfrac{\sqrt{3}}{3}, \\ \dfrac{1}{18}\sqrt{3}\,h^2 & \text{dla (for)} \quad \operatorname{ar\,tgh}\dfrac{\sqrt{3}}{3}-h \leqslant x_0 \leqslant \operatorname{ar\,tgh}\dfrac{\sqrt{3}}{3}, \end{cases}$
ar tgh x	$\frac{x_0+h}{4[1-(x_0+h)^2]^2}h^2,$
ctgh x	$\frac{1}{4}(\operatorname{ctgh}^3 x_0-\operatorname{ctgh} x_0)h^2,$
ar ctgh x	$\frac{x_0}{4(x_0^2-1)^2}h^3,$
$\sin x^\circ$	$0{,}0000002+\sin(x_0^\circ+h)\cdot 0{,}000000010577,$
$\cos x^\circ$	$0{,}0000002+\cos x_0^\circ\cdot 0{,}000000010577,$
arc° sin x	$25784\cdot\frac{x_0+h}{[1-(x_0+h)^2]^{3/2}}h^2$ sek,
arc° cos x	$25784\cdot\frac{x_0+h}{[1-(x_0+h)^2]^{3/2}}h^2$ sek,
$\sin x^{\mathrm{r}}$	$\frac{1}{8}\sin(x_0^{\mathrm{r}}+h)h^2,$
arc$^{\mathrm{r}}$ sin x	$\frac{x_0+h}{8[1-(x_0+h)^2]^{3/2}}h^2$ rad,
$\cos x^{\mathrm{r}}$	$\frac{1}{8}\cos x_0^{\mathrm{r}}\,h^2,$
arc$^{\mathrm{r}}$ cos x	$\frac{x_0+h}{8[1-(x_0+h)^2]^{3/2}}h^2$ rad,
tg x°	$\begin{cases} 0{,}000000021154[\operatorname{tg}(x_0^\circ+h)+\operatorname{tg}^3(x_0^\circ+h)] & \text{dla (for)} \quad h=1', \\ 0{,}00000000058762[\operatorname{tg}(x_0^\circ+h)+\operatorname{tg}^3(x_0^\circ+h)] & \text{dla (for)} \quad h=10'', \\ 0{,}0000000000058762[\operatorname{tg}(x_0^\circ+h)+\operatorname{tg}^3(x_0^\circ+h)] & \text{dla (for)} \quad h=1'', \end{cases}$
arc° tg x	$\begin{cases} 51567\cdot\dfrac{x_0+h}{[1+(x_0+h)^2]^2}h^2 \text{ sek} & \text{dla (for)} \quad x_0 \leqslant \dfrac{\sqrt{3}}{3}-h, \\ 51567\cdot\dfrac{x_0}{(1+x_0^2)^2}h^2 \text{ sek} & \text{dla (for)} \quad x_0 \geqslant \dfrac{\sqrt{3}}{3}, \\ 9668{,}68\sqrt{3}h^2 < 16747\,h^2 \text{ sek} & \text{dla (for)} \quad \dfrac{\sqrt{3}}{3}-h \leqslant x_0 \leqslant \dfrac{\sqrt{3}}{3}, \end{cases}$

$\operatorname{ctg} x^{\circ}$

$$\begin{cases} 0{,}000000021154\,(\operatorname{ctg} x_0^{\circ} + \operatorname{ctg}^3 x_0^{\circ}) & \text{dla (for)} \quad h = 1', \\ 0{,}00000000058762\,(\operatorname{ctg} x_0^{\circ} + \operatorname{ctg}^3 x_0^{\circ}) & \text{dla (for)} \quad h = 10'', \\ 0{,}0000000000058762\,(\operatorname{ctg} x_0^{\circ} + \operatorname{ctg}^3 x_0^{\circ}) & \text{dla (for)} \quad h = 1'', \end{cases}$$

$\operatorname{arc}^{\circ} \operatorname{ctg} x$

$$\begin{cases} 51567 \cdot \dfrac{x_0 + h}{[1 + (x_0 + h)^2]^2}\, h^2 \text{ sek} & \text{dla (for)} \quad x_0 \leqslant \dfrac{\sqrt{3}}{3} - h, \\ 51567 \cdot \dfrac{x_0}{(1 + x_0^2)^2}\, h^2 \text{ sek} & \text{dla (for)} \quad x_0 \geqslant \dfrac{\sqrt{3}}{3}, \\ 16747\, h^2 \text{ sek} & \text{dla (fot)} \quad \dfrac{\sqrt{3}}{3} - h \leqslant x_0 \leqslant \dfrac{\sqrt{3}}{3}, \end{cases}$$

$\operatorname{tg} x^{\mathrm{r}}$

$$\frac{1}{4}\left[\operatorname{tg}\left(x_0^{\mathrm{r}} + h\right) + \operatorname{tg}^3\left(x_0^{\mathrm{r}} + h\right)\right] h^2,$$

$\operatorname{arc}^{\mathrm{r}} \operatorname{tg} x$

$$\begin{cases} \dfrac{x_0 + h}{4\,[1 + (x_0 + h)^2]^2}\, h^2 \text{ rad} & \text{dla (for)} \quad x_0 \leqslant \dfrac{\sqrt{3}}{3} - h, \\ \dfrac{x_0}{4\,(1 + x_0^2)^2}\, h^2 \text{ rad} & \text{dla (for)} \quad x_0 \geqslant \dfrac{\sqrt{3}}{3}, \\ \dfrac{3}{64}\sqrt{3}\, h^2 \text{ rad} & \text{dla (for)} \quad \dfrac{\sqrt{3}}{3} - h \leqslant x_0 \leqslant \dfrac{\sqrt{3}}{3}, \end{cases}$$

$\operatorname{ctg} x^{\mathrm{r}}$

$$\frac{1}{4}\left(\operatorname{ctg} x_0^{\mathrm{r}} + \operatorname{ctg}^3 x_0^{\mathrm{r}}\right) h^2,$$

$\operatorname{arc}^{\mathrm{r}} \operatorname{ctg} x$

$$\begin{cases} \dfrac{x_0 + h}{4\,[1 + (x_0 + h)^2]^2}\, h^2 \text{ rad} & \text{dla (for)} \quad x_0 \leqslant \dfrac{\sqrt{3}}{3} - h, \\ \dfrac{x_0}{4\,(1 + x_0^2)^2}\, h^2 \text{ rad} & \text{dla (for)} \quad x_0 \geqslant \dfrac{\sqrt{3}}{3}, \\ \dfrac{3}{64}\sqrt{3}\, h^2 \text{ rad} & \text{dla (for)} \quad \dfrac{\sqrt{3}}{3} - h \leqslant x_0 \leqslant \dfrac{\sqrt{3}}{3}. \end{cases}$$

SPIS RZECZY

CONTENTS

$\prod_i p_i$

ar ctgh x

$\frac{1}{x}$

°,′,″→rad

$\log_{10} x$

rad→°,′,″

10^x

sin x°

cos x°

x^2

arc° sin x

$\sqrt{x}$

arc° cos x

x^3

sin x^r

$\sqrt[3]{x}$

arc^r sin x

e^x

cos x^r

e^{-x}

arc^rcos x

$\log_e x$

tg x°

ctg x°

sinh x

arc° tg x

ar sinh x

arc° ctg x

cosh x

tg x^r

ar cosh x

arc^r tg x

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ctg x^r

ar tgh x

arc^r ctg x

ctgh x

const