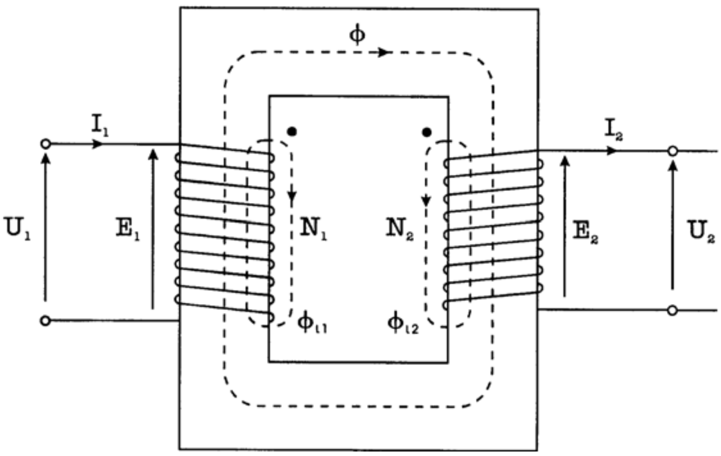
TRANSFORMERS

EQUIVALENT CIRCUIT OF TWO-WINDING TRANSFORMER



Full representation of the real transformer

Dots • show the points of higher potential.

There are applied following conventions of arrow directions:
-for primary circuit -the passive sign convention (receiver, the power is absorbed),

-for secondary circuit - the active sign convention (source, the power is released).

Features: Real transformer with two windings (primary & secondary) of different numbers of turns (usually). Sinusoidal voltage supply. The windings are characterised by their resistances R_1 and R_2. The magnetic core with power losses and with the main (mutual) flux Φ and leakage fluxes Φ_{l1} and Φ_{l2}. Primary current I_1 and secondary current I_2 flow.

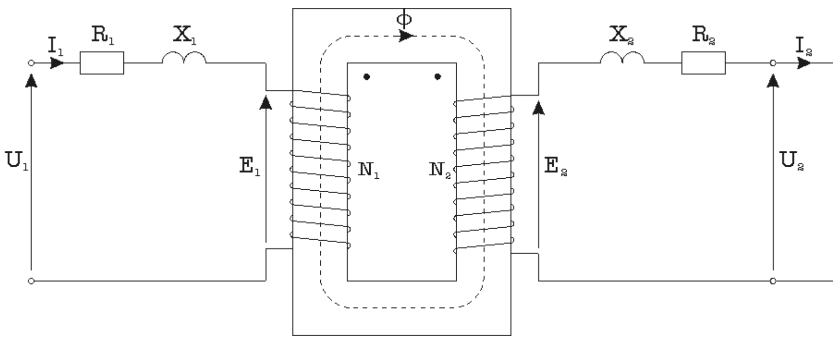
Electromotive forces due to mutual flux are induced accordingly:

$E_1 = 4.44 N_1 f \Phi$ $E_2 = 4.44 N_2 f \Phi$

Emfs due to leakage fluxes:

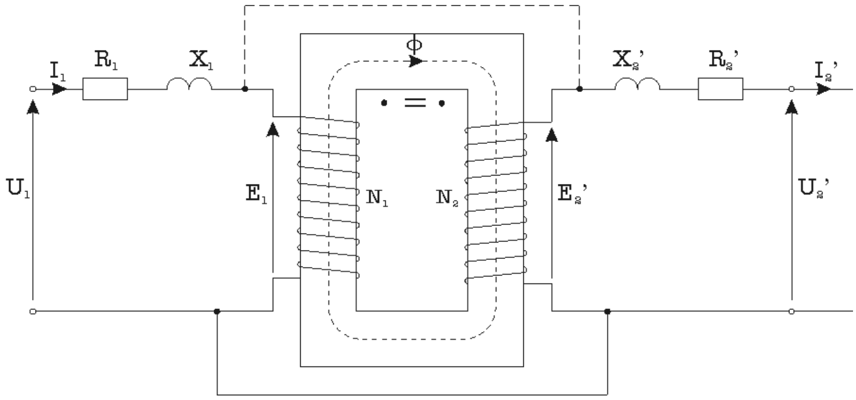
$E_{x1} = 4.44 N_1 f \Phi_{l1}$ $E_{x2} = 4.44 N_2 f \Phi_{l2}$

Leakage fluxes are directly proportional to currents I_1 and I_2 and emfs corresponding to leakage fluxes can be represented by the voltage drops at leakage reactances X_1 and X_2



Representation of the transformer by semi-ideal transformer (with no leakage fluxes and zero-resistance winding)

REFERRING SECONDARY WINDING TO PRIMARY



Secondary circuit referred (recalculated) to primary so as to have the same potential differences

Assume that we recalculate secondary emf into new value that will be called secondary emf referred to primary

$$E_2^{'} = E_2 \frac{N_1}{N_2} = 4.44 N_2 f\Phi \cdot \frac{N_1}{N_2} = 4.44 N_1 f\Phi = E_1$$

and in the same manner secondary output voltage referred to primary

$$U_2^{'} = U_2 \frac{N_1}{N_2}$$

Now the beginnings of primary and secondary windings have the same potentials ($E_2'=E_1$)

Referred secondary circuit has to represent the real quantities of energy conversion. It means that the values of power and losses cannot be changed in the new model of transformer. Therefore:

secondary current referred to primary:

$$U_2^{'} I_2^{'} = U_2 I_2 \qquad \Rightarrow \qquad I_2^{'} = I_2 \frac{N_2}{N_1}$$

secondary resistance referred to primary:

$$R_2^{'} (I_2^{'})^2 = R_2 I_2^2 \qquad \Rightarrow \qquad R_2^{'} = R_2 (\frac{N_1}{N_2})^2$$

and in the same manner secondary reactance referred to primary:

$$X_2^{'} = X_2 (\frac{N_1}{N_2})^2$$

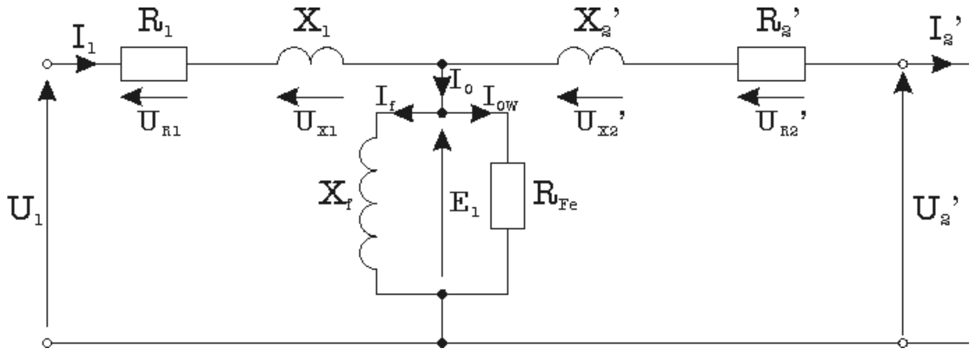
The points having the same potentials ($E_1=E_2$') can be connected without any current flow. We have now only one electrical circuit without electrical separation between primary and secondary circuits.

How we can represent (by means of electrical circuit) the semi-ideal magnetic core with two identical emfs induced in the windings and with power losses in magnetic circuit?

1. Connection of two bottom points of the circuit can be done without any consequence as they belong to two separated circuits. The only result of such connection is the equalisation of their potentials.
2. After recalculation of emfs ($E_1=E_2$') the potentials of the upper points become equal each other. Their connection doesn't lead to the flow of current between them!

All "referred" values are indicated with primes ' for example R_2' ; X_2'

EQUIVALENT (ELECTRICAL) CIRCUIT OF TWO-WINDING TRANSFORMER



I_o - no-load current

I_f - magnetizing current (reactive component of no-load current)

I_{ow} - active component of no-load current

Parallel branch of this T-type circuit represents the magnetic core of the transformer:

<u>iron-core resistance</u> R_{Fe} - the resistance having the value corresponding to power loss in the magnetic circuit, according to relation:

$$\Delta P_o = \Delta P_{Fe} = \Delta P_h + \Delta P_e = R_{Fe} I_{ow}^2$$

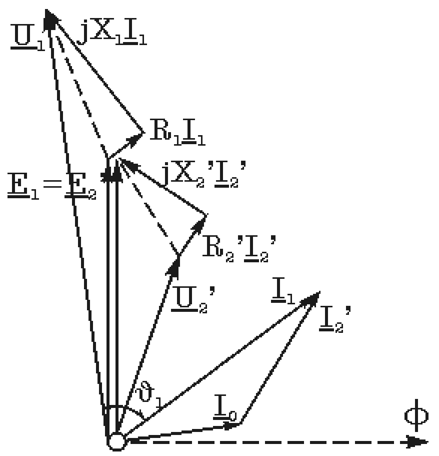
<u>magnetizing reactance</u> X_f - the reactance of primary circuit corresponding to mutual flux and representing primary emf due to relation:

$$E_1 = I_f X_f \qquad \text{or} \qquad \underline{E}_1 = \mathrm{j} X_f \underline{I}_f = \underline{E}_2^{'}$$

where:

$$\underline{I}_o = \underline{I}_f + \underline{I}_{ow} \qquad \text{or} \qquad I_o = \sqrt{I_f^2 + I_{ow}^2}$$

Phasor diagram for transformer equivalent circuit:



and voltage balance equations:

$$\boxed{\underline{U}_1 = R_1 \underline{I}_1 + \mathrm{j} X_1 \underline{I}_1 + \underline{E}_1 = \underline{I}_1 (R_1 + \mathrm{j} X_1) + \underline{I}_2^{'} (R_2^{'} + \mathrm{j} X_2^{'}) + \underline{U}_2^{'}}$$

NO-LOAD OPERATION AND NO-LOAD TEST

At no-load state the secondary circuit of the transformer is opened:

$$I_2=0; \qquad I_1=I_o$$

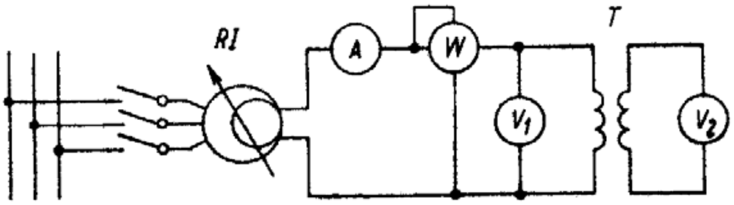
Active power supplied to the transformer at no-load state:

$$P_{1o}=\Delta P_{Cuo}+\Delta P_{Fe}=R_1 I_o^2+\Delta P_{Fe}$$

No-load loss (power loss not depending on the value of current):

$$\Delta P_o=\Delta P_{Fe}= P_{1o}- R_1 I_o^2$$

No-load test of the transformer is easy to be made in laboratory or even in a substation:



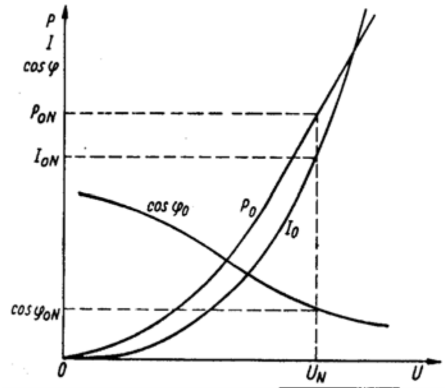
Measuring circuit diagram

RI – induction regulator (voltage source of variable output),
A – ammeter,
V – voltmeter,
W - wattmeter

During test the supplying voltage is varied from 0 up to U_N (sometimes more). The following quantities are measured:

$$U_1 \; ; \; U_2 \; ; \; I_o \; ; \; P_{1o}$$

Appropriate characteristics are drawn in the function of primary voltage:



Basic no-load characteristics

Observe the course of I_o characteristic! Doesn't it look as magnetizing curve? Try to explain why?

Observe also very interesting course of no-load power factor curve. How it can be explain from the physical meaning point of view?

Notice that no-load power factor value for rated voltage is very low!

From the no-load test results we can determine some parameters of the transformer being tested or its equivalent circuit:

$$\cos\varphi = \frac{P_{1o}}{U_1 I_o} \qquad I_{ow} = I_o \cos\varphi_o \qquad I_f = I_o \sin\varphi_o$$

$$R_{Fe} = \frac{\Delta P_o}{I_{ow}^2} \approx \frac{U_1^2}{\Delta P_o} = \frac{U_1^2}{\Delta P_{Fe}} \qquad X_f = \frac{U_1}{I_f}$$

The value of ΔP_{FeN} (iron core loss at nominal voltage) is always given in ratings of transformers or at the nameplates.

Rated no-load current (i.e. the value of no-load current at rated voltage) expressed in p.u. or in percent

$$I_{oNr} = \frac{I_{oN}}{I_N} \quad \text{or} \quad I_{oN\%} = \frac{I_{oN}}{I_N}\times 100\%$$

is rather of small value: from a few % in large power transformers to 20-30% in small transformers.

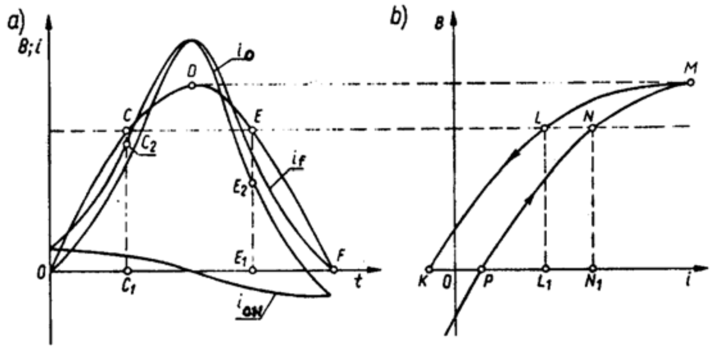
MAGNETIZING CURRENT HARMONICS

Assumption: transformer (at no-load = electromagnetic coil) is supplied with the voltage of sinusoidal waveform.
This voltage must be balanced by sinusoidal emf induced in the primary winding, hence the flux (flux density) has to be of sine waveform.

OCDEF – magnetic flux density waveform

B(t) – sinusoidal
Φ=f(i) and B=f(i) must be determined from the characteristic of electromagnetic circuit with- saturation of the core and- hysteresis phenomena taken into consideration.

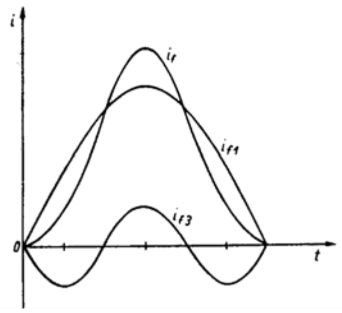
$C_1C_2=ON_1$ – from rising-up part of hysteresis loop;
$E_1E_2=OL_1$ – from falling-down part of hysteresis loop



Determination of no-load current waveform

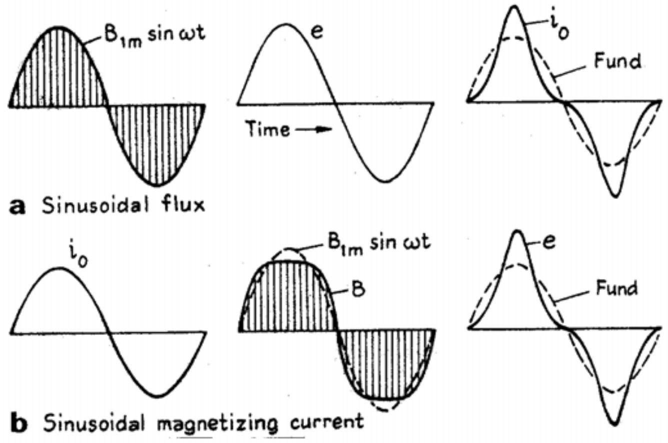
From the figure:

- i_0 is a sum of active (i_{0w}) and reactive (i_f – magnetising current) components.
- i_f is of non-sinusoidal waveform: $i_f=i_{f1}+i_{f3}+i_{f5}+i_{f7}+...$ *(odd harmonics, the 3rd one is the most important from higher order harmonics).*



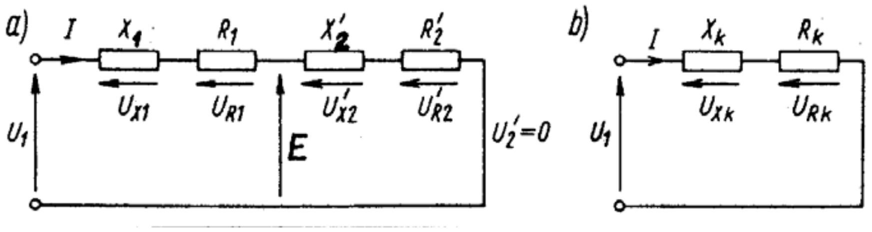
<u>Conclusion</u>: In the transformer supplied from sinusoidal voltage source the waveform of magnetizing current absorbed from the same source tends to be disturbed from the sinusoidal pattern – at least 3rd harmonic is required!

<u>Question</u>: What can happen when the source cannot provide higher harmonics of current to flow?
In such situation the flux (flux density) waveform contains 3rd and other odd harmonics, therefore emf *e* waveform is affected by higher harmonics (first of all by 3rd harmonic). See next figure:



SHORT-CIRCUIT STATE, SHORT-CIRCUIT TEST AND PARAMETERS

At short circuit the secondary terminals of transformer are short-circuited. U_2 is equal to zero and I_2 and I_1 can be very high (depending on voltage). They are much higher than I_o, therefore, after assuming $I_o \approx 0$ the equivalent circuit can be simplified:



Due to $I_o \approx 0$ the parallel branch of the circuit can be neglected. Only series resistances and reactances appear in equivalent circuit.

We define the following parameters:
<u>short-circuit resistance and reactance</u>

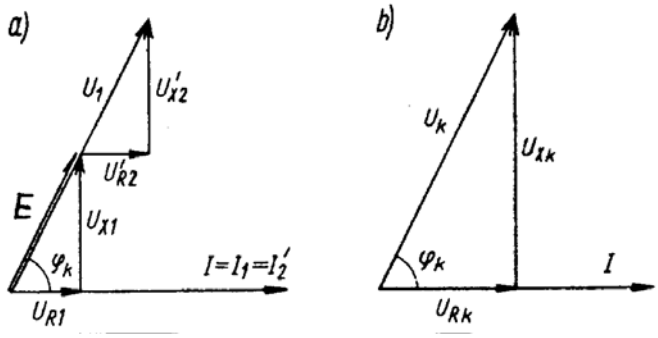
$$R_k = R_1 + R_2' \quad \text{and} \quad X_k = X_1 + X_2'$$

As X_k corresponds to leakage fluxes, the saturation effect doesn't affect its value and we can assume that X_k is constant, independently on current or voltage. It means also that Z_k=const.

<u>short-circuit impedance</u> (sometimes called *equivalent impedance of transformer)*

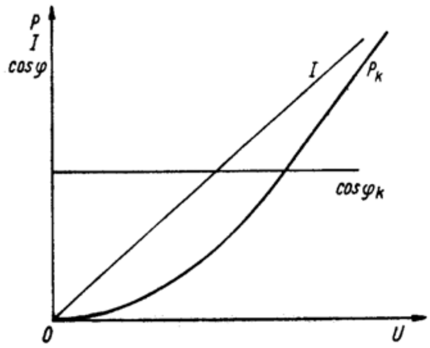
$$\underline{Z}_k = R_k + \mathrm{j}X_k \qquad Z_k = \sqrt{R_k^2 + X_k^2} \qquad (Z_k = \frac{U}{I} \quad \text{in short-circuited transf.})$$

Phasor diagram for short-circuited transformer:



can be supplemented by R_k, X_k, Z_k triangle (short-circuit impedance) triangle.

In laboratory or during field tests the short-circuit tests are very often performed. For the voltage varied from zero up to the value providing the flow of current not too much higher than I_N they are measured: P_1, I_1, $\cos\varphi$ and their characteristics drawn:



For $I_1 = I_N$ we determine and specify:

$U=U_k$ – <u>short-circuit voltage</u> (having two components: U_{Rk} and U_{Xk});
$P_1=P_k$ – <u>short-circuit power;</u>
$\cos\varphi = \cos\varphi_k$ – <u>short-circuit power factor</u>.

During test the value of $\cos\varphi_k$ can be measured directly by power factor meter or calculated from other values readings:

$$\cos\varphi = \frac{P}{S} = \frac{P}{UI}$$

Short-circuit voltage is the value of supplying voltage in short-circuited transformer that provides a flow of rated currents (I_2 and I_1). Its value is always shown in ratings of transformers or at the nameplates (usually in p.u. or in %).

Application of per unit (or relative) values:

$$I_r = \frac{I}{I_N}; \qquad U_r = \frac{U}{U_N}; \qquad P_r = \frac{P}{S_N}; \qquad Z_r = \frac{Z}{Z_N} \quad \text{where } Z_N = \frac{U_N}{I_N}$$

yields:

$$U_{kr} = \frac{U_k}{U_N} = \frac{\dfrac{U_k}{I_N}}{\dfrac{U_N}{I_N}} = \frac{Z_k}{Z_N} = Z_{kr}$$

Typical percent value of short-circuit voltage is of the following level:
- several % for small and medium power transformers,
- 10 to 20% for large power transformers (hundreds of MVA).

Short-circuit power, i.e. the power absorbed by short-circuited transformer is totally “lost” in the transformer (the output power P_2=0). The iron losses in such transformer are much smaller when compared to copper loss and can be neglected. Hence, *short-circuit power* (absorbed by the transformer with rated current) can be calculated as follows:

$$P_k = I_N^2 R_k = I_{1N}^2 + \left(I_{2N}^{'}\right)^2 R_2^{'} = \Delta P_{CuN}$$

The value of ΔP_{CuN} (rated copper loss, or – in other words – copper loss at rated load) is always shown in ratings of transformers or at the nameplates.

It is easy to prove that:

Try to prove it!

$$\Delta P_{CuN\%} = U_{Rk\%} \qquad \left(\Delta P_{CuN\%} = \frac{\Delta P_{CuN}}{S_N} 100\%\right)$$

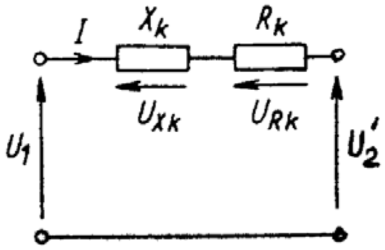
and other interesting relation concerning the value of short-circuit current flowing in short-circuited transformer supplied with rated voltage (*rated short-circuit current)*:

Practical meaning of this relation is as follows: imagine the transformer having short-circuit voltage=10%. In such transformer the short-circuit current at nominal supplying voltage is 10×I_N.

$$I_{kNr} = \frac{I_{kN}}{I_N} = \frac{\dfrac{U_N}{Z_k}}{I_N} = \frac{1}{\dfrac{Z_k I_N}{U_N}} = \frac{1}{U_{kr}}$$

TRANSFORMER UNDER LOAD

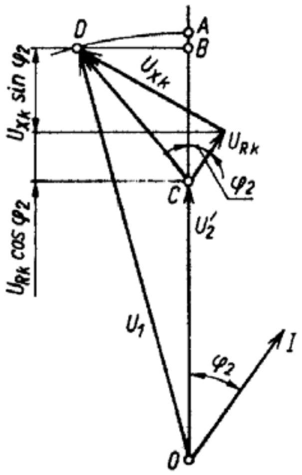
For simplified consideration of the transformer operation under load we can apply its simplified equivalent circuit.



Simplification of equivalent circuit (EC) leads to some error – much lower when I_o is smaller in comparison to I_1 or I_2 currents. Hence, we can use such EC for discussing power transformers fully loaded.

Using such model we can consider the problem of output voltage variation for different value and character of the load current. Let us consider the following problem: transformer is supplied with constant primary voltage U_1 and the loading current I_2 varies. The character of load current is given by its phase angle φ_2 varying at closed interval [-90°, +90°].
For $I_2'=I=0$ the output voltage $U'_{2o}=U_1$. For $I>0$ and some φ_2

At this figure the voltage drops triangle is drawn not to the scale. The hypotenuse DC in case of $I=I_N$ corresponds to the value of U_k and can be few percent of U_1.



we observe the voltage drop ΔU (difference in voltage values):

$$\Delta U = U_1 - U_2^{'} = U_{2o}^{'} - U_2^{'} = \overline{CA} \approx \overline{CB} = IR_k \cos\varphi_2 + IX_k \sin\varphi_2$$

$U_{2oN}'=U_{1N}$

Assuming $U_1=U_{1N}$ and expressing ΔU in per units

$$\Delta U_r = \frac{U_{2oN}^{'} - U_2^{'}}{U_{2oN}^{'}} = \frac{U_{2oN} - U_2}{U_{2oN}} = \frac{I}{I_N}\left(R_{kr}\cos\varphi_2 + X_{kr}\sin\varphi_2\right)$$

$U_{Rkr}=R_{kr}$

$U_{Xkr}=X_{kr}$

and for $I=I_N$

$$\Delta U_{rN} = R_{kr}\cos\varphi_2 + X_{kr}\sin\varphi_2 = U_{Rkr}\cos\varphi_2 + U_{Xkr}\sin\varphi_2$$

U_{2oN} is no-load output voltage when $U_1=U_{1N}$

and final expression for the value of output voltage

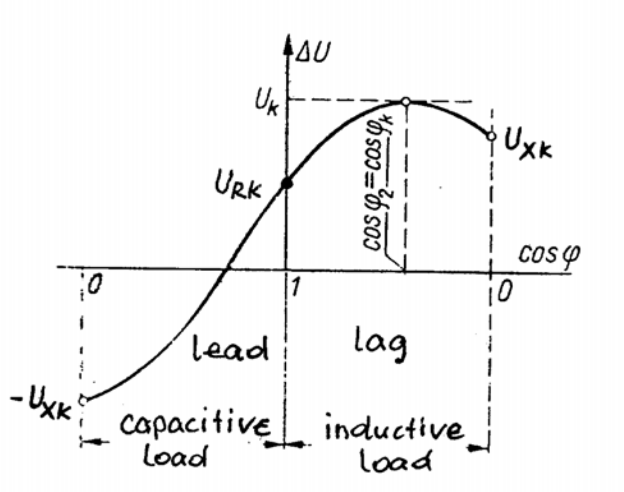
$$\boxed{U_2 = U_{2oN}\left(1 - \frac{I}{I_N}\Delta U_{rN}\right)}$$

U_{2o} is no-load output voltage for $U_1 \neq U_{1N}$

In similar manner, for any value of supply voltage U_1 and corresponding value of U_{2o} we can calculate the output voltage

$$\boxed{U_2 = U_{2o}\left(1 - \frac{U_{1N}}{U_1}\frac{I}{I_N}\Delta U_{rN}\right)} \quad \text{or} \quad \boxed{U_2 = U_{2o}\left[1 - \frac{S}{S_N}\left(\frac{U_{1N}}{U_1}\right)^2 \Delta U_{rN}\right]}$$

The function ΔU=f($\cos\varphi_2$) for $I=I_N$ is shown in following figure



It is seen that max(ΔU)=U_k occurs at $\cos\varphi_2=\cos\varphi_k$ i.e. for very special value being a known parameter of the transformer (short-circuit power factor).

Some interesting practical observations can be concluded:

1. in the transformer fully loaded ($I=I_N$ – it is very much required not to overload the transformer above its rated current) the voltage drop at internal impedances of the transformer can be maximum U_k (or in p.u. – U_{kr} or $U_{k\%}$). For example in the transformer having $U_{k\%}$=18% the maximum voltage drop (assuming $I \le I_N$) can be 18%, i.e.
   $$\max\left(\frac{U_{2o}-U_2}{U_{2o}}\times 100\%\right)=18\%$$
   from where it follows that minimum output voltage could be of the value of 82% of U_{2o} and it would appear for $\cos\varphi_2=\cos\varphi_k$

2. For some capacitive loads (leading character of the current) the voltage drop can be of negative value, what means the output voltage at such load can be higher than at no load.

Summarising above we can conclude that in the transformer having high value of short-circuit voltage one can expect correspondingly large variation of output voltage with varying load.

lead ⇒ leading character of the current, i.e. the current of capacitive character, or current phasor leads the voltage phasor;

lag ⇒ lagging character of the load, inductive load, current phasor lags the voltage phasor.

Imagine your transformer having rated voltages 15000/220 V. These voltages are voltage values at no-load. Let your transformer operates as step-down and let it to be supplied with 15 kV and fully loaded. Would you like to have your home appliances to be supplied with the voltage 0.82×220=180 V?