

XIII. Funkcja $\operatorname{ar} \sinh x$, czyli $\sinh^{-1} x$

x	0	δ	1	δ	2	δ	3	δ	4	δ
0,0	0,00000	1000	0,01000	1000	0,02000	1000	0,03000	999	0,03999	999
0,1	0,09983	995	0,10978	993	0,11971	993	0,12964	991	0,13955	989
0,2	0,19869	980	0,20849	977	0,21826	976	0,22802	973	0,23775	972
0,3	0,29567	957	0,30524	954	0,31478	951	0,32429	948	0,33377	945
0,4	0,39004	926	0,39930	924	0,40854	920	0,41774	917	0,42691	914
0,5	0,48121	893	0,49014	889	0,49903	885	0,50788	882	0,51670	878
0,6	0,56882	856	0,57738	852	0,58590	848	0,59438	844	0,60282	840
0,7	0,65267	817	0,66084	813	0,66897	810	0,67707	806	0,68513	802
0,8	0,73267	779	0,74046	775	0,74821	771	0,75592	768	0,76360	764
0,9	0,80887	741	0,81628	738	0,82366	734	0,83100	730	0,83830	727
1,0	0,88137	706	0,88843	702	0,89545	698	0,90243	695	0,90938	691
1,1	0,95035	671	0,95706	667	0,96373	665	0,97038	661	0,97699	658
1,2	1,01597	639	1,02236	635	1,02871	633	1,03504	629	1,04133	626
1,3	1,07845	608	1,08453	606	1,09059	602	1,09661	600	1,10261	596
1,4	1,13798	580	1,14378	577	1,14955	575	1,15530	571	1,16101	569
1,5	1,19476	554	1,20030	551	1,20581	548	1,21129	546	1,21675	543
1,6	1,24898	529	1,25427	527	1,25954	524	1,26478	521	1,26999	520
1,7	1,30082	506	1,30588	504	1,31092	501	1,31593	500	1,32093	497
1,8	1,35044	485	1,35529	482	1,36011	481	1,36492	478	1,36970	477
1,9	1,39800	465	1,40265	463	1,40728	460	1,41188	460	1,41648	457
2,0	1,44364	446	1,44810	444	1,45254	443	1,45697	441	1,46138	440
2,1	1,48748	429	1,49177	428	1,49605	426	1,50031	424	1,50455	422
2,2	1,52966	413	1,53379	412	1,53791	409	1,54200	409	1,54609	407
2,3	1,57028	398	1,57426	396	1,57822	396	1,58218	393	1,58611	392
2,4	1,60944	384	1,61328	382	1,61710	382	1,62092	379	1,62471	379
2,5	1,64723	371	1,65094	369	1,65463	369	1,65832	367	1,66199	365
2,6	1,68374	359	1,68733	357	1,69090	356	1,69446	355	1,69801	353
2,7	1,71905	347	1,72252	346	1,72598	344	1,72942	343	1,73285	343
2,8	1,75323	336	1,75659	334	1,75993	334	1,76327	333	1,76660	331
2,9	1,78634	326	1,78960	324	1,79284	323	1,79607	323	1,79930	322
3,0	1,81845	315	1,82160	315	1,82475	314	1,82789	313	1,83102	312
3,1	1,84960	307	1,85267	306	1,85573	304	1,85877	304	1,86181	303
3,2	1,87986	298	1,88284	297	1,88581	296	1,88877	296	1,89173	294
3,3	1,90927	290	1,91217	289	1,91506	288	1,91794	287	1,92081	286
3,4	1,93788	282	1,94070	281	1,94351	280	1,94631	280	1,94911	278
3,5	1,96572	274	1,96846	274	1,97120	273	1,97393	272	1,97665	272
3,6	1,99284	267	1,99551	266	1,99817	266	2,00083	266	2,00349	264
3,7	2,01926	261	2,02187	260	2,02447	259	2,02706	258	2,02964	258
3,8	2,04503	254	2,04757	254	2,05011	252	2,05263	253	2,05516	252
3,9	2,07017	248	2,07265	247	2,07512	247	2,07759	247	2,08006	245
4,0	2,09471	243	2,09714	241	2,09955	241	2,10196	241	2,10437	240
4,1	2,11869	236	2,12105	236	2,12341	236	2,12577	235	2,12812	235
4,2	2,14211	232	2,14443	230	2,14673	231	2,14904	230	2,15134	229
4,3	2,16502	226	2,16728	226	2,16954	225	2,17179	225	2,17404	224
4,4	2,18742	222	2,18964	220	2,19184	221	2,19405	220	2,19625	219
4,5	2,20935	216	2,21151	217	2,21368	216	2,21584	215	2,21799	215
4,6	2,23081	213	2,23294	211	2,23505	212	2,23717	211	2,23928	210
4,7	2,25184	208	2,25392	207	2,25599	207	2,25806	207	2,26013	206
4,8	2,27244	204	2,27448	203	2,27651	203	2,27854	203	2,28057	202
4,9	2,29264	199	2,29463	200	2,29663	199	2,29862	198	2,30060	199

Błąd przybliżeń podanych na str. 234 i 235 jest nie większy niż 0,000005. Błąd przybliżenia otrzymanego przez interpolację liniową jest dla $0,00 < x < 2,00$ mniejszy niż 0,00000867 + błąd zaokrąglenia wyniku, a dla $2,00 < x < 5,00$ mniejszy niż 0,00000725 + błąd zaokrąglenia wyniku.

Przykład interpolacji na str. 240.

XIII. Function $\operatorname{ar} \sinh x$, i. e. $\sinh^{-1} x$

5	δ	6	δ	7	δ	8	δ	9	δ	x
0,04998	998	0,05996	998	0,06994	997	0,07991	997	0,08988	995	0,0
0,14944	989	0,15933	986	0,16919	985	0,17904	984	0,18888	981	0,1
0,24747	969	0,25716	966	0,26682	964	0,27646	962	0,28608	959	0,2
0,34322	943	0,35265	939	0,36204	936	0,37140	933	0,38073	931	0,3
0,43605	910	0,44515	907	0,45422	903	0,46325	900	0,47225	896	0,4
0,52548	874	0,53422	871	0,54293	867	0,55160	863	0,56023	859	0,5
0,61122	837	0,61959	833	0,62792	828	0,63620	826	0,64446	821	0,6
0,69315	798	0,70113	794	0,70907	790	0,71697	787	0,72484	783	0,7
0,77124	760	0,77884	756	0,78640	753	0,79393	749	0,80142	745	0,8
0,84557	723	0,85280	720	0,86000	716	0,86716	712	0,87428	709	0,9
0,91629	688	0,92317	685	0,93002	681	0,93683	677	0,94360	675	1,0
0,98357	654	0,99011	652	0,99663	648	1,00311	645	1,00956	641	1,1
1,04759	623	1,05382	621	1,06003	617	1,06620	614	1,07234	611	1,2
1,10857	594	1,11451	591	1,12042	588	1,12630	586	1,13216	582	1,3
1,16670	567	1,17237	564	1,17801	561	1,18362	558	1,18920	556	1,4
1,22218	541	1,22759	539	1,23298	536	1,23834	533	1,24367	531	1,5
1,27519	517	1,28036	515	1,28551	513	1,29064	510	1,29574	508	1,6
1,32590	495	1,33085	493	1,33578	491	1,34069	488	1,34557	487	1,7
1,37447	474	1,37921	473	1,38394	470	1,38864	469	1,39333	467	1,8
1,42105	455	1,42560	454	1,43014	452	1,43466	449	1,43915	449	1,9
1,46578	437	1,47015	436	1,47451	434	1,47885	433	1,48318	430	2,0
1,50877	421	1,51298	420	1,51718	417	1,52135	416	1,52551	415	2,1
1,55016	405	1,55421	404	1,55825	402	1,56227	401	1,56628	400	2,2
1,59003	391	1,59394	390	1,59784	388	1,60172	386	1,60558	386	2,3
1,62850	377	1,63227	376	1,63603	375	1,63978	373	1,64351	372	2,4
1,66564	365	1,66929	363	1,67292	362	1,67654	361	1,68015	359	2,5
1,70154	353	1,70507	351	1,70858	350	1,71208	349	1,71557	348	2,6
1,73628	341	1,73969	340	1,74309	339	1,74648	338	1,74986	337	2,7
1,76991	331	1,77322	330	1,77652	328	1,77980	328	1,78308	326	2,8
1,80252	320	1,80572	320	1,80892	318	1,81210	318	1,81528	317	2,9
1,83414	311	1,83725	310	1,84035	310	1,84345	308	1,84653	307	3,0
1,86484	302	1,86786	302	1,87088	300	1,87388	300	1,87688	298	3,1
1,89467	294	1,89761	293	1,90054	292	1,90346	291	1,90637	290	3,2
1,92367	286	1,92653	285	1,92938	284	1,93222	283	1,93505	283	3,3
1,95189	278	1,95467	278	1,95745	276	1,96021	276	1,96297	275	3,4
1,97937	270	1,98207	271	1,98478	269	1,98747	269	1,99016	268	3,5
2,00613	264	2,00877	263	2,01140	263	2,01403	262	2,01665	261	3,6
2,03222	258	2,03480	257	2,03737	256	2,03993	255	2,04248	255	3,7
2,05768	251	2,06019	250	2,06269	250	2,06519	249	2,06768	249	3,8
2,08251	246	2,08497	244	2,08741	244	2,08985	243	2,09228	243	3,9
2,10677	239	2,10916	239	2,11155	238	2,11393	238	2,11631	238	4,0
2,13047	234	2,13281	233	2,13514	233	2,13747	232	2,13979	232	4,1
2,15363	229	2,15592	228	2,15820	228	2,16048	227	2,16275	227	4,2
2,17628	224	2,17852	223	2,18075	223	2,18298	222	2,18520	222	4,3
2,19844	219	2,20063	219	2,20282	218	2,20500	218	2,20718	217	4,4
2,22014	214	2,22228	214	2,22442	214	2,22656	213	2,22869	212	4,5
2,24138	210	2,24348	210	2,24558	209	2,24767	209	2,24976	208	4,6
2,26219	206	2,26425	205	2,26630	205	2,26835	205	2,27040	204	4,7
2,28259	202	2,28461	201	2,28662	201	2,28863	200	2,29063	201	4,8
2,30259	197	2,30456	198	2,30654	197	2,30851	197	2,31048	196	4,9

$\operatorname{ar} \sinh x$

The error of the approximations given on pp. 234 and 235 is not greater than 0,000005. The error of an approximation obtained by linear interpolation is for $0,00 < x < 2,00$ less than 0,00000867 + the error of rounding off the result and for $2,00 < x < 5,00$ less than 0,00000725 + the error of rounding off the result.

An example of interpolation is given on p. 241.

XIII. Funkcja $\operatorname{ar} \sinh x$, czyli $\sinh^{-1} x$

x	0	δ	1	δ	2	δ	3	δ	4	δ
5,0	2,31244	196	2,31440	195	2,31635	195	2,31830	195	2,32025	195
5,1	2,33186	193	2,33379	191	2,33570	192	2,33762	191	2,33953	191
5,2	2,35093	188	2,35281	189	2,35470	188	2,35658	187	2,35845	187
5,3	2,36964	185	2,37149	185	2,37334	184	2,37518	185	2,37703	184
5,4	2,38801	182	2,38983	182	2,39165	181	2,39346	181	2,39527	181
5,5	2,40606	179	2,40785	178	2,40963	178	2,41141	178	2,41319	177
5,6	2,42379	176	2,42555	175	2,42730	175	2,42905	175	2,43080	174
5,7	2,44122	173	2,44295	172	2,44467	172	2,44639	172	2,44811	171
5,8	2,45836	169	2,46005	170	2,46175	169	2,46344	169	2,46513	169
5,9	2,47521	166	2,47687	167	2,47854	167	2,48021	166	2,48187	166
6,0	2,49178	164	2,49342	164	2,49506	164	2,49670	163	2,49833	164
6,1	2,50809	161	2,50970	162	2,51132	161	2,51293	161	2,51454	160
6,2	2,52414	159	2,52573	159	2,52732	158	2,52890	159	2,53049	158
6,3	2,53994	156	2,54150	157	2,54307	156	2,54463	156	2,54619	156
6,4	2,55549	155	2,55704	154	2,55858	153	2,56011	154	2,56165	153
6,5	2,57081	152	2,57233	152	2,57385	152	2,57537	151	2,57688	151
6,6	2,58591	149	2,58740	150	2,58890	149	2,59039	149	2,59188	149
6,7	2,60078	147	2,60225	148	2,60373	147	2,60520	147	2,60667	146
6,8	2,61543	146	2,61689	145	2,61834	145	2,61979	145	2,62124	144
6,9	2,62988	143	2,63131	143	2,63274	143	2,63417	143	2,63560	142
7,0	2,64412	141	2,64553	142	2,64695	140	2,64835	141	2,64976	141
7,1	2,65816	140	2,65956	139	2,66095	139	2,66234	139	2,66373	138
7,2	2,67202	137	2,67339	137	2,67476	137	2,67613	137	2,67750	137
7,3	2,68568	136	2,68704	135	2,68839	135	2,68974	135	2,69109	135
7,4	2,69916	134	2,70050	134	2,70184	133	2,70317	133	2,70450	134
7,5	2,71247	132	2,71379	132	2,71511	131	2,71642	132	2,71774	131
7,6	2,72560	130	2,72690	130	2,72820	130	2,72950	130	2,73080	130
7,7	2,73856	128	2,73984	129	2,74113	128	2,74241	129	2,74370	128
7,8	2,75135	128	2,75263	127	2,75390	126	2,75516	127	2,75643	126
7,9	2,76399	126	2,76525	125	2,76650	125	2,76775	125	2,76900	125
8,0	2,77647	124	2,77771	124	2,77895	124	2,78019	123	2,78142	124
8,1	2,78880	122	2,79002	123	2,79125	122	2,79247	122	2,79369	122
8,2	2,80098	121	2,80219	121	2,80340	120	2,80460	121	2,80581	120
8,3	2,81301	120	2,81421	119	2,81540	119	2,81659	120	2,81779	119
8,4	2,82490	118	2,82608	118	2,82726	118	2,82844	118	2,82962	118
8,5	2,83666	116	2,83782	117	2,83899	116	2,84015	117	2,84132	116
8,6	2,84827	116	2,84943	115	2,85058	115	2,85173	115	2,85288	115
8,7	2,85976	114	2,86090	114	2,86204	114	2,86318	113	2,86431	114
8,8	2,87111	113	2,87224	113	2,87337	112	2,87449	113	2,87562	112
8,9	2,88234	112	2,88346	111	2,88457	111	2,88568	112	2,88680	111
9,0	2,89344	111	2,89455	110	2,89565	110	2,89675	110	2,89785	110
9,1	2,90443	109	2,90552	109	2,90661	109	2,90770	109	2,90879	108
9,2	2,91529	108	2,91637	108	2,91745	108	2,91853	107	2,91960	108
9,3	2,92604	107	2,92711	107	2,92818	106	2,92924	107	2,93031	106
9,4	2,93667	106	2,93773	106	2,93879	105	2,93984	106	2,94090	105
9,5	2,94720	104	2,94824	105	2,94929	104	2,95033	105	2,95138	104
9,6	2,95761	104	2,95865	103	2,95968	104	2,96072	103	2,96175	103
9,7	2,96792	102	2,96894	103	2,96997	102	2,97099	102	2,97201	102
9,8	2,97812	102	2,97914	101	2,98015	101	2,98116	101	2,98217	102
9,9	2,98822	101	2,98923	100	2,99023	100	2,99123	100	2,99223	101

Błąd przybliżeń podanych na str. 236 i 237 jest nie większy niż 0,000005, a błąd przybliżenia otrzymanego przez interpolację liniową jest mniejszy niż 0,00000548 + błąd zaokrąglenia wyniku.

Przykład interpolacji na str. 240.

XIII. Function $\operatorname{ar} \sinh x$, i. e. $\sinh^{-1} x$

5	δ	6	δ	7	δ	8	δ	9	δ	x
2,32220	194	2,32414	193	2,32607	194	2,32801	193	2,32994	192	5,0
2,34144	190	2,34334	190	2,34524	190	2,34714	190	2,34904	189	5,1
2,36032	187	2,36219	187	2,36406	186	2,36592	186	2,36778	186	5,2
2,37887	183	2,38070	183	2,38253	183	2,38436	183	2,38619	182	5,3
2,39708	180	2,39888	180	2,40068	180	2,40248	179	2,40427	179	5,4
2,41496	178	2,41674	176	2,41850	177	2,42027	176	2,42203	176	5,5
2,43254	175	2,43429	173	2,43602	174	2,43776	173	2,43949	173	5,6
2,44982	172	2,45154	171	2,45325	170	2,45495	170	2,45665	171	5,7
2,46682	168	2,46850	168	2,47018	168	2,47186	167	2,47353	168	5,8
2,48353	165	2,48518	166	2,48684	165	2,48849	164	2,49013	165	5,9
2,49997	163	2,50160	162	2,50322	163	2,50485	162	2,50647	162	6,0
2,51614	161	2,51775	160	2,51935	160	2,52095	159	2,52254	160	6,1
2,53207	158	2,53365	157	2,53522	158	2,53680	157	2,53837	157	6,2
2,54775	155	2,54930	155	2,55085	155	2,55240	155	2,55395	154	6,3
2,56318	153	2,56471	153	2,56624	153	2,56777	152	2,56929	152	6,4
2,57839	151	2,57990	150	2,58140	151	2,58291	150	2,58441	150	6,5
2,59337	149	2,59486	148	2,59634	148	2,59782	148	2,59930	148	6,6
2,60813	147	2,60960	146	2,61106	146	2,61252	146	2,61398	145	6,7
2,62268	145	2,62413	144	2,62557	144	2,62701	143	2,62844	144	6,8
2,63702	143	2,63845	142	2,63987	142	2,64129	142	2,64271	141	6,9
2,65117	140	2,65257	140	2,65397	140	2,65537	140	2,65677	139	7,0
2,66511	139	2,66650	138	2,66788	138	2,66926	138	2,67064	138	7,1
2,67887	137	2,68024	136	2,68160	136	2,68296	136	2,68432	136	7,2
2,69244	135	2,69379	135	2,69514	134	2,69648	134	2,69782	134	7,3
2,70584	132	2,70716	133	2,70849	133	2,70982	132	2,71114	133	7,4
2,71905	131	2,72036	131	2,72167	131	2,72298	131	2,72429	131	7,5
2,73210	129	2,73339	130	2,73469	129	2,73598	129	2,73727	129	7,6
2,74498	128	2,74626	127	2,74753	128	2,74881	127	2,75008	127	7,7
2,75769	127	2,75896	126	2,76022	126	2,76148	126	2,76274	125	7,8
2,77025	125	2,77150	124	2,77274	125	2,77399	124	2,77523	124	7,9
2,78266	123	2,78389	123	2,78512	123	2,78635	122	2,78757	123	8,0
2,79491	121	2,79612	122	2,79734	121	2,79855	122	2,79977	121	8,1
2,80701	121	2,80822	120	2,80942	120	2,81062	120	2,81182	119	8,2
2,81898	118	2,82016	119	2,82135	119	2,82254	118	2,82372	118	8,3
2,83080	117	2,83197	117	2,83314	118	2,83432	117	2,83549	117	8,4
2,84248	116	2,84364	116	2,84480	116	2,84596	116	2,84712	115	8,5
2,85403	115	2,85518	115	2,85633	114	2,85747	114	2,85861	115	8,6
2,86545	114	2,86659	113	2,86772	113	2,86885	113	2,86998	113	8,7
2,87674	112	2,87786	112	2,87898	112	2,88010	112	2,88122	112	8,8
2,88791	111	2,88902	111	2,89013	110	2,89123	111	2,89234	110	8,9
2,89895	110	2,90005	109	2,90114	110	2,90224	109	2,90333	110	9,0
2,90987	109	2,91096	108	2,91204	109	2,91313	108	2,91421	108	9,1
2,92068	107	2,92175	108	2,92283	107	2,92390	107	2,92497	107	9,2
2,93137	106	2,93243	107	2,93350	106	2,93456	106	2,93562	105	9,3
2,94195	105	2,94300	105	2,94405	105	2,94510	105	2,94615	105	9,4
2,95242	104	2,95346	104	2,95450	104	2,95554	104	2,95658	103	9,5
2,96278	103	2,96381	103	2,96484	103	2,96587	102	2,96689	103	9,6
2,97303	102	2,97405	102	2,97507	102	2,97609	102	2,97711	101	9,7
2,98319	100	2,98419	101	2,98520	101	2,98621	101	2,98722	100	9,8
2,99324	099	2,99423	100	2,99523	100	2,99623	100	2,99723	099	9,9

$\operatorname{ar} \sinh x$

The error of the approximations given on pp. 236 and 237 is not greater than 0,000005 and the error of an approximation obtained by linear interpolation is less than 0,00000548 + the error of rounding off the result.

An example of interpolation is given on p. 241.

XIII. Funkcja $\operatorname{ar} \sinh x$, czyli $\sinh^{-1} x$

x	0	δ	1	δ	2	δ	3	δ	4	δ
10,0	2,99822	100	2,99922	099	3,00021	099	3,00120	100	3,00220	099
10,1	3,00812	099	3,00911	098	3,01009	099	3,01108	098	3,01206	098
10,2	3,01793	097	3,01890	098	3,01988	097	3,02085	097	3,02182	098
10,3	3,02764	097	3,02861	096	3,02957	096	3,03053	097	3,03150	096
10,4	3,03726	095	3,03821	096	3,03917	095	3,04012	096	3,04108	095
10,5	3,04678	095	3,04773	095	3,04868	094	3,04962	095	3,05057	094
10,6	3,05622	094	3,05716	094	3,05810	093	3,05903	094	3,05997	093
10,7	3,06557	093	3,06650	093	3,06743	093	3,06836	092	3,06928	093
10,8	3,07483	092	3,07575	092	3,07667	092	3,07759	092	3,07851	092
10,9	3,08401	091	3,08492	091	3,08583	091	3,08674	092	3,08766	091
11,0	3,09310	091	3,09401	090	3,09491	090	3,09581	091	3,09672	090
11,1	3,10212	089	3,10301	090	3,10391	089	3,10480	090	3,10570	089
11,2	3,11105	089	3,11194	089	3,11283	088	3,11371	089	3,11460	088
11,3	3,11990	088	3,12078	088	3,12166	088	3,12254	088	3,12342	088
11,4	3,12868	087	3,12955	087	3,13042	088	3,13130	087	3,13217	087
11,5	3,13738	087	3,13825	086	3,13911	086	3,13997	087	3,14084	086
11,6	3,14601	085	3,14686	086	3,14772	086	3,14858	085	3,14943	086
11,7	3,15456	085	3,15541	085	3,15626	085	3,15711	085	3,15796	085
11,8	3,16304	084	3,16388	084	3,16472	085	3,16557	084	3,16641	084
11,9	3,17145	083	3,17228	084	3,17312	084	3,17396	083	3,17479	083
12,0	3,17979	083	3,18062	082	3,18144	083	3,18227	083	3,18310	083
12,1	3,18806	082	3,18888	082	3,18970	082	3,19052	082	3,19134	083
12,2	3,19626	082	3,19708	081	3,19789	082	3,19871	081	3,19952	081
12,3	3,20439	081	3,20520	081	3,20601	081	3,20682	081	3,20763	081
12,4	3,21247	080	3,21327	080	3,21407	080	3,21487	081	3,21568	080
12,5	3,22047	080	3,22127	080	3,22207	079	3,22286	080	3,22366	079
12,6	3,22841	080	3,22921	079	3,23000	079	3,23079	078	3,23157	079
12,7	3,23630	078	3,23708	078	3,23786	079	3,23865	078	3,23943	078
12,8	3,24411	078	3,24489	078	3,24567	078	3,24645	078	3,24723	077
12,9	3,25187	078	3,25265	077	3,25342	077	3,25419	077	3,25496	077
13,0	3,25957	077	3,26034	077	3,26111	076	3,26187	077	3,26264	076
13,1	3,26721	076	3,26797	076	3,26873	076	3,26949	076	3,27025	076
13,2	3,27480	075	3,27555	076	3,27631	075	3,27706	075	3,27781	076
13,3	3,28232	075	3,28307	075	3,28382	075	3,28457	075	3,28532	074
13,4	3,28979	075	3,29054	074	3,29128	074	3,29202	074	3,29276	075
13,5	3,29721	073	3,29794	074	3,29868	074	3,29942	074	3,30016	073
13,6	3,30457	073	3,30530	073	3,30603	073	3,30676	073	3,30749	074
13,7	3,31187	073	3,31260	073	3,31333	072	3,31405	073	3,31478	073
13,8	3,31913	072	3,31985	072	3,32057	072	3,32129	072	3,32201	072
13,9	3,32633	071	3,32704	072	3,32776	072	3,32848	071	3,32919	072
14,0	3,33348	071	3,33419	071	3,33490	071	3,33561	071	3,33632	071
14,1	3,34058	070	3,34128	071	3,34199	071	3,34270	070	3,34340	071
14,2	3,34763	070	3,34833	070	3,34903	070	3,34973	070	3,35043	070
14,3	3,35463	069	3,35532	070	3,35602	070	3,35672	069	3,35741	070
14,4	3,36158	069	3,36227	069	3,36296	070	3,36366	069	3,36435	069
14,5	3,36848	069	3,36917	069	3,36986	068	3,37054	069	3,37123	069
14,6	3,37534	068	3,37602	069	3,37671	068	3,37739	068	3,37807	068
14,7	3,38215	068	3,38283	068	3,38351	067	3,38418	068	3,38486	068
14,8	3,38891	068	3,38959	067	3,39026	067	3,39093	068	3,39161	067
14,9	3,39563	067	3,39630	067	3,39697	067	3,39764	067	3,39831	067

Błąd przybliżeń podanych na str. 238 i 239 jest nie większy niż 0,000005, a błąd przybliżenia otrzymanego przez interpolację liniową jest mniejszy niż 0,00000513 + błąd zaokrąglenia wyniku.

Przykład interpolacji na str. 240.

XIII. Function $\operatorname{ar} \sinh x$, i. e. $\sinh^{-1} x$

5	δ	6	δ	7	δ	8	δ	9	δ	x
3,00319	099	3,00418	098	3,00516	099	3,00615	099	3,00714	098	10,0
3,01304	098	3,01402	098	3,01500	098	3,01598	097	3,01695	098	10,1
3,02280	097	3,02377	097	3,02474	096	3,02570	097	3,02667	097	10,2
3,03246	096	3,03342	096	3,03438	096	3,03534	096	3,03630	096	10,3
3,04203	095	3,04298	095	3,04393	095	3,04488	095	3,04583	095	10,4
3,05151	094	3,05245	095	3,05340	094	3,05434	094	3,05528	094	10,5
3,06090	094	3,06184	093	3,06277	093	3,06370	094	3,06464	093	10,6
3,07021	093	3,07114	092	3,07206	092	3,07298	093	3,07391	092	10,7
3,07943	092	3,08035	091	3,08126	092	3,08218	091	3,08309	092	10,8
3,08857	090	3,08947	091	3,09038	091	3,09129	091	3,09220	090	10,9
3,09762	090	3,09852	090	3,09942	090	3,10032	090	3,10122	090	11,0
3,10659	089	3,10748	090	3,10838	089	3,10927	089	3,11016	089	11,1
3,11548	089	3,11637	088	3,11725	089	3,11814	088	3,11902	088	11,2
3,12430	088	3,12518	087	3,12605	088	3,12693	087	3,12780	088	11,3
3,13304	087	3,13391	087	3,13478	087	3,13565	086	3,13651	087	11,4
3,14170	086	3,14256	087	3,14343	086	3,14429	086	3,14515	086	11,5
3,15029	086	3,15115	085	3,15200	085	3,15285	086	3,15371	085	11,6
3,15881	084	3,15965	085	3,16050	085	3,16135	084	3,16219	085	11,7
3,16725	084	3,16809	084	3,16893	084	3,16977	084	3,17061	084	11,8
3,17562	084	3,17646	083	3,17729	083	3,17812	083	3,17895	084	11,9
3,18393	083	3,18476	082	3,18558	083	3,18641	082	3,18723	083	12,0
3,19217	082	3,19299	081	3,19380	082	3,19462	082	3,19544	082	12,1
3,20033	082	3,20115	081	3,20196	081	3,20277	081	3,20358	081	12,2
3,20844	081	3,20925	080	3,21005	081	3,21086	080	3,21166	081	12,3
3,21648	080	3,21728	080	3,21808	080	3,21888	079	3,21967	080	12,4
3,22445	080	3,22525	079	3,22604	079	3,22683	079	3,22762	079	12,5
3,23236	079	3,23315	079	3,23394	078	3,23472	079	3,23551	079	12,6
3,24021	078	3,24099	079	3,24178	078	3,24256	078	3,24334	077	12,7
3,24800	078	3,24878	077	3,24955	078	3,25033	077	3,25110	077	12,8
3,25573	077	3,25650	077	3,25727	077	3,25804	077	3,25881	076	12,9
3,26340	076	3,26416	077	3,26493	076	3,26569	076	3,26645	076	13,0
3,27101	076	3,27177	076	3,27253	075	3,27328	076	3,27404	076	13,1
3,27857	075	3,27932	075	3,28007	075	3,28082	075	3,28157	075	13,2
3,28606	075	3,28681	075	3,28756	074	3,28830	075	3,28905	074	13,3
3,29351	074	3,29425	074	3,29499	074	3,29573	074	3,29647	074	13,4
3,30089	074	3,30163	073	3,30236	074	3,30310	073	3,30383	074	13,5
3,30823	073	3,30896	073	3,30969	073	3,31042	072	3,31114	073	13,6
3,31551	072	3,31623	073	3,31696	072	3,31768	072	3,31840	073	13,7
3,32273	072	3,32345	072	3,32417	072	3,32489	072	3,32561	072	13,8
3,32991	071	3,33062	072	3,33134	071	3,33205	071	3,33276	072	13,9
3,33703	071	3,33774	071	3,33845	071	3,33916	071	3,33987	071	14,0
3,34411	070	3,34481	071	3,34552	070	3,34622	070	3,34692	071	14,1
3,35113	070	3,35183	070	3,35253	070	3,35323	070	3,35393	070	14,2
3,35811	069	3,35880	070	3,35950	069	3,36019	070	3,36089	069	14,3
3,36504	069	3,36573	069	3,36642	069	3,36711	068	3,36779	069	14,4
3,37192	068	3,37260	069	3,37329	068	3,37397	069	3,37466	068	14,5
3,37875	068	3,37943	068	3,38011	068	3,38079	068	3,38147	068	14,6
3,38554	067	3,38621	068	3,38689	067	3,38756	068	3,38824	067	14,7
3,39228	067	3,39295	067	3,39362	067	3,39429	067	3,39496	067	14,8
3,39898	066	3,39964	067	3,40031	067	3,40098	066	3,40164	067	14,9

$\operatorname{ar} \sinh x$

The error of the approximations given on pp. 238 and 239 is not greater than 0,000005 and the error of an approximation obtained by linear interpolation is less than 0,00000513 + the error of rounding off the result.

An example of interpolation is given on p. 241.

XIII. Funkcja $\operatorname{ar} \sinh x$, czyli $\sinh^{-1} x$

x	0	δ	1	δ	2	δ	3	δ	4	δ
15,0	3,40231	066	3,40297	067	3,40364	066	3,40430	066	3,40496	067
15,1	3,40894	066	3,40960	066	3,41026	066	3,41092	066	3,41158	066
15,2	3,41552	066	3,41618	066	3,41684	065	3,41749	066	3,41815	065
15,3	3,42207	065	3,42272	065	3,42337	065	3,42402	065	3,42467	065
15,4	3,42857	064	3,42921	065	3,42986	065	3,43051	065	3,43116	064
15,5	3,43503	064	3,43567	064	3,43631	065	3,43696	064	3,43760	064
15,6	3,44144	064	3,44208	064	3,44272	064	3,44336	064	3,44400	064
15,7	3,44782	064	3,44846	063	3,44909	064	3,44973	063	3,45036	063
15,8	3,45416	063	3,45479	063	3,45542	063	3,45605	063	3,45668	063
15,9	3,46045	063	3,46108	063	3,46171	063	3,46234	062	3,46296	063
16,	3,46671	622	3,47293	618	3,47911	614	3,48525	611	3,49136	606
17,	3,52722	586	3,53308	582	3,53890	579	3,54469	575	3,55044	572
18,	3,58429	553	3,58982	550	3,59532	547	3,60079	545	3,60624	541
19,	3,63828	524	3,64352	521	3,64873	519	3,65392	516	3,65908	514
20,	3,68950	499	3,69449	495	3,69944	493	3,70437	491	3,70928	489
21,	3,73824	474	3,74298	472	3,74770	470	3,75240	468	3,75708	466
22,	3,78471	453	3,78924	451	3,79375	449	3,79824	447	3,80271	445
23,	3,82911	434	3,83345	431	3,83776	430	3,84206	428	3,84634	426
24,	3,87163	416	3,87579	414	3,87993	412	3,88405	410	3,88815	409
25,	3,91242	399	3,91641	397	3,92038	396	3,92434	394	3,92828	393
26,	3,95161	384	3,95545	382	3,95927	381	3,96308	379	3,96687	378
27,	3,98933	369	3,99302	368	3,99670	367	4,00037	365	4,00402	364
28,	4,02567	356	4,02923	355	4,03278	354	4,03632	353	4,03985	351
29,	4,06074	344	4,06418	343	4,06761	342	4,07103	340	4,07443	339
30,	4,09462	333	4,09795	331	4,10126	331	4,10457	329	4,10786	328
31,	4,12739	322	4,13061	321	4,13382	320	4,13702	319	4,14021	318
32,	4,15913	312	4,16225	310	4,16535	310	4,16845	309	4,17154	308
33,	4,18988	303	4,19291	301	4,19592	301	4,19893	300	4,20193	299
34,	4,21972	294	4,22266	293	4,22559	291	4,22850	291	4,23141	291
35,	4,24870	285	4,25155	284	4,25439	284	4,25723	283	4,26006	282
36,	4,27686	277	4,27963	277	4,28240	275	4,28515	275	4,28790	275
37,	4,30425	270	4,30695	269	4,30964	268	4,31232	268	4,31500	267
38,	4,33091	262	4,33353	262	4,33615	262	4,33877	260	4,34137	260
39,	4,35687	256	4,35943	256	4,36199	254	4,36453	254	4,36707	254
40,	4,38218	250	4,38468	249	4,38717	248	4,38965	248	4,39213	247
41,	4,40687	243	4,40930	243	4,41173	243	4,41416	241	4,41657	242
42,	4,43096	238	4,43334	237	4,43571	236	4,43807	236	4,44043	236
43,	4,45448	232	4,45680	232	4,45912	231	4,46143	231	4,46374	230
44,	4,47747	227	4,47974	226	4,48200	226	4,48426	225	4,48651	225
45,	4,49993	222	4,50215	222	4,50437	221	4,50658	220	4,50878	220
46,	4,52191	217	4,52408	216	4,52624	217	4,52841	215	4,53056	216
47,	4,54341	212	4,54553	212	4,54765	212	4,54977	211	4,55188	211
48,	4,56446	208	4,56654	207	4,56861	208	4,57069	206	4,57275	207
49,	4,58507	204	4,58711	203	4,58914	203	4,59117	203	4,59320	202

Błąd przybliżeń podanych na str. 240 i 241 jest nie większy niż 0,000005. Błąd przybliżenia otrzymanego przez interpolację liniową jest dla $15,00 < x < 16,00$ mniejszy niż 0,00000506 + błąd zaokrąglenia wyniku, dla $16,00 < x < 25,0$ mniejszy niż 0,00000912 + błąd zaokrąglenia wyniku, a dla $25,0 < x < 50,0$ mniejszy niż 0,00000700 + błąd zaokrąglenia wyniku.

Przykład. Obliczmy $\operatorname{ar} \sinh 27,8419$. W tablicy odczytujemy, że $\operatorname{ar} \sinh 27,8 \approx 4,01851$ i $\delta = 358$. W tablicy poprawek odczytujemy dla $\delta = 358$ i cyfr 4, 1 i 9 liczby 143,2, 35,8 i 322,2. Zatem $\operatorname{ar} \sinh 27,8419 \approx 4,01851 + 0,001432 + 0,0000358 + 0,00003222 = 4,02001002$ z dokładnością do 0,00000700, czyli $\operatorname{ar} \sinh 27,8419 \approx 4,02001$ z dokładnością do 0,0000071. (Poprawkę do liczby $\operatorname{ar} \sinh 27,8 \approx 4,01851$ można również obliczyć mnożąc $0,419 \cdot \delta = 0,419 \cdot 358 = 150,002$. Mamy wtedy $\operatorname{ar} \sinh 27,8419 \approx 4,01851 + 0,00150002 = 4,02001002$, jak poprzednio).

XIII. Function $\operatorname{ar} \sinh x$, i. e. $\sinh^{-1} x$

5	δ	6	δ	7	δ	8	δ	9	δ	x
3,40563	066	3,40629	066	3,40695	066	3,40761	067	3,40828	066	15,0
3,41224	065	3,41289	066	3,41355	066	3,41421	066	3,41487	065	15,1
3,41880	065	3,41945	066	3,42011	065	3,42076	065	3,42141	066	15,2
3,42532	065	3,42597	065	3,42662	065	3,42727	065	3,42792	065	15,3
3,43180	065	3,43245	064	3,43309	065	3,43374	064	3,43438	065	15,4
3,43824	064	3,43888	064	3,43952	064	3,44016	064	3,44080	064	15,5
3,44464	063	3,44527	064	3,44591	064	3,44655	063	3,44718	064	15,6
3,45099	064	3,45163	063	3,45226	063	3,45289	064	3,45353	063	15,7
3,45731	063	3,45794	063	3,45857	063	3,45920	063	3,45983	062	15,8
3,46359	062	3,46421	063	3,46484	062	3,46546	063	3,46609	062	15,9
3,49742	604	3,50346	599	3,50945	596	3,51541	592	3,52133	589	16,
3,55616	569	3,56185	566	3,56751	562	3,57313	560	3,57873	556	17,
3,61165	538	3,61703	535	3,62238	533	3,62771	530	3,63301	527	18,
3,66422	511	3,66933	508	3,67441	506	3,67947	503	3,68450	500	19,
3,71417	486	3,71903	483	3,72386	482	3,72868	479	3,73347	477	20,
3,76174	464	3,76638	461	3,77099	459	3,77558	457	3,78015	456	21,
3,80716	443	3,81159	441	3,81600	439	3,82039	437	3,82476	435	22,
3,85060	424	3,85484	423	3,85907	420	3,86327	419	3,86746	417	23,
3,89224	407	3,89631	405	3,90036	404	3,90440	402	3,90842	400	24,
3,93221	391	3,93612	390	3,94002	388	3,94390	386	3,94776	385	25,
3,97065	376	3,97441	375	3,97816	374	3,98190	372	3,98562	371	26,
4,00766	363	4,01129	362	4,01491	360	4,01851	358	4,02209	358	27,
4,04336	350	4,04686	349	4,05035	347	4,05382	347	4,05729	345	28,
4,07782	339	4,08121	337	4,08458	336	4,08794	335	4,09129	333	29,
4,11114	327	4,11441	326	4,11767	326	4,12093	324	4,12417	322	30,
4,14339	316	4,14655	316	4,14971	315	4,15286	314	4,15600	313	31,
4,17462	307	4,17769	307	4,18076	305	4,18381	304	4,18685	303	32,
4,20492	297	4,20789	298	4,21087	296	4,21383	295	4,21678	294	33,
4,23432	289	4,23721	288	4,24009	288	4,24297	287	4,24584	286	34,
4,26288	281	4,26569	280	4,26849	280	4,27129	279	4,27408	278	35,
4,29065	273	4,29338	273	4,29611	272	4,29883	271	4,30154	271	36,
4,31767	266	4,32033	265	4,32298	265	4,32563	264	4,32827	264	37,
4,34397	260	4,34657	258	4,34915	258	4,35173	258	4,35431	256	38,
4,36961	253	4,37214	252	4,37466	251	4,37717	251	4,37968	250	39,
4,39460	247	4,39707	246	4,39953	245	4,40198	245	4,40443	244	40,
4,41899	240	4,42139	240	4,42379	240	4,42619	239	4,42858	238	41,
4,44279	235	4,44514	234	4,44748	234	4,44982	233	4,45215	233	42,
4,46604	230	4,46834	229	4,47063	228	4,47291	228	4,47519	228	43,
4,48876	225	4,49101	224	4,49325	223	4,49548	223	4,49771	222	44,
4,51098	220	4,51318	219	4,51537	218	4,51755	218	4,51973	218	45,
4,53272	214	4,53486	215	4,53701	213	4,53914	214	4,54128	213	46,
4,55399	210	4,55609	210	4,55819	209	4,56028	209	4,56237	209	47,
4,57482	206	4,57688	205	4,57893	205	4,58098	205	4,58303	204	48,
4,59522	202	4,59724	201	4,59925	201	4,60126	201	4,60327	200	49,

$\operatorname{ar} \sinh x$

The error of the approximations given on pp. 240 and 241 is not greater than 0,000005. The error of an approximation obtained by linear interpolation is for $15,00 < x < 16,00$ less than 0,00000506 + the error of rounding off the result, for $16,0 < x < 25,0$ less than 0,00000912 + the error of rounding off the result and for $25,0 < x < 50,0$ less than 0,00000700 + the error of rounding off the result.

Example. To find $\operatorname{ar} \sinh 27,8419$ we read in the table that $\operatorname{ar} \sinh 27,8 \approx 4,01851$ and $\delta = 358$. In the table of proportional parts we find for $\delta = 358$ and the figures 4, 1 and 9 the numbers 143,2 and 35,8 and 322,2. Consequently $\operatorname{ar} \sinh 27,8419 \approx 4,01851 + 0,001432 + 0,0000358 + 0,00003222 = 4,02001002$ with error less than 0,00000700, i. e. $\operatorname{ar} \sinh 27,8419 \approx 4,02001$ with error less than 0,0000071. (The correction for the number $\operatorname{ar} \sinh 27,8 \approx 4,01851$ can also be found by multiplying $0,419 \cdot \delta = 0,419 \cdot 358 = 150,002$. We then have $\operatorname{ar} \sinh 27,8419 \approx 4,01851 + 0,00150002 = 4,02001002$ as before).

XIII. Funkcja $\operatorname{ar} \sinh x$, czyli $\sinh^{-1} x$

x	0	δ	1	δ	2	δ	3	δ	4	δ
50,	4,60527	200	4,60727	199	4,60926	199	4,61125	199	4,61324	198
51,	4,62507	196	4,62703	195	4,62898	195	4,63093	195	4,63288	194
52,	4,64448	192	4,64640	192	4,64832	191	4,65023	191	4,65214	191
53,	4,66353	188	4,66541	188	4,66729	188	4,66917	188	4,67105	187
54,	4,68222	185	4,68407	184	4,68591	185	4,68776	184	4,68960	183
55,	4,70056	182	4,70238	181	4,70419	181	4,70600	181	4,70781	180
56,	4,71858	178	4,72036	178	4,72214	178	4,72392	177	4,72569	178
57,	4,73628	175	4,73803	175	4,73978	174	4,74152	175	4,74327	174
58,	4,75366	173	4,75539	172	4,75711	171	4,75882	172	4,76054	171
59,	4,77076	169	4,77245	169	4,77414	169	4,77583	168	4,77751	168
60,	4,78756	167	4,78923	166	4,79089	166	4,79255	165	4,79420	166
61,	4,80409	164	4,80573	163	4,80736	163	4,80899	163	4,81062	163
62,	4,82035	161	4,82196	161	4,82357	160	4,82517	161	4,82678	160
63,	4,83634	159	4,83793	158	4,83951	158	4,84109	158	4,84267	158
64,	4,85209	156	4,85365	156	4,85521	156	4,85677	155	4,85832	155
65,	4,86759	154	4,86913	154	4,87067	153	4,87220	153	4,87373	153
66,	4,88286	151	4,88437	151	4,88588	151	4,88739	151	4,88890	151
67,	4,89790	149	4,89939	149	4,90088	148	4,90236	149	4,90385	148
68,	4,91271	147	4,91418	147	4,91565	146	4,91711	146	4,91857	146
69,	4,92731	144	4,92875	145	4,93020	144	4,93164	145	4,93309	144
70,	4,94169	143	4,94312	143	4,94455	142	4,94597	142	4,94739	142
71,	4,95588	140	4,95728	141	4,95869	140	4,96009	140	4,96149	140
72,	4,96986	139	4,97125	139	4,97264	138	4,97402	138	4,97540	138
73,	4,98365	137	4,98502	137	4,98639	136	4,98775	137	4,98912	136
74,	4,99726	135	4,99861	135	4,99996	134	5,00130	135	5,00265	134
75,	5,01068	133	5,01201	133	5,01334	133	5,01467	133	5,01600	132
76,	5,02392	132	5,02524	131	5,02655	131	5,02786	131	5,02917	131
77,	5,03699	130	5,03829	130	5,03959	129	5,04088	130	5,04218	129
78,	5,04990	128	5,05118	128	5,05246	128	5,05374	127	5,05501	128
79,	5,06264	126	5,06390	126	5,06516	127	5,06643	126	5,06769	125
80,	5,07521	125	5,07646	125	5,07771	125	5,07896	124	5,08020	124
81,	5,08763	124	5,08887	123	5,09010	123	5,09133	123	5,09256	123
82,	5,09990	122	5,10112	122	5,10234	122	5,10356	121	5,10477	121
83,	5,11202	121	5,11323	120	5,11443	120	5,11563	120	5,11683	120
84,	5,12400	119	5,12519	119	5,12638	118	5,12756	119	5,12875	118
85,	5,13583	118	5,13701	117	5,13818	118	5,13936	117	5,14053	117
86,	5,14753	116	5,14869	116	5,14985	116	5,15101	116	5,15217	116
87,	5,15909	115	5,16024	114	5,16138	115	5,16253	115	5,16368	114
88,	5,17052	113	5,17165	114	5,17279	113	5,17392	113	5,17505	113
89,	5,18182	112	5,18294	112	5,18406	112	5,18518	112	5,18630	112
90,	5,19299	111	5,19410	111	5,19521	111	5,19632	110	5,19742	111
91,	5,20404	110	5,20514	109	5,20623	110	5,20733	109	5,20842	110
92,	5,21497	108	5,21605	109	5,21714	108	5,21822	108	5,21930	109
93,	5,22578	107	5,22685	107	5,22792	108	5,22900	107	5,23007	107
94,	5,23647	106	5,23753	107	5,23860	106	5,23966	106	5,24072	106
95,	5,24705	105	5,24810	105	5,24915	105	5,25020	105	5,25125	105
96,	5,25752	104	5,25856	104	5,25960	104	5,26064	104	5,26168	104
97,	5,26788	104	5,26892	102	5,26994	103	5,27097	103	5,27200	103
98,	5,27814	102	5,27916	102	5,28018	102	5,28120	101	5,28221	102
99,	5,28829	101	5,28930	101	5,29031	101	5,29132	100	5,29232	101

Błąd przybliżeń podanych na str. 242 i 243 jest nie większy niż 0,000005, a błąd przybliżenia otrzymanego przez interpolację liniową jest mniejszy niż 0,00000550 + błąd zaokrąglenia wyniku.

Przykład interpolacji na str. 240.

XIII. Function $\operatorname{ar} \sinh x$, i. e. $\sinh^{-1} x$

5	δ	6	δ	7	δ	8	δ	9	δ	x
4,61522	198	4,61720	197	4,61917	197	4,62114	197	4,62311	196	50,
4,63482	194	4,63676	194	4,63870	193	4,64063	193	4,64256	192	51,
4,65405	190	4,65595	190	4,65785	190	4,65975	189	4,66164	189	52,
4,67292	186	4,67478	187	4,67665	186	4,67851	185	4,68036	186	53,
4,69143	183	4,69326	183	4,69509	183	4,69692	182	4,69874	182	54,
4,70961	180	4,71141	180	4,71321	179	4,71500	179	4,71679	179	55,
4,72747	176	4,72923	177	4,73100	176	4,73276	176	4,73452	176	56,
4,74501	174	4,74675	173	4,74848	173	4,75021	173	4,75194	172	57,
4,76225	170	4,76395	171	4,76566	170	4,76736	170	4,76906	170	58,
4,77919	168	4,78087	168	4,78255	167	4,78422	167	4,78589	167	59,
4,79586	165	4,79751	165	4,79916	164	4,80080	165	4,80245	164	60,
4,81225	162	4,81387	163	4,81550	162	4,81712	161	4,81873	162	61,
4,82838	160	4,82998	159	4,83157	160	4,83317	159	4,83476	158	62,
4,84425	157	4,84582	157	4,84739	157	4,84896	157	4,85053	156	63,
4,85987	155	4,86142	155	4,86297	154	4,86451	154	4,86605	154	64,
4,87526	152	4,87678	152	4,87830	152	4,87982	152	4,88134	152	65,
4,89041	150	4,89191	150	4,89341	150	4,89491	149	4,89640	150	66,
4,90533	148	4,90681	148	4,90829	147	4,90976	148	4,91124	147	67,
4,92003	146	4,92149	146	4,92295	145	4,92440	146	4,92586	145	68,
4,93453	143	4,93596	144	4,93740	143	4,93883	143	4,94026	143	69,
4,94881	142	4,95023	141	4,95164	142	4,95306	141	4,95447	141	70,
4,96289	140	4,96429	140	4,96569	139	4,96708	139	4,96847	139	71,
4,97678	138	4,97816	138	4,97954	137	4,98091	137	4,98228	137	72,
4,99048	136	4,99184	136	4,99320	135	4,99455	136	4,99591	135	73,
5,00399	134	5,00533	134	5,00667	134	5,00801	134	5,00935	133	74,
5,01732	133	5,01865	132	5,01997	132	5,02129	132	5,02261	131	75,
5,03048	131	5,03179	130	5,03309	130	5,03439	131	5,03570	129	76,
5,04347	129	5,04476	128	5,04604	129	5,04733	128	5,04861	129	77,
5,05629	127	5,05756	127	5,05883	127	5,06010	127	5,06137	127	78,
5,06894	126	5,07020	126	5,07146	125	5,07271	125	5,07396	125	79,
5,08144	124	5,08268	124	5,08392	124	5,08516	124	5,08640	123	80,
5,09379	122	5,09501	123	5,09624	122	5,09746	122	5,09868	122	81,
5,10598	121	5,10719	121	5,10840	121	5,10961	121	5,11082	120	82,
5,11803	120	5,11923	119	5,12042	120	5,12162	119	5,12281	119	83,
5,12993	119	5,13112	118	5,13230	118	5,13348	118	5,13466	117	84,
5,14170	117	5,14287	116	5,14403	117	5,14520	116	5,14636	117	85,
5,15333	115	5,15448	115	5,15563	116	5,15679	115	5,15794	115	86,
5,16482	114	5,16596	114	5,16710	114	5,16824	114	5,16938	114	87,
5,17618	113	5,17731	113	5,17844	113	5,17957	112	5,18069	113	88,
5,18742	111	5,18853	112	5,18965	111	5,19076	112	5,19188	111	89,
5,19853	110	5,19963	110	5,20073	111	5,20184	110	5,20294	110	90,
5,20952	109	5,21061	109	5,21170	109	5,21279	109	5,21388	109	91,
5,22039	108	5,22147	107	5,22254	108	5,22362	108	5,22470	108	92,
5,23114	107	5,23221	106	5,23327	107	5,23434	107	5,23541	106	93,
5,24178	105	5,24283	106	5,24389	105	5,24494	106	5,24600	105	94,
5,25230	105	5,25335	104	5,25439	105	5,25544	104	5,25648	104	95,
5,26272	103	5,26375	104	5,26479	103	5,26582	103	5,26685	103	96,
5,27303	102	5,27405	102	5,27507	103	5,27610	102	5,27712	102	97,
5,28323	101	5,28424	102	5,28526	101	5,28627	101	5,28728	101	98,
5,29333	100	5,29433	101	5,29534	100	5,29634	100	5,29734	100	99,

$\operatorname{ar} \sinh x$

The error of the approximations given on pp. 242 and 243 is not greater than 0,000005 and the error of an approximation obtained by linear interpolation is less than 0,00000550 + the error of rounding off the result.

An example of interpolation is given on p. 241.

XIII. Funkcja $\operatorname{ar} \sinh x$, czyli $\sinh^{-1} x$

x	0	δ	1	δ	2	δ	3	δ	4	δ
100,	5,29834	100	5,29934	100	5,30034	100	5,30134	099	5,30233	100
101,	5,30829	099	5,30928	099	5,31027	099	5,31126	098	5,31224	099
102,	5,31814	098	5,31912	098	5,32010	098	5,32108	098	5,32206	097
103,	5,32790	097	5,32887	097	5,32984	097	5,33081	097	5,33178	096
104,	5,33756	096	5,33852	096	5,33948	096	5,34044	096	5,34140	096
105,	5,34713	095	5,34808	095	5,34903	095	5,34998	095	5,35093	095
106,	5,35661	094	5,35755	094	5,35849	094	5,35943	094	5,36037	094
107,	5,36600	093	5,36693	094	5,36787	093	5,36880	093	5,36973	093
108,	5,37530	093	5,37623	092	5,37715	092	5,37807	093	5,37900	092
109,	5,38452	091	5,38543	092	5,38635	091	5,38726	092	5,38818	091
110,	5,39365	091	5,39456	090	5,39546	091	5,39637	091	5,39728	090
111,	5,40270	090	5,40360	090	5,40450	090	5,40540	089	5,40629	090
112,	5,41167	089	5,41256	089	5,41345	089	5,41434	089	5,41523	089
113,	5,42055	089	5,42144	088	5,42232	089	5,42321	088	5,42409	088
114,	5,42936	088	5,43024	088	5,43112	087	5,43199	088	5,43287	087
115,	5,43810	087	5,43897	087	5,43984	086	5,44070	087	5,44157	087
116,	5,44676	086	5,44762	086	5,44848	086	5,44934	086	5,45020	086
117,	5,45534	085	5,45619	086	5,45705	085	5,45790	085	5,45875	085
118,	5,46385	085	5,46470	084	5,46554	085	5,46639	084	5,46723	085
119,	5,47229	084	5,47313	084	5,47397	084	5,47481	083	5,47564	084
120,	5,48066	083	5,48149	083	5,48232	083	5,48315	083	5,48398	083
121,	5,48895	083	5,48978	083	5,49061	082	5,49143	083	5,49226	082
122,	5,49719	081	5,49800	082	5,49882	082	5,49964	082	5,50046	081
123,	5,50535	081	5,50616	081	5,50697	081	5,50778	081	5,50859	081
124,	5,51345	080	5,51425	081	5,51506	080	5,51586	081	5,51667	080
125,	5,52148	080	5,52228	080	5,52308	079	5,52387	080	5,52467	080
126,	5,52944	080	5,53024	079	5,53103	079	5,53182	079	5,53261	080
127,	5,53735	079	5,53814	078	5,53892	079	5,53971	078	5,54049	079
128,	5,54519	078	5,54597	078	5,54675	078	5,54753	078	5,54831	078
129,	5,55297	078	5,55375	077	5,55452	078	5,55530	077	5,55607	077
130,	5,56070	077	5,56147	076	5,56223	077	5,56300	077	5,56377	077
131,	5,56836	076	5,56912	076	5,56988	077	5,57065	076	5,57141	076
132,	5,57596	076	5,57672	076	5,57748	075	5,57823	076	5,57899	075
133,	5,58351	075	5,58426	075	5,58501	075	5,58576	075	5,58651	075
134,	5,59100	075	5,59175	074	5,59249	075	5,59324	074	5,59398	075
135,	5,59844	074	5,59918	074	5,59992	074	5,60066	073	5,60139	074
136,	5,60582	073	5,60655	074	5,60729	073	5,60802	073	5,60875	074
137,	5,61314	073	5,61387	073	5,61460	073	5,61533	073	5,61606	072
138,	5,62041	073	5,62114	072	5,62186	073	5,62259	072	5,62331	072
139,	5,62763	072	5,62835	072	5,62907	072	5,62979	072	5,63051	071
140,	5,63480	072	5,63552	071	5,63623	071	5,63694	072	5,63766	071
141,	5,64192	071	5,64263	071	5,64334	070	5,64404	071	5,64475	071
142,	5,64899	070	5,64969	070	5,65039	071	5,65110	070	5,65180	070
143,	5,65600	070	5,65670	070	5,65740	070	5,65810	070	5,65880	069
144,	5,66297	070	5,66367	069	5,66436	069	5,66505	070	5,66575	069
145,	5,66989	069	5,67058	069	5,67127	069	5,67196	069	5,67265	069
146,	5,67677	068	5,67745	068	5,67813	069	5,67882	068	5,67950	068
147,	5,68359	068	5,68427	068	5,68495	068	5,68563	068	5,68631	068
148,	5,69037	068	5,69105	067	5,69172	068	5,69240	067	5,69307	067
149,	5,69710	068	5,69778	067	5,69845	067	5,69912	067	5,69979	066

Błąd przybliżeń podanych na str. 244 i 245 jest nie większy niż 0,000005, a błąd przybliżenia otrzymanego przez interpolację liniową jest mniejszy niż 0,00000513 + błąd zaokrąglenia wyniku.

Przykład interpolacji na str. 240.

XIII. Function $\operatorname{ar} \sinh x$, i. e. $\sinh^{-1} x$

5	δ	6	δ	7	δ	8	δ	9	δ	x
5,30333	099	5,30432	100	5,30532	099	5,30631	099	5,30730	099	100,
5,31323	098	5,31421	099	5,31520	098	5,31618	098	5,31716	098	101,
5,32303	098	5,32401	097	5,32498	098	5,32596	097	5,32693	097	102,
5,33274	097	5,33371	096	5,33467	097	5,33564	096	5,33660	096	103,
5,34236	095	5,34331	096	5,34427	095	5,34522	096	5,34618	095	104,
5,35188	095	5,35283	094	5,35377	095	5,35472	094	5,35566	095	105,
5,36131	094	5,36225	094	5,36319	094	5,36413	093	5,36506	094	106,
5,37066	093	5,37159	093	5,37252	093	5,37345	092	5,37437	093	107,
5,37992	092	5,38084	092	5,38176	092	5,38268	092	5,38360	092	108,
5,38909	092	5,39001	091	5,39092	091	5,39183	091	5,39274	091	109,
5,39818	091	5,39909	090	5,39999	090	5,40089	091	5,40180	090	110,
5,40719	090	5,40809	089	5,40898	090	5,40988	089	5,41077	090	111,
5,41612	089	5,41701	089	5,41790	088	5,41878	089	5,41967	088	112,
5,42497	088	5,42585	088	5,42673	088	5,42761	088	5,42849	087	113,
5,43374	087	5,43461	088	5,43549	087	5,43636	087	5,43723	087	114,
5,44244	086	5,44330	087	5,44417	086	5,44503	086	5,44589	087	115,
5,45106	085	5,45191	086	5,45277	086	5,45363	085	5,45448	086	116,
5,45960	085	5,46045	085	5,46130	085	5,46215	085	5,46300	085	117,
5,46808	084	5,46892	084	5,46976	085	5,47061	084	5,47145	084	118,
5,47648	084	5,47732	083	5,47815	084	5,47899	083	5,47982	084	119,
5,48481	083	5,48564	083	5,48647	083	5,48730	083	5,48813	082	120,
5,49308	082	5,49390	082	5,49472	082	5,49554	083	5,49637	082	121,
5,50127	082	5,50209	082	5,50291	081	5,50372	081	5,50453	082	122,
5,50940	081	5,51021	081	5,51102	081	5,51183	081	5,51264	081	123,
5,51747	080	5,51827	080	5,51907	081	5,51988	080	5,52068	080	124,
5,52547	080	5,52627	079	5,52706	080	5,52786	079	5,52865	079	125,
5,53341	079	5,53420	078	5,53498	079	5,53577	079	5,53656	079	126,
5,54128	078	5,54206	079	5,54285	078	5,54363	078	5,54441	078	127,
5,54909	078	5,54987	078	5,55065	077	5,55142	078	5,55220	077	128,
5,55684	077	5,55761	078	5,55839	077	5,55916	077	5,55993	077	129,
5,56454	076	5,56530	077	5,56607	076	5,56683	077	5,56760	076	130,
5,57217	076	5,57293	076	5,57369	076	5,57445	076	5,57521	075	131,
5,57974	076	5,58050	075	5,58125	076	5,58201	075	5,58276	075	132,
5,58726	075	5,58801	075	5,58876	075	5,58951	074	5,59025	075	133,
5,59473	074	5,59547	074	5,59621	074	5,59695	074	5,59769	075	134,
5,60213	074	5,60287	074	5,60361	073	5,60434	074	5,60508	074	135,
5,60949	073	5,61022	073	5,61095	073	5,61168	073	5,61241	073	136,
5,61678	073	5,61751	073	5,61824	072	5,61896	073	5,61969	072	137,
5,62403	072	5,62475	072	5,62547	072	5,62619	072	5,62691	072	138,
5,63122	072	5,63194	072	5,63266	071	5,63337	072	5,63409	071	139,
5,63837	071	5,63908	071	5,63979	071	5,64050	071	5,64121	071	140,
5,64546	071	5,64617	070	5,64687	071	5,64758	070	5,64828	071	141,
5,65250	070	5,65320	070	5,65390	070	5,65460	070	5,65530	070	142,
5,65949	070	5,66019	070	5,66089	069	5,66158	070	5,66228	069	143,
5,66644	069	5,66713	069	5,66782	069	5,66851	069	5,66920	069	144,
5,67334	068	5,67402	069	5,67471	068	5,67539	069	5,67608	069	145,
5,68018	069	5,68087	068	5,68155	068	5,68223	068	5,68291	068	146,
5,68699	067	5,68766	068	5,68834	068	5,68902	067	5,68969	068	147,
5,69374	068	5,69442	067	5,69509	067	5,69576	067	5,69643	067	148,
5,70045	067	5,70112	067	5,70179	067	5,70246	067	5,70313	066	149,

$\operatorname{ar} \sinh x$

The error of the approximations given on pp. 244 and 245 is not greater than 0,000005 and the error of an approximation obtained by linear interpolation is less than 0,00000513 + the error of rounding off the result.

An example of interpolation is given on p. 241.

XIII. Funkcja $\operatorname{ar} \sinh x$, czyli $\sinh^{-1} x$

x	0	δ	1	δ	2	δ	3	δ	4	δ
150,	5,70379	067	5,70446	067	5,70513	066	5,70579	067	5,70646	066
151,	5,71044	066	5,71110	066	5,71176	066	5,71242	066	5,71308	066
152,	5,71704	066	5,71770	065	5,71835	066	5,71901	066	5,71967	065
153,	5,72360	065	5,72425	065	5,72490	065	5,72555	066	5,72621	065
154,	5,73011	065	5,73076	065	5,73141	065	5,73206	064	5,73270	065
<hr/>										
155,	5,73658	065	5,73723	064	5,73787	065	5,73852	064	5,73916	064
156,	5,74301	064	5,74365	064	5,74429	064	5,74493	064	5,74557	064
157,	5,74940	064	5,75004	064	5,75068	063	5,75131	064	5,75195	063
158,	5,75575	063	5,75638	064	5,75702	063	5,75765	063	5,75828	063
159,	5,76206	063	5,76269	063	5,76332	063	5,76395	062	5,76457	063
<hr/>										
16	5,76833	623	5,77456	619	5,78075	616	5,78691	611	5,79302	608
17	5,82895	587	5,83482	583	5,84065	580	5,84645	576	5,85221	573
18	5,88611	554	5,89165	551	5,89716	548	5,90264	545	5,90809	542
19	5,94018	525	5,94543	522	5,95065	519	5,95584	517	5,96101	514
<hr/>										
20	5,99147	499	5,99646	496	6,00142	494	6,00636	491	6,01127	489
21	6,04026	475	6,04501	473	6,04974	470	6,05444	469	6,05913	466
22	6,08678	454	6,09132	451	6,09583	449	6,10032	448	6,10480	445
23	6,13123	434	6,13557	432	6,13989	430	6,14419	428	6,14847	427
24	6,17379	416	6,17795	414	6,18209	412	6,18621	411	6,19032	409
<hr/>										
25	6,21461	399	6,21860	398	6,22258	396	6,22654	395	6,23049	392
26	6,25383	384	6,25767	383	6,26150	380	6,26530	380	6,26910	378
27	6,29157	370	6,29527	368	6,29895	367	6,30262	366	6,30628	364
28	6,32794	357	6,33151	355	6,33506	354	6,33860	352	6,34212	352
29	6,36303	344	6,36647	343	6,36990	342	6,37332	341	6,37673	340
<hr/>										
30	6,39693	333	6,40026	332	6,40358	330	6,40688	330	6,41018	328
31	6,42972	322	6,43294	321	6,43615	320	6,43935	319	6,44254	318
32	6,46147	312	6,46459	311	6,46770	310	6,47080	309	6,47389	308
33	6,49224	303	6,49527	301	6,49828	301	6,50129	300	6,50429	299
34	6,52209	294	6,52503	293	6,52796	292	6,53088	291	6,53379	290
<hr/>										
35	6,55108	286	6,55394	284	6,55678	284	6,55962	283	6,56245	282
36	6,57925	278	6,58203	276	6,58479	276	6,58755	275	6,59030	275
37	6,60665	270	6,60935	269	6,61204	269	6,61473	267	6,61740	267
38	6,63332	263	6,63595	262	6,63857	261	6,64118	261	6,64379	260
39	6,65930	256	6,66186	255	6,66441	255	6,66696	254	6,66950	253
<hr/>										
40	6,68461	250	6,68711	249	6,68960	249	6,69209	247	6,69456	248
41	6,70931	243	6,71174	243	6,71417	243	6,71660	241	6,71901	242
42	6,73340	238	6,73578	237	6,73815	237	6,74052	236	6,74288	236
43	6,75693	233	6,75926	231	6,76157	232	6,76389	230	6,76619	230
44	6,77992	227	6,78219	227	6,78446	226	6,78672	225	6,78897	225
<hr/>										
45	6,80240	222	6,80462	221	6,80683	221	6,80904	221	6,81125	220
46	6,82437	218	6,82655	216	6,82871	217	6,83088	215	6,83303	216
47	6,84588	213	6,84801	212	6,85013	211	6,85224	212	6,85436	210
48	6,86693	209	6,86902	207	6,87109	207	6,87316	207	6,87523	207
49	6,88755	204	6,88959	204	6,89163	203	6,89366	202	6,89568	203
50	6,90776									

Dla $x > 500$ korzystamy ze wzoru

$$\operatorname{ar} \sinh x = \log_e 2x + 0,0000005$$

z dokładnością do 0,0000005.

Błąd przybliżeń podanych na str. 246 i 247 jest nie większy niż 0,000005. Błąd przybliżenia otrzymanego przez interpolację liniową jest dla $150,0 < x < 160,0$ mniejszy niż $0,00000506 + \text{błąd zaokrąglenia wyniku}$, dla $160 < x < 250$ mniejszy niż $0,00000890 + \text{błąd zaokrąglenia wyniku}$, a dla $250 < x < 500$ mniejszy niż $0,00000700 + \text{błąd zaokrąglenia wyniku}$.

Przykład interpolacji na str. 240.

XIII. Function $\operatorname{ar} \sinh x$, i. e. $\sinh^{-1} x$

5	δ	6	δ	7	δ	8	δ	9	δ	x
5,70712	067	5,70779	066	5,70845	066	5,70911	067	5,70978	066	150,
5,71374	066	5,71440	066	5,71506	066	5,71572	066	5,71638	066	151,
5,72032	066	5,72098	065	5,72163	066	5,72229	065	5,72294	066	152,
5,72686	065	5,72751	065	5,72816	065	5,72881	065	5,72946	065	153,
5,73335	065	5,73400	065	5,73465	064	5,73529	065	5,73594	064	154,
5,73980	065	5,74045	064	5,74109	064	5,74173	064	5,74237	064	155,
5,74621	064	5,74685	064	5,74749	064	5,74813	064	5,74877	063	156,
5,75258	064	5,75322	063	5,75385	064	5,75449	063	5,75512	063	157,
5,75891	063	5,75954	063	5,76017	063	5,76080	063	5,76143	063	158,
5,76520	063	5,76583	062	5,76645	063	5,76708	063	5,76771	062	159,
5,79910	604	5,80514	601	5,81115	597	5,81712	593	5,82305	590	16
5,85794	570	5,86364	566	5,86930	564	5,87494	560	5,88054	557	17
5,91351	539	5,91890	536	5,92426	534	5,92960	530	5,93490	528	18
5,96615	512	5,97127	509	5,97636	506	5,98142	504	5,98646	501	19
6,01616	487	6,02103	484	6,02587	482	6,03069	480	6,03549	477	20
6,06379	464	6,06843	462	6,07305	460	6,07765	457	6,08222	456	21
6,10925	444	6,11369	441	6,11810	440	6,12250	437	6,12687	436	22
6,15274	424	6,15698	423	6,16121	421	6,16542	420	6,16962	417	23
6,19441	407	6,19848	406	6,20254	404	6,20658	402	6,21060	401	24
6,23441	392	6,23833	390	6,24223	388	6,24611	387	6,24998	385	25
6,27288	377	6,27665	375	6,28040	374	6,28414	372	6,28786	371	26
6,30992	363	6,31355	362	6,31717	360	6,32077	359	6,32436	358	27
6,34564	350	6,34914	349	6,35263	348	6,35611	347	6,35958	345	28
6,38013	338	6,38351	337	6,38688	336	6,39024	335	6,39359	334	29
6,41346	327	6,41673	327	6,42000	325	6,42325	324	6,42649	323	30
6,44572	317	6,44889	316	6,45205	315	6,45520	314	6,45834	313	31
6,47697	308	6,48005	306	6,48311	305	6,48616	305	6,48921	303	32
6,50728	298	6,51026	297	6,51323	297	6,51620	295	6,51915	294	33
6,53669	290	6,53959	288	6,54247	288	6,54535	287	6,54822	286	34
6,56527	281	6,56808	280	6,57088	280	6,57368	279	6,57647	278	35
6,59305	273	6,59578	273	6,59851	272	6,60123	272	6,60395	270	36
6,62007	267	6,62274	265	6,62539	265	6,62804	265	6,63069	263	37
6,64639	260	6,64899	258	6,65157	258	6,65415	258	6,65673	257	38
6,67203	253	6,67456	253	6,67709	251	6,67960	251	6,68211	250	39
6,69704	246	6,69950	246	6,70196	246	6,70442	244	6,70686	245	40
6,72143	240	6,72383	240	6,72623	240	6,72863	239	6,73102	238	41
6,74524	235	6,74759	234	6,74993	234	6,75227	234	6,75461	232	42
6,76849	230	6,77079	229	6,77308	229	6,77537	228	6,77765	227	43
6,79122	225	6,79347	224	6,79571	223	6,79794	223	6,80017	223	44
6,81345	219	6,81564	219	6,81783	219	6,82002	218	6,82220	217	45
6,83519	214	6,83733	215	6,83948	214	6,84162	213	6,84375	213	46
6,85646	211	6,85857	209	6,86066	210	6,86276	209	6,86485	208	47
6,87730	206	6,87936	205	6,88141	205	6,88346	205	6,88551	204	48
6,89771	201	6,89972	202	6,90174	201	6,90375	200	6,90575	201	49

$\operatorname{ar} \sinh x$

For $x > 500$ we use the formula

$$\operatorname{ar} \sinh x \approx \log_e 2x + 0,0000005$$

with error less than 0,0000005.

The error of the approximations given on pp. 246 and 247 is not greater than 0,000005. The error of an approximation obtained by linear interpolation is for $150,0 < x < 160,0$ less than 0,00000506 + the error of rounding off the result, for $160 < x < 250$ less than 0,00000890 + the error of rounding off the result and for $250 < x < 500$ less than 0,00000700 + the error of rounding off the result.

An example of interpolation is given on p. 241.