

WZORY — FORMULAE

Wzory redukcyjne — Reduction formulae

$$\begin{aligned}
 e^{a+x} &= e^a \cdot e^x, & a^x &= e^{x \log_e a} = 10^{x \log_{10} a} \quad (a > 0), \\
 \log(ax) &= \log a + \log x, & \log_a x &= \frac{\log_e x}{\log_e a} = \frac{\log_{10} x}{\log_{10} a} \quad (a > 0, a \neq 1, x > 0), \\
 \sinh x &= \frac{1}{2} (e^x - e^{-x}), & \cosh x &= \frac{1}{2} (e^x + e^{-x}), \\
 \operatorname{tgh} x &= \frac{\sinh x}{\cosh x}, & \operatorname{ctgh} x &= \frac{1}{\operatorname{tgh} x} = \frac{\cosh x}{\sinh x}, \\
 \cosh^2 x - \sinh^2 x &= 1, \\
 \sinh(a+x) &= \sinh a \cosh x + \cosh a \sinh x = \sinh a + 2 \sinh \frac{x}{2} \cosh \left(a + \frac{x}{2}\right), \\
 \cosh(a+x) &= \cosh a \cosh x + \sinh a \sinh x = \cosh a + 2 \sinh \frac{x}{2} \sinh \left(a + \frac{x}{2}\right), \\
 \operatorname{tgh}(a+x) &= \frac{\operatorname{tgh} a + \operatorname{tgh} x}{1 + \operatorname{tgh} a \operatorname{tgh} x} = \operatorname{tgh} a + \frac{\sinh x}{\cosh a \cosh(a+x)}, \\
 \operatorname{ctgh}(a+x) &= \frac{1 + \operatorname{ctgh} a \operatorname{ctgh} x}{\operatorname{ctgh} a + \operatorname{ctgh} x} = \operatorname{ctgh} a - \frac{\sinh x}{\sinh a \sinh(a+x)}, \\
 \sinh 2x &= 2 \sinh x \cosh x, & \cosh 2x &= 2 \cosh^2 x - 1, \\
 \sinh 3x &= 3 \sinh x + 4 \sinh^3 x, & \cosh 3x &= 4 \cosh^3 x - 3 \cosh x, \\
 \sinh 4x &= \cosh x (4 \sinh x + 8 \sinh^3 x), & \cosh 4x &= 8 \cosh^4 x - 8 \cosh^2 x + 1, \\
 \sinh 5x &= 5 \sinh x + 20 \sinh^3 x + 16 \sinh^5 x, & \cosh 5x &= 16 \cosh^5 x - 20 \cosh^3 x + 5 \cosh x, \\
 \operatorname{ar} \sinh x &= \log_e (x + \sqrt{x^2 + 1}), \\
 \operatorname{ar} \cosh x &= \log_e (x + \sqrt{x^2 - 1}) \quad (x > 1), \\
 \operatorname{ar} \operatorname{tgh} x &= \log_e \sqrt{\frac{1+x}{1-x}} = \frac{1}{2} (\log_e (1+x) - \log_e (1-x)) \quad (-1 < x < 1), \\
 \operatorname{ar} \operatorname{ctgh} x &= \log_e \sqrt{\frac{x+1}{x-1}} = \frac{1}{2} (\log_e |x+1| - \log_e |x-1|) \quad (|x| > 1), \\
 \operatorname{ar} \sinh x &= \operatorname{sgn} x \cdot \operatorname{ar} \cosh \sqrt{x^2 + 1} = \operatorname{ar} \operatorname{tgh} \frac{x}{\sqrt{x^2 + 1}}, & \operatorname{sgn} x &= \begin{cases} 1 & \text{dla (for) } x > 0, \\ 0 & \text{dla (for) } x = 0, \\ -1 & \text{dla (for) } x < 0. \end{cases} \\
 \operatorname{ar} \cosh x &= \operatorname{ar} \sinh \sqrt{x^2 - 1} = \operatorname{ar} \operatorname{tgh} \frac{\sqrt{x^2 - 1}}{x} \quad (x \geq 1), \\
 \operatorname{ar} \operatorname{tgh} x &= \operatorname{ar} \sinh \frac{x}{\sqrt{1-x^2}} = \operatorname{sgn} x \cdot \operatorname{ar} \cosh \frac{x}{\sqrt{1-x^2}} = \operatorname{ar} \operatorname{ctgh} \frac{1}{x} \quad (|x| < 1), \\
 \operatorname{ar} \operatorname{ctgh} x &= \operatorname{ar} \sinh \frac{1}{\sqrt{x^2 - 1}} = \operatorname{sgn} x \cdot \operatorname{ar} \cosh \frac{x}{\sqrt{x^2 - 1}} = \operatorname{ar} \operatorname{tgh} \frac{1}{x} \quad (|x| > 1),
 \end{aligned}$$

Wzory redukcyjne — Reduction formulae

$$\begin{aligned} \operatorname{ar sinh}(a+x) &= \operatorname{ar sinh} a + \operatorname{ar sinh}((a+x)\sqrt{1+a^2} - a\sqrt{1+(a+x)^2}), \\ \operatorname{ar cosh}(a+x) &= \operatorname{ar cosh} a + \operatorname{ar cosh}(a(a+x) - \sqrt{(a^2-1)[(a+x)^2-1]}) \quad (a > 1, a+x > 1), \end{aligned}$$

$$\operatorname{ar tgh}(a+x) = \operatorname{ar tgh} a + \operatorname{ar tgh} \frac{x}{1-a(a+x)} \quad (|a| < 1, |a+x| < 1),$$

$$\operatorname{ar ctgh}(a+x) = \operatorname{ar ctgh} a - \operatorname{ar ctgh} \frac{a(a+x)-1}{x} \quad (|a| > 1, |a+x| > 1),$$

$$\cos^2 x + \sin^2 x = 1,$$

$$\operatorname{tg} x = \frac{\sin x}{\cos x},$$

$$\operatorname{ctg} x = \frac{1}{\operatorname{tg} x} = \frac{\cos x}{\sin x},$$

$$\sin(a+x) = \sin a \cos x + \cos a \sin x = \sin a + 2 \sin \frac{x}{2} \cos\left(a + \frac{x}{2}\right),$$

$$\cos(a+x) = \cos a \cos x - \sin a \sin x = \cos a - 2 \sin \frac{x}{2} \sin\left(a + \frac{x}{2}\right),$$

$$\operatorname{tg}(a+x) = \frac{\operatorname{tg} a + \operatorname{tg} x}{1 - \operatorname{tg} a \operatorname{tg} x} = \operatorname{tg} a + \frac{\sin x}{\cos a \cos(a+x)},$$

$$\operatorname{ctg}(a+x) = \frac{\operatorname{ctg} a \operatorname{ctg} x - 1}{\operatorname{ctg} a + \operatorname{ctg} x} = \operatorname{ctg} a - \frac{\sin x}{\sin a \sin(a+x)},$$

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1}{2}(1 - \cos x)},$$

$$\operatorname{tg} \frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x},$$

$$\cos \frac{x}{2} = \pm \sqrt{\frac{1}{2}(1 + \cos x)},$$

$$\operatorname{ctg} \frac{x}{2} = \frac{\sin x}{1 - \cos x} = \frac{1 + \cos x}{\sin x},$$

$$\sin 2x = 2 \sin x \cos x,$$

$$\sin 3x = 3 \sin x - 4 \sin^3 x,$$

$$\sin 4x = \cos x (4 \sin x - 8 \sin^3 x),$$

$$\sin 5x = 5 \sin x - 20 \sin^3 x + 16 \sin^5 x,$$

$$\cos 2x = 2 \cos^2 x - 1,$$

$$\cos 3x = 4 \cos^3 x - 3 \cos x,$$

$$\cos 4x = 8 \cos^4 x - 8 \cos^2 x + 1,$$

$$\cos 5x = 16 \cos^5 x - 20 \cos^3 x + 5 \cos x,$$

$$\operatorname{arc sin} x + \operatorname{arc cos} x = \frac{\pi}{2},$$

$$\operatorname{arc tg} x + \operatorname{arc ctg} x = \frac{\pi}{2},$$

$$\begin{aligned} \operatorname{arc sin} x &= \begin{cases} \operatorname{arc cos} \sqrt{1-x^2} & \text{dla (for) } 0 \leq x \leq 1, \\ -\operatorname{arc cos} \sqrt{1-x^2} & \text{dla (for) } -1 \leq x < 0, \end{cases} \\ &= \operatorname{arc tg} \frac{x}{\sqrt{1-x^2}} \end{aligned}$$

$$\begin{aligned} &= \begin{cases} \operatorname{arc ctg} \frac{\sqrt{1-x^2}}{x} & \text{dla (for) } 0 < x \leq 1, \\ \operatorname{arc ctg} \frac{\sqrt{1-x^2}}{x} - \pi & \text{dla (for) } -1 \leq x < 0, \end{cases} \end{aligned}$$

$$\operatorname{arc cos} x = \begin{cases} \operatorname{arc sin} \sqrt{1-x^2} & \text{dla (for) } 0 \leq x \leq 1, \\ \pi - \operatorname{arc sin} \sqrt{1-x^2} & \text{dla (for) } -1 \leq x < 0, \end{cases}$$

$$\begin{aligned} &= \begin{cases} \operatorname{arc tg} \frac{\sqrt{1-x^2}}{x} & \text{dla (for) } 0 < x \leq 1, \\ \pi + \operatorname{arc tg} \frac{\sqrt{1-x^2}}{x} & \text{dla (for) } -1 \leq x < 0, \end{cases} \end{aligned}$$

$$= \operatorname{arc ctg} \frac{x}{\sqrt{1-x^2}} \quad \text{dla (for) } x^2 < 1,$$

Wzory redukcyjne — Reduction formulae

$$\begin{aligned}
 \operatorname{arctg} x &= \arcsin \frac{x}{\sqrt{1+x^2}} \\
 &= \begin{cases} \arccos \frac{1}{\sqrt{1+x^2}} & \text{dla (for) } x \geq 0, \\ -\arccos \frac{1}{\sqrt{1+x^2}} & \text{dla (for) } x \leq 0, \end{cases} \\
 &= \begin{cases} \operatorname{arctg} \frac{1}{x} & \text{dla (for) } x > 0, \\ \operatorname{arctg} \frac{1}{x} - \pi & \text{dla (for) } x < 0, \end{cases} \\
 \operatorname{arctg} x &= \begin{cases} \arcsin \frac{1}{\sqrt{1+x^2}} & \text{dla (for) } x \geq 0, \\ \pi - \arcsin \frac{1}{\sqrt{1+x^2}} & \text{dla (for) } x \leq 0, \end{cases} \\
 &= \arccos \frac{x}{\sqrt{1+x^2}} \\
 &= \begin{cases} \operatorname{arctg} \frac{1}{x} & \text{dla (for) } x > 0, \\ \pi + \operatorname{arctg} \frac{1}{x} & \text{dla (for) } x < 0. \end{cases} \\
 \arcsin(a+x) &= \arcsin a + \begin{cases} \arcsin((a+x)\sqrt{1-a^2} - a\sqrt{1-(a+x)^2}) & \text{dla (for) } a(a+x) \geq 0 \text{ albo (or) } a^2 + (a+x)^2 \leq 1, \\ \pi - \arcsin((a+x)\sqrt{1-a^2} - a\sqrt{1-(a+x)^2}) & \text{dla (for) } a+x > 0, a < 0, a^2 + (a+x)^2 > 1, \\ -\pi - \arcsin((a+x)\sqrt{1-a^2} - a\sqrt{1-(a+x)^2}) & \text{dla (for) } a+x < 0, a > 0, a^2 + (a+x)^2 > 1, \end{cases} \\
 &= \arcsin a + \begin{cases} \arccos(\sqrt{(1-a^2)[1-(a+x)^2]} + a(a+x)) & \text{dla (for) } x > 0, \\ -\arccos(\sqrt{(1-a^2)[1-(a+x)^2]} + a(a+x)) & \text{dla (for) } x < 0, \end{cases} \\
 \arccos(a+x) &= \arccos a + \begin{cases} -\arccos(a(a+x) + \sqrt{(1-a^2)[1-(a+x)^2]}) & \text{dla (for) } x > 0, \\ \arccos(a(a+x) + \sqrt{(1-a^2)[1-(a+x)^2]}) & \text{dla (for) } x < 0, \end{cases} \\
 \operatorname{arctg}(a+x) &= \operatorname{arctg} a + \begin{cases} \operatorname{arctg} \frac{x}{1+a(a+x)} & \text{dla (for) } a(a+x) > -1, \\ \pi + \operatorname{arctg} \frac{x}{1+a(a+x)} & \text{dla (for) } x > 0, a(a+x) < -1, \\ -\pi + \operatorname{arctg} \frac{x}{1+a(a+x)} & \text{dla (for) } x < 0, a(a+x) < -1, \end{cases} \\
 \operatorname{arctg}(a+x) &= \operatorname{arctg} a + \begin{cases} -\operatorname{arctg} \frac{1+a(a+x)}{x} & \text{dla (for) } x > 0, \\ \pi - \operatorname{arctg} \frac{1+a(a+x)}{x} & \text{dla (for) } x < 0. \end{cases}
 \end{aligned}$$

Rozwinięcia w szereg potęgowy — Expansions in a power series

$$\begin{aligned}
 \frac{1}{a+x} &= \frac{1}{a} \left[1 - \frac{x}{a} + \left(\frac{x}{a}\right)^2 - \left(\frac{x}{a}\right)^3 + \dots \right], \quad -1 < \frac{x}{a} \leq 1, \\
 \sqrt{a+x} &= \sqrt{a} \left[1 + \frac{1}{2} \left(\frac{x}{a}\right) - \frac{1}{2 \cdot 4} \left(\frac{x}{a}\right)^2 + \frac{1 \cdot 3}{2 \cdot 4 \cdot 6} \left(\frac{x}{a}\right)^3 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8} \left(\frac{x}{a}\right)^4 + \dots \right], \quad -1 < \frac{x}{a} \leq 1, \\
 \frac{1}{\sqrt{a+x}} &= \frac{1}{\sqrt{a}} \left[1 - \frac{1}{2} \left(\frac{x}{a}\right) + \frac{1 \cdot 3}{2 \cdot 4} \left(\frac{x}{a}\right)^2 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \left(\frac{x}{a}\right)^3 + \dots \right], \quad -1 < \frac{x}{a} \leq 1, \\
 \sqrt[3]{a+x} &= \sqrt[3]{a} \left[1 + \frac{1}{3} \left(\frac{x}{a}\right) - \frac{2}{3 \cdot 6} \left(\frac{x}{a}\right)^2 + \frac{2 \cdot 5}{3 \cdot 6 \cdot 9} \left(\frac{x}{a}\right)^3 - \frac{2 \cdot 5 \cdot 8}{3 \cdot 6 \cdot 9 \cdot 12} \left(\frac{x}{a}\right)^4 + \dots \right], \quad -1 < \frac{x}{a} \leq 1,
 \end{aligned}$$

$$\frac{1}{\sqrt[n]{a+x}} = \frac{1}{\sqrt[n]{a}} \left[1 - \frac{1}{n} \left(\frac{x}{a} \right) + \frac{1 \cdot 4}{3 \cdot 6} \left(\frac{x}{a} \right)^2 - \frac{1 \cdot 4 \cdot 7}{3 \cdot 6 \cdot 9} \left(\frac{x}{a} \right)^3 + \dots \right], \quad -1 < \frac{x}{a} \leq 1,$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots,$$

$$\log_e(a+x) = \log_e a + \frac{x}{a} - \frac{1}{2} \left(\frac{x}{a} \right)^2 + \frac{1}{3} \left(\frac{x}{a} \right)^3 - \frac{1}{4} \left(\frac{x}{a} \right)^4 + \dots, \quad -1 < \frac{x}{a} \leq 1,$$

$$\log_e \frac{a+x}{a-x} = 2 \left[\frac{x}{a} + \frac{1}{3} \left(\frac{x}{a} \right)^3 + \frac{1}{5} \left(\frac{x}{a} \right)^5 + \dots \right], \quad \left(\frac{x}{a} \right)^2 < 1,$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots,$$

$$\operatorname{ar} \sinh x = \begin{cases} x - \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{x^5}{5} - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{x^7}{7} + \dots, & x^2 \leq 1, \\ \log_e 2x + \frac{1}{2} \cdot \frac{1}{2x^2} - \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{1}{4x^4} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{1}{6x^6} - \dots, & x \geq 1, \end{cases}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots,$$

$$\operatorname{ar} \cosh x = \begin{cases} \sqrt{2(x-1)} \left[1 - \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{x-1}{2} + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{1}{5} \cdot \left(\frac{x-1}{2} \right)^2 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{1}{7} \left(\frac{x-1}{2} \right)^3 + \dots \right], & 1 \leq x \leq 3, \\ \log_e 2x - \frac{1}{2} \cdot \frac{1}{2x^2} - \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{1}{4x^4} - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{1}{6x^6} - \dots, & x > 1, \end{cases}$$

$$\operatorname{tgh} x = x - \frac{1}{3} x^3 + \frac{2}{15} x^5 - \frac{17}{315} x^7 + \frac{62}{2835} x^9 - \frac{1382}{155925} x^{11} + \dots, \quad x^2 < \frac{\pi^2}{4},$$

$$\operatorname{ar} \operatorname{tgh} x = \begin{cases} x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots, & x^2 < 1, \\ -\frac{1}{2} \left[\ln \frac{1-x}{2} + \frac{1-x}{2} + \frac{1}{2} \left(\frac{1-x}{2} \right)^2 + \frac{1}{3} \left(\frac{1-x}{2} \right)^3 + \dots \right], & x^2 < 1, \end{cases}$$

$$\operatorname{ctgh} x = \frac{1}{x} + \frac{x}{3} - \frac{x^3}{45} + \frac{2x^5}{945} - \frac{x^7}{4725} + \frac{2x^9}{93555} - \frac{1382x^{11}}{638512875} + \dots, \quad x^2 < \pi^2,$$

$$\operatorname{ar} \operatorname{ctgh} x = \begin{cases} -\frac{1}{2} \left[\ln \frac{x-1}{2} - \frac{x-1}{2} + \frac{1}{2} \left(\frac{x-1}{2} \right)^2 - \frac{1}{3} \left(\frac{x-1}{2} \right)^3 + \dots \right], & 1 < x \leq 3, \\ \frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + \frac{1}{7x^7} + \dots, & x^2 > 1, \end{cases}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots,$$

$$\operatorname{arc} \sin x = \begin{cases} x + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{x^7}{7} + \dots, & x^2 < 1, \\ \frac{\pi}{2} - \sqrt{2(1-x)} \left[1 + \frac{1}{2} \cdot \frac{1}{3} \left(\frac{1-x}{2} \right) + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{1}{5} \left(\frac{1-x}{2} \right)^2 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{1}{7} \left(\frac{1-x}{2} \right)^3 + \right. \\ \left. + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8} \cdot \frac{1}{9} \left(\frac{1-x}{2} \right)^4 + \dots \right], & x^2 < 1, \end{cases}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots,$$

$$\operatorname{arc} \cos x = \begin{cases} \frac{\pi}{2} - x - \frac{1}{2} \cdot \frac{x^3}{3} - \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{x^5}{5} - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{x^7}{7} - \dots, & x^2 < 1, \\ \sqrt{2(1-x)} \left[1 + \frac{1}{2} \cdot \frac{1}{3} \left(\frac{1-x}{2} \right) + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{1}{5} \left(\frac{1-x}{2} \right)^2 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{1}{7} \left(\frac{1-x}{2} \right)^3 + \right. \\ \left. + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8} \cdot \frac{1}{9} \left(\frac{1-x}{2} \right)^4 + \dots \right], & x^2 < 1, \end{cases}$$

$$\begin{aligned} \operatorname{tg} x &= \begin{cases} x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \frac{62}{2835}x^9 + \frac{1382}{155925}x^{11} + \dots, & x^2 < \frac{\pi^2}{4}, \\ \frac{1}{\frac{\pi}{2} - x} - \frac{1}{3}\left(\frac{\pi}{2} - x\right) - \frac{1}{45}\left(\frac{\pi}{2} - x\right)^3 - \frac{2}{945}\left(\frac{\pi}{2} - x\right)^5 - \frac{1}{4725}\left(\frac{\pi}{2} - x\right)^7 - \\ - \frac{2}{93555}\left(\frac{\pi}{2} - x\right)^9 - \frac{1382}{638512875}\left(\frac{\pi}{2} - x\right)^{11} - \dots, & \left(\frac{\pi}{2} - x\right)^2 < \frac{\pi^2}{4}, \end{cases} \\ \operatorname{arc} \operatorname{tg} x &= \begin{cases} x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots, & x^2 \leq 1, \\ \frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \frac{1}{7x^7} - \dots, & x > 1, \\ -\frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \frac{1}{7x^7} - \dots, & x \leq -1, \end{cases} \\ \operatorname{ctg} x &= \begin{cases} \frac{1}{x} - \frac{1}{3}x - \frac{1}{45}x^3 - \frac{2x^5}{945} - \frac{x^7}{4725} - \frac{2x^9}{93555} - \frac{1382x^{11}}{638512875} - \dots, & x^2 < \frac{\pi^2}{4}, \\ \frac{\pi}{2} - x + \frac{1}{3}\left(\frac{\pi}{2} - x\right)^3 + \frac{2}{15}\left(\frac{\pi}{2} - x\right)^5 + \frac{17}{315}\left(\frac{\pi}{2} - x\right)^7 + \frac{62}{2835}\left(\frac{\pi}{2} - x\right)^9 + \\ + \frac{1382}{155925}\left(\frac{\pi}{2} - x\right)^{11} + \dots, & \left(\frac{\pi}{2} - x\right)^2 < \frac{\pi^2}{4}, \end{cases} \\ \operatorname{arc} \operatorname{ctg} x &= \begin{cases} \frac{\pi}{2} - x + \frac{x^3}{3} - \frac{x^5}{5} + \frac{x^7}{7} - \frac{x^9}{9} + \dots, & x^2 \leq 1, \\ \frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} - \frac{1}{7x^7} + \dots, & x > 1, \\ \pi + \frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} - \frac{1}{7x^7} + \dots, & x \leq -1. \end{cases} \end{aligned}$$

Błędy interpolacji funkcji $f(x)$ w przedziale $0 \leq x_0 < x < x_0 + h$
Errors of interpolation of function $f(x)$ in the interval $0 \leq x_0 < x < x_0 + h$

Funkcja Function	Błąd interpolacji Error of interpolation
$\frac{1}{x}$	$\frac{h^2}{4x_0^3},$
$\log_{10} x$	$0,054287 \frac{h^2}{x_0^2}$
10^x	$0,66274 \cdot 10^{x_0+h} \cdot h^2,$
x^3	$\frac{1}{4} h^2,$
\sqrt{x}	$\frac{h^2}{32x_0\sqrt{x_0}},$
x^3	$\frac{3}{4} (x_0 + h) h^2,$
$\sqrt[3]{x}$	$\frac{h^2}{36x_0\sqrt[3]{x_0^2}},$
e^x	$\frac{1}{8} e^{x_0+h} \cdot h^2,$
e^{-x}	$\frac{1}{8} e^{-x_0} \cdot h^2,$
$\sinh x$	$\frac{1}{8} \sinh(x_0 + h) \cdot h^2,$

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$\operatorname{ar sinh} x$	$\begin{cases} \frac{x_0 + h}{[(x_0 + h)^2 + 1]^{3/2}} \cdot \frac{h^2}{8} & \text{dla (for) } x_0 \leq \frac{\sqrt{2}}{2} - h, \\ \frac{x_0}{(x_0^2 + 1)^{3/2}} \cdot \frac{h^2}{8} & \text{dla (for) } x_0 \geq \frac{\sqrt{2}}{2}, \\ \frac{1}{36} \sqrt{3} h^2 & \text{dla (for) } \frac{\sqrt{2}}{2} - h \leq x_0 \leq \frac{\sqrt{2}}{2}, \end{cases}$
$\cosh x$	$\frac{1}{8} \cosh(x_0 + h) \cdot h^2,$
$\operatorname{ar cosh} x$	$\frac{x_0}{(x_0^2 - 1)^{3/2}} \cdot \frac{h^2}{8},$
$\operatorname{tgh} x$	$\begin{cases} \frac{1}{4} [\operatorname{tgh}(x_0 + h) - \operatorname{tgh}^3(x_0 + h)] & \text{dla (for) } x_0 \leq \operatorname{ar tgh} \frac{\sqrt{3}}{3} - h, \\ \frac{1}{4} (\operatorname{tgh} x_0 - \operatorname{tgh}^3 x_0) & \text{dla (for) } x_0 \geq \operatorname{ar tgh} \frac{\sqrt{3}}{3}, \\ \frac{1}{18} \sqrt{3} h^2 & \text{dla (for) } \operatorname{ar tgh} \frac{\sqrt{3}}{3} - h \leq x_0 \leq \operatorname{ar tgh} \frac{\sqrt{3}}{3}, \end{cases}$
$\operatorname{ar tgh} x$	$\frac{x_0 + h}{4 [1 - (x_0 + h)^2]^2} h^2,$
$\operatorname{ctgh} x$	$\frac{1}{4} (\operatorname{ctgh}^3 x_0 - \operatorname{ctgh} x_0) h^2,$
$\operatorname{ar ctgh} x$	$\frac{x_0}{4 (x_0^2 - 1)^2} h^3,$
$\sin x^\circ$	$0,0000002 + \sin(x_0^\circ + h) \cdot 0,000000010577,$
$\cos x^\circ$	$0,0000002 + \cos x_0^\circ \cdot 0,000000010577,$
$\operatorname{arc}^\circ \sin x$	$25784 \cdot \frac{x_0 + h}{[1 - (x_0 + h)^2]^{3/2}} h^2 \text{ sek},$
$\operatorname{arc}^\circ \cos x$	$25784 \cdot \frac{x_0 + h}{[1 - (x_0 + h)^2]^{3/2}} h^2 \text{ sek},$
$\sin x^r$	$\frac{1}{8} \sin(x_0^r + h) h^2,$
$\operatorname{arc}^r \sin x$	$\frac{x_0 + h}{8 [1 - (x_0 + h)^2]^{3/2}} h^2 \text{ rad},$
$\cos x^r$	$\frac{1}{8} \cos x_0^r h^2,$
$\operatorname{arc}^r \cos x$	$\frac{x_0 + h}{8 [1 - (x_0 + h)^2]^{3/2}} h^2 \text{ rad},$
$\operatorname{tg} x^\circ$	$\begin{cases} 0,000000021154 [\operatorname{tg}(x_0^\circ + h) + \operatorname{tg}^3(x_0^\circ + h)] & \text{dla (for) } h = 1', \\ 0,00000000058762 [\operatorname{tg}(x_0^\circ + h) + \operatorname{tg}^3(x_0^\circ + h)] & \text{dla (for) } h = 10'', \\ 0,000000000058762 [\operatorname{tg}(x_0^\circ + h) + \operatorname{tg}^3(x_0^\circ + h)] & \text{dla (for) } h = 1'', \end{cases}$
$\operatorname{arc}^\circ \operatorname{tg} x$	$\begin{cases} 51567 \cdot \frac{x_0 + h}{[1 + (x_0 + h)^2]^2} h^2 \text{ sek} & \text{dla (for) } x_0 \leq \frac{\sqrt{3}}{3} - h, \\ 51567 \cdot \frac{x_0}{(1 + x_0^2)^2} h^2 \text{ sek} & \text{dla (for) } x_0 \geq \frac{\sqrt{3}}{3}, \\ 9668,68 \sqrt{3} h^2 < 16747 h^2 \text{ sek} & \text{dla (for) } \frac{\sqrt{3}}{3} - h \leq x_0 \leq \frac{\sqrt{3}}{3}, \end{cases}$

$$\begin{aligned}
& \text{ctg } x^{\circ} \quad \begin{cases} 0,000000021154 (\text{ctg } x_0^{\circ} + \text{ctg}^3 x_0^{\circ}) & \text{dla (for) } h = 1', \\ 0,00000000058762 (\text{ctg } x_0^{\circ} + \text{ctg}^3 x_0^{\circ}) & \text{dla (for) } h = 10'', \\ 0,0000000000058762 (\text{ctg } x_0^{\circ} + \text{ctg}^3 x_0^{\circ}) & \text{dla (for) } h = 1'', \end{cases} \\
& \text{arc}^{\circ} \text{ctg } x \quad \begin{cases} 51567 \cdot \frac{x_0 + h}{[1 + (x_0 + h)^2]^2} h^2 \text{ sek} & \text{dla (for) } x_0 \leq \frac{\sqrt{3}}{3} - h, \\ 51567 \cdot \frac{x_0}{(1 + x_0^2)^2} h^2 \text{ sek} & \text{dla (for) } x_0 \geq \frac{\sqrt{3}}{3}, \\ 16747 h^2 \text{ sek} & \text{dla (for) } \frac{\sqrt{3}}{3} - h \leq x_0 \leq \frac{\sqrt{3}}{3}, \end{cases} \\
& \text{tg } x^{\text{r}} \quad \frac{1}{4} [\text{tg}(x_0^{\text{r}} + h) + \text{tg}^3(x_0^{\text{r}} + h)] h^2, \\
& \text{arc}^{\text{r}} \text{tg } x \quad \begin{cases} \frac{x_0 + h}{4 [1 + (x_0 + h)^2]^2} h^2 \text{ rad} & \text{dla (for) } x_0 \leq \frac{\sqrt{3}}{3} - h, \\ \frac{x_0}{4 (1 + x_0^2)^2} h^2 \text{ rad} & \text{dla (for) } x_0 \geq \frac{\sqrt{3}}{3}, \\ \frac{3}{64} \sqrt{3} h^2 \text{ rad} & \text{dla (for) } \frac{\sqrt{3}}{3} - h \leq x_0 \leq \frac{\sqrt{3}}{3}, \end{cases} \\
& \text{ctg } x^{\text{r}} \quad \frac{1}{4} (\text{ctg } x_0^{\text{r}} + \text{ctg}^3 x_0^{\text{r}}) h^2, \\
& \text{arc}^{\text{r}} \text{ctg } x \quad \begin{cases} \frac{x_0 + h}{4 [1 + (x_0 + h)^2]^2} h^2 \text{ rad} & \text{dla (for) } x_0 \leq \frac{\sqrt{3}}{3} - h, \\ \frac{x_0}{4 (1 + x_0^2)^2} h^2 \text{ rad} & \text{dla (for) } x_0 \geq \frac{\sqrt{3}}{3}, \\ \frac{3}{64} \sqrt{3} h^2 \text{ rad} & \text{dla (for) } \frac{\sqrt{3}}{3} - h \leq x_0 \leq \frac{\sqrt{3}}{3}. \end{cases}
\end{aligned}$$



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_____	$\prod p_i$
_____	$\arctg x^i$
_____	$\frac{1}{x}$
_____	$^{\circ}, ', '' \rightarrow \text{rad}$
_____	$\log_{10} x$
_____	$\text{rad} \rightarrow ^{\circ}, ', ''$
_____	10^x
_____	$\sin x^{\circ}$
_____	$\cos x^{\circ}$
_____	x^2
_____	$\arcsin x$
_____	\sqrt{x}
_____	$\arccos x$
_____	x^3
_____	$\sin x^r$
_____	$\sqrt[3]{x}$
_____	$\arcsin x^r$
_____	e^x
_____	$\cos x^r$
_____	e^{-x}
_____	$\arccos x^r$
_____	$\log_e x$
_____	$\tg x^{\circ}$
_____	$\ctg x^{\circ}$
_____	$\sinh x$
_____	$\arcsin x$
_____	$\arcsin x$
_____	$\arcsin x$
_____	$\cosh x$
_____	$\tg x^r$
_____	$\text{arccosh } x$
_____	$\arcsin x$
_____	$\tgh x$
_____	$\ctg x^r$
_____	$\text{artgh } x$
_____	$\arcsin x$
_____	$\ctgh x$
_____	const