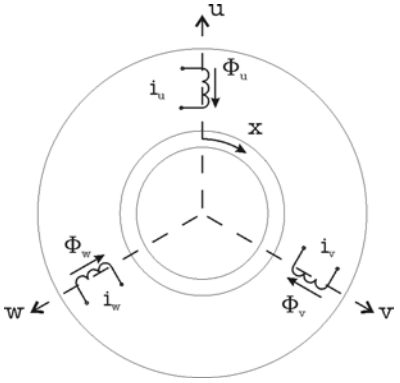
INDUCTION (ASYNCHRONOUS) MACHINES THEORY

MAGNETIC FIELD IN THE AIR GAP OF 3-PHASE MACHINE

The idea of magnetic field rotating along the air gap.



For instantaneous values:

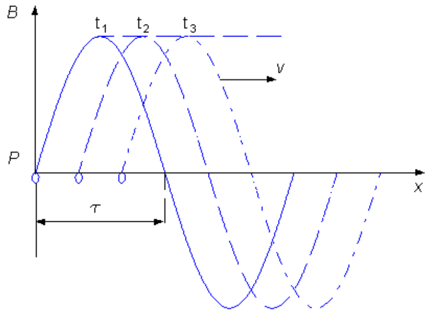
$$i_u = I_m \sin \omega t$$
$$i_v = I_m \sin(\omega t - 2/3\pi)$$
$$i_w = I_m \sin(\omega t - 4/3\pi)$$
$$\omega = 2\pi f \text{ - pulsation}$$
$$i_u + i_v + i_w = 0$$
$$\boxed{\phi_u + \phi_v + \phi_w = 0}$$

i_u i_v i_w - symmetrical 3–phase currents;

three identical coils (phase windings) displaced from each other by 120^0

f - frequency, [Hz]

But the resultant spatial distribution of flux density along the air-gap periphery *x*, being also the function of time, has the following form:



The halves of sine function represent the magnetic poles: N and S respectively, moving along axis x, as it is shown for:
t_1, t_2, t_3 – consecutive moments of time

This spatial distribution can be expressed by the function:

$$B(x,t) = B_m \sin\left(\omega t - \frac{\pi}{\tau} x\right)$$

where $\tau = \dfrac{\pi D}{2p}$ is called *pole pitch* (the width of one magnetic pole), *D* is diameter of the air gap, *p* is the number of pole pairs.

τ - pole pitch (the width of 1 pole), [m],
D - diameter of the air gap,
p - number of pole pairs

Such kind of the field is known as travelling (rotating) magnetic field. As it can be seen from diagram the sinusoidal wave moves along axis *x*. To determine the velocity of the travelling field (the speed of rotating field) imagine an observer travelling with the *B*–wave (for example with point P). For the observer:

$$B_m \sin\left(\omega t - \frac{\pi}{\tau} x\right) = 0 \;\rightarrow\; \sin\left(\omega t - \frac{\pi}{\tau} x\right) = 0 \;\rightarrow\; \omega t - \frac{\pi}{\tau} x = 0$$

Differentiating both sides with respect to time yields:

$$\omega - \frac{\pi}{\tau}\frac{dx}{dt} = 0 \qquad \rightarrow \qquad \boxed{\frac{dx}{dt} = v_1 = \frac{\omega \tau}{\pi} = 2\tau f \left[\frac{\text{m}}{\text{s}}\right]}$$

v_1 – linear speed (velocity) of travelling (rotating) field

Linear velocity of travelling magnetic field = 2 τ f

Angular speed of rotating magnetic field:

$$\Omega_{m1} = \frac{v_1}{\frac{D}{2}} = \frac{2\tau f}{\frac{D}{2}} = \frac{\omega}{p} \quad \left[\frac{\text{rad}}{\text{s}}\right]$$

or rotational speed of magnetic field (synchronous speed):

$$n_1 = \frac{f}{p}\left[\frac{\text{rev}}{\text{s}}\right] \qquad \text{or} \qquad \boxed{n_1 = \frac{60f}{p}\left[\frac{\text{rev}}{\text{min}}\right]}$$

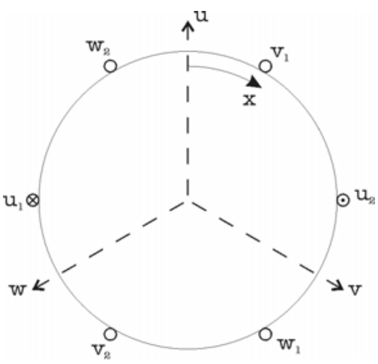
The term "synchronous speed" is very important. It will be often used!

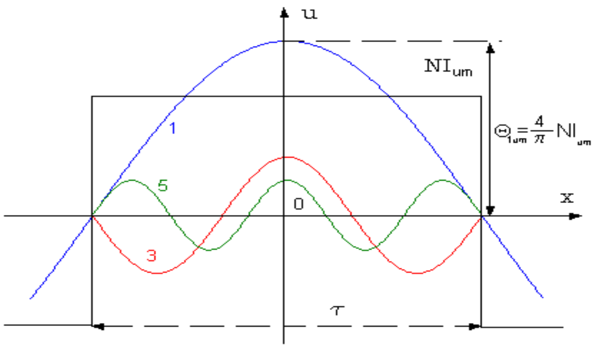
Instead of rev/min *other symbol of the same unit is also used:* min^{-1}.

Value of p can be only integer and not smaller than 1. For smallest possible p the corresponding rotational speeds of magnetic field are given in the table for f=50 Hz.

p	$n_1\left[\frac{\text{rev}}{\text{min}}\right]$
1	3000
2	1500
3	1000
4	750
⋮	⋮

The realisation





MMF of phase u winding (elementary winding)

Spatial distribution of MMF Θ is a rectangular curve that can be presented as a sum of sinusoidal harmonic components.

[Notice the mistake at the diagram: 3rd harmonic's curve should be drawn with negative amplitude; compare with corresponding relation at the next page]

$$i_u = I_{um}\sin\omega t$$

$$i_v = I_{vm}\sin\left(\omega t - \frac{2}{3}\pi\right)$$

$$i_w = I_{wm}\sin\left(\omega t - \frac{4}{3}\pi\right)$$

Due to Fourier analysis

$$\Theta_u(x,t) = \Theta_{1u}(x,t) + \Theta_{3u}(x,t) + \Theta_{5u}(x,t) + \ldots$$

all odd harmonics

For 1st harmonics only, for all 3 phases:

$$\begin{cases} \Theta_{1u}(x,t) = \Theta_{1um}\cos\frac{\pi}{\tau}x\sin\omega t \\ \Theta_{1v}(x,t) = \Theta_{1vm}\cos\left(\frac{\pi}{\tau}x - \frac{2}{3}\pi\right)\sin\left(\omega t - \frac{2}{3}\pi\right) \\ \Theta_{1w}(x,t) = \Theta_{1wm}\cos\left(\frac{\pi}{\tau}x - \frac{4}{3}\pi\right)\sin\left(\omega t - \frac{4}{3}\pi\right) \end{cases}$$

$$\Theta_1(x,t) = \Theta_{1u}(x,t) + \Theta_{1v}(x,t) + \Theta_{1w}(x,t)$$

$$\boxed{\Theta_1(x,t) = \frac{3}{2}\Theta_{1m}\sin\left(\omega t - \frac{\pi}{\tau}x\right)}$$

Assuming the symmetry of currents and phase windings:
$\Theta_{1um} = \Theta_{1vm} = \Theta_{1wm} = \Theta_{1m}$

Taking into account the following relations:

$$\phi = \frac{\Theta(x,t)}{R_\mu} \qquad B = \frac{\phi}{s_\delta} = \frac{\Theta(x,t)}{\dfrac{2\delta}{\mu_0 s_\delta} s_\delta} = \frac{\mu_0}{2\delta}\Theta(x,t)$$

we get final expression of flux density spatial distribution

$$\boxed{B_1(x,t) = \frac{3}{2} B_{1m} \sin\left(\omega t - \frac{\pi}{\tau} x\right)}$$

Interpretation of the relation as before: rotating magnetic field (1st harmonic of spatial distribution) rotating with the speed $n_1 = \dfrac{60 f}{p}$.

<u>For 3rd harmonics</u> of spatial distribution

$$\Theta_{3u}(x,t) = -\frac{1}{3}\Theta_{1m} \cos\left(3\frac{\pi}{\tau}x\right)\sin\omega t$$

$$\Theta_{3v}(x,t) = -\frac{1}{3}\Theta_{1m} \cos\left(3\left(\frac{\pi}{\tau}x - \frac{2}{3}\pi\right)\right)\sin\left(\omega t - \frac{2}{3}\pi\right)$$

$$\Theta_{3w}(x,t) = -\frac{1}{3}\Theta_{1m} \cos\left(3\left(\frac{\pi}{\tau}x - \frac{4}{3}\pi\right)\right)\sin\left(\omega t - \frac{4}{3}\pi\right)$$

$$\sum = 0$$

Third spatial harmonics of 3–phase winding don’t produce resultant magnetic field.
The same result is for all odd harmonics of $3k \quad (k = 1,2,3,...)$ number ($3k$ order).

<u>For 5th harmonics</u> of spatial distribution $(\nu = 5)$

$$\begin{cases} \Theta_{5u}(x,t) = \dfrac{1}{5}\Theta_{1m} \cos\left(5\dfrac{\pi}{\tau}x\right)\sin\omega t \\ \Theta_{5v}(x,t) = \dfrac{1}{5}\Theta_{1m} \cos 5\left(\dfrac{\pi}{\tau}x - \dfrac{2}{3}\pi\right)\sin\left(\omega t - \dfrac{2}{3}\pi\right) \\ \Theta_{5w}(x,t) = \dfrac{1}{5}\Theta_{1m} \cos 5\left(\dfrac{\pi}{\tau}x - \dfrac{4}{3}\pi\right)\sin\left(\omega t - \dfrac{4}{3}\pi\right) \end{cases}$$

$$\boxed{B_5(x,t) = \frac{3}{2}\cdot\frac{1}{5}B_{1m}\sin\left(\omega t + 5\frac{\pi}{\tau}x\right)}$$

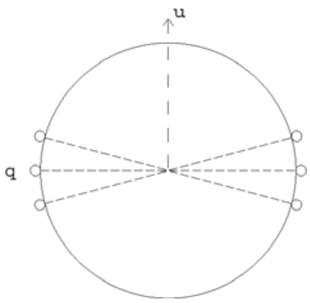
This wave of magnetic field has 5*p* pole pairs, rotates with the speed

$$\boxed{n_5 = -\frac{1}{5}\cdot\frac{60 f}{p}} \text{ or } \boxed{\Omega_{m5} = -\frac{1}{5}\Omega_{m1}} \text{ or } \boxed{\upsilon_5 = -2\tau_s f = -\frac{1}{5}\upsilon_1}$$

and its amplitude is 5 time lower than that of 1st harmonic.
Negative value of speed means that the direction of rotation is opposite to that of the first harmonic (opposite to the phase sequence).
Other space harmonics: $\nu = 1 \pm 6k \quad (k = 0,1,2,3,...)$

R_μ is the value of reluctance for the flux flowing two times across the air gap, through the stator yoke, stator teeth, rotor yoke and teeth. Reluctance of the flux path in magnetic core is negligible comparing to reluctance of the air gap. Therefore, the latter has been taken into consideration only.

When the winding isn't "elementary" but multi-coils one (distributed!):



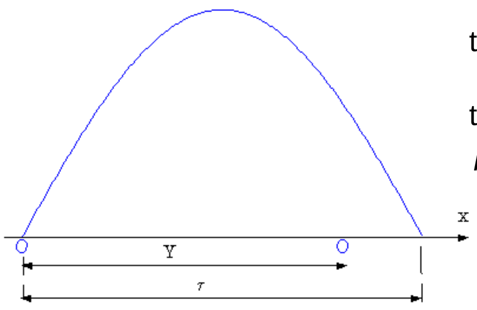
(q- number of slots per pole and per phase)

the resultant amplitude of $B_{\nu m}$ is $k_{b\nu}N$ times bigger, where

$$k_{b\nu} = \frac{\sin\frac{\pi}{2m}\nu}{q\sin\frac{\pi}{2mq}\nu}$$

is the breadth factor or distribution factor for ν-th harmonic (m – number of phases)

and when the coils are chorded (chorded winding):



the coil width (coil span) $Y < \tau$

the resultant amplitude of $B_{\nu m}$ is $k_{p\nu}N$ times bigger, where

$$k_{p\nu} = \sin\left(\nu\frac{Y}{\tau}\cdot\frac{\pi}{2}\right)$$

is the pitch factor or coil-span factor for ν-th harmonic.

Y – coil width
τ - pole pitch

full–pitch winding: $Y = \tau$

fractional–pitch winding: $Y < \tau$

The winding factor $\boxed{k_{w\nu} = k_{b\nu}\cdot k_{p\nu}}$

General relation for flux density spatial harmonics

$$\boxed{B_\nu(x,t) = \operatorname{sign}a\cdot\frac{1}{\nu}\cdot\frac{3}{2}\cdot k_{w\nu}\cdot B_{1m}\sin\left(\omega t - \operatorname{sign}\nu\cdot\frac{\nu\pi}{\tau}x\right)}$$

where

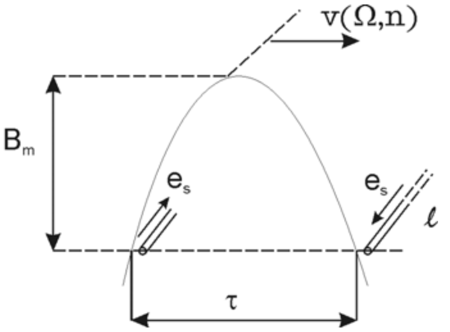
ν	1	5	7	11	13	17	19	23	...
sign$a$	+	+	-	-	+	+	-	-	...
signν	+	-	+	-	+	-	+	-	...

(positive or negative direction of rotation)

Magnetic field in the air gap of 3-phase machine produced there due to 3-phase current flowing in 3-phase symmetrical winding can be understood as the sum of component spatial harmonic fields. There exist only odd number harmonics not being multiple of 3 (1, 5, 7, 11, ...). They have amplitudes decreasing according to relation B_{1m}/ν and that decrease can be even higher depending on winding construction. Harmonic fields rotate in positive or negative direction with the speed n_1/ν where n_1 is the synchronous speed of first harmonic, rotating in the direction corresponding to phase sequence.

EMF INDUCED BY ROTATING (TRAVELLING) MAGNETIC FIELD

Let us consider the case of elementary winding in form of one turn embedded in two slots of stationary magnetic core. Magnetic flux linked with the winding is sinusoidally distributed along the turn width and moves (rotates) with respect to the winding.



Sinusoidal flux density wave moving with the speed υ (or Ω or n) induces emfs in both sides of the turn (coil)

$$e_s = Bl\upsilon \qquad e_t = 2Bl\upsilon$$

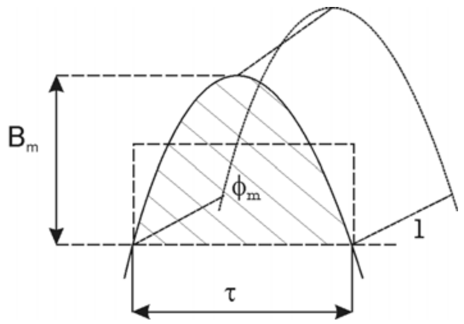
$$B(t) = B_m \sin\omega t = B_m \sin 2\pi f t$$

l is the length of active part of turn (axial length of the core)

$$\tau = \upsilon \cdot \frac{T}{2} = \frac{\upsilon}{2f} \;\rightarrow\; f = \frac{\upsilon}{2\tau} = \frac{\Omega D}{2 \cdot 2\dfrac{\pi D}{2p}} = \frac{p\Omega}{2\pi}$$

For N turns

$$e_{Nt} = \underbrace{2Nl \cdot 2\tau f \cdot B_m}_{E_{1m}} \sin\omega t$$



From the figure we can easily find out the relationship between the amplitude of flux density B_m and full magnetic flux of one pole:

$$\Phi_m = \frac{2}{\pi} B_m \tau l$$

and then calculate rms value of emf e_{Nt}

$$E_1 = \frac{2}{\sqrt{2}} N \cdot l \cdot 2\tau f \cdot \frac{\phi_m \pi}{2\tau \cdot l}$$

where Φ_m is the full magnetic flux of one pole. After reconstruction

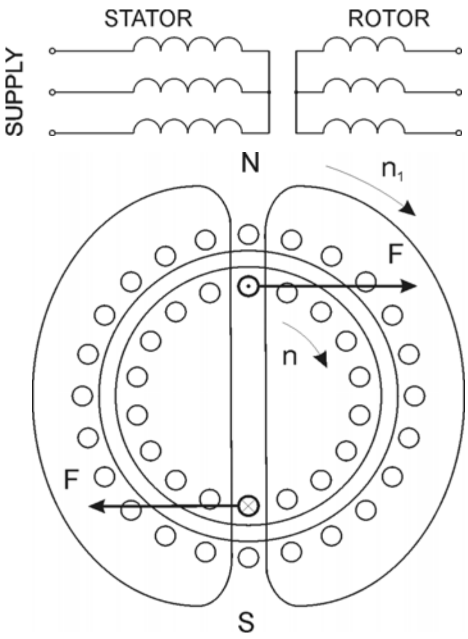
$$E_1 = \frac{2}{\sqrt{2}} \pi \cdot f \cdot N \cdot \phi_m = 4.44 f N \phi_m$$

In distributed and chorded winding (described by the winding factor k_w)

$$\boxed{E_1 = 4.44 f\, N\, k_w\, \phi_m}$$

where $f = \dfrac{p\Omega}{2\pi} = \dfrac{pn}{60}$ and speed is expressed in $n\left[\dfrac{\text{rev}}{\text{min}}\right]$ or in [min^{-1}].

PRINCIPLES OF OPERATION OF INDUCTION MACHINE



Assumptions:

1. There are 3–phase windings at stator & rotor (here connected in stars). The rotor winding circuit is closed. The rotor winding is of the same number of pole pairs *p* as stator.

2. The currents (sinusoidal) of stator winding involve the magnetic field rotating in the air gap. We take into consideration only 1st harmonic (space harmonic) rotating with the speed n_1 (synchronous speed).

3. The rotor rotates with the speed n.

Basing on the knowledge of phenomena of rotating magnetic field and emfs induced we can observe several effects appearing in such arrangement.

At the beginning let us consider $\boxed{n < n_1}$.

The rotor winding moves with respect to stator field. The relative rotational speed of this motion is

$$n_1 - n$$

and is counter–clockwise for the case shown in the figure.

Due to the motion of rotor winding with respect to stator field the emfs are induced in the rotor and corresponding currents flow (Fig).

The rotor currents react with the stator field and forces F and torque $T = F \cdot D$ are produced (*D* is the diameter of the rotor).

Slip is the difference of speeds $n_1 - n$ usually expressed in p.u. or %

$$\boxed{s = \frac{n_1 - n}{n_1}} \quad \text{or} \quad s = \frac{n_1 - n}{n_1} 100\%$$

$$\boxed{n = n_1(1 - s)}$$

n	...	$-n_1$	0	n_1	$2n_1$	...
s	...	2	1	0	-1	...

For $n < n_1$ the direction of torque is the same as speed n (of the rotor) It is driving torque – the machine operates as a motor.

It is quite easy – after determination of rotor emfs & currents – to notice, that for $\boxed{n > n_1}$ the torque produced is opposite to n - therefore it is braking torque. If we want to keep the speed constant at such level, we must drive the machine – the machine operates as a generator (the mechanical energy must be supplied to the machine).

For $\boxed{n = n_1}$ emfs = 0, rotor currents = 0, T = 0.

Machine has to operate asynchronously → ASYNCHRONOUS MACHINE.

For operation of the machine the rotor currents must flow. They appear in rotor winding by induction principle → INDUCTION MACHINE.

For $\boxed{n < 0}$ - machine rotates in opposite direction to n_1, while the torque produced is "positive". Such rotation is forced by external torque, while the machine torque opposes to it. The machine operates as a brake.

EMF INDUCED IN ROTOR WINDING

3–phase rotor winding is linked with sinusoidally distributed stator rotating field. The speed of stator field with respect to the rotor winding is

$$n_1 - n$$

therefore, rotor emfs have so called slip frequency

$$f_2 = \frac{p(n_1 - n)}{60} = \frac{pn_1}{60} \cdot \frac{n_1 - n}{n_1} = f_1 \cdot s$$

$$\boxed{f_2 = sf_1}$$

For n=0 (locked rotor) the frequency of rotor emf = f_1.

For n=n_1 the frequency of rotor emf = 0.

where: f_1 - stator voltage & currents frequency, f_2 - rotor voltage & currents frequency, s - slip.

Rms value of rotor emf

$$E_{2s} = 4.44 N_2 \cdot f_2 \cdot k_{wr} \cdot \phi_m = 4.44 N_2 \cdot s \cdot f_1 \cdot k_{wr} \cdot \phi_m = sE_2$$

E_{2s} - emf induced in the rotor when $f_2 = f_1$ ie when $n = 0$ (locked rotor).

ROTOR ROTATING FIELD

3-phase currents flowing in 3-phase rotor winding excite the rotor rotating field. It rotates with respect to the rotor winding (rotor body) with the speed

$$n_2 = \frac{60 f_2}{p} = s \cdot n_1$$

The rotor rotating field rotates with respect to stator with the speed being a sum of n and n_2:

$$n_{\Sigma r} = n + n_2 = n + s \cdot n_1 = n_1(1 - s) + sn_1 = n_1 \qquad \textbf{!}$$

Notice please a very interesting conclusion: rotor magnetic field rotates with respect to the stator with synchronous speed n_1 – the same as stator rotating field. Both fields rotate synchronously in the air gap and only the rotor rotates in this resultant field *asynchronously* – with the speed *n*.

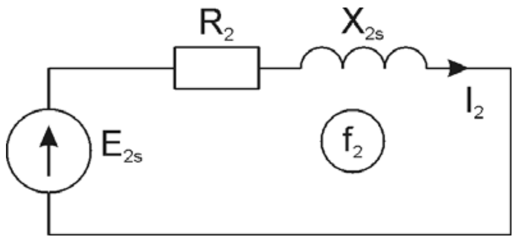
ROTOR ELECTRICAL CIRCUIT – EMF, CURRENT, IMPEDANCE

Besides the main magnetic fluxes of rotor & stator there exist also leakage fluxes – excited by rotor & stator currents but linked only with their own circuits.

Therefore, the rotor circuit is characterized by its resistance and leakage reactance X_{2s} which value depends on rotor frequency $\left(X_{2s} = 2\pi f_2 L_{2l}\right)$
$\rightarrow X_{2s} = s \cdot 2\pi f_1 L_{2l} = sX_2$

X_2 can be understood as the rotor reactance at frequency f_1 (i.e. for locked rotor).

Rotor circuit for one phase:

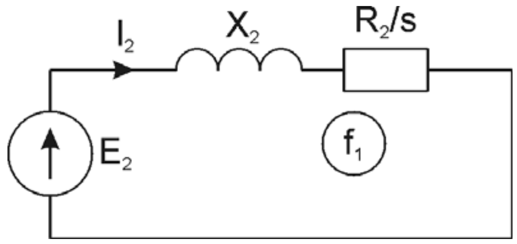


$$I_2 = \frac{E_{2s}}{\sqrt{R_2^2 + X_{2s}^2}} = \frac{sE_2}{\sqrt{R_2^2 + \left(sX_2\right)^2}}$$

or, after reconstruction

$$I_2 = \frac{E_2}{\sqrt{\left(\frac{R_2}{s}\right)^2 + X_2^2}}$$

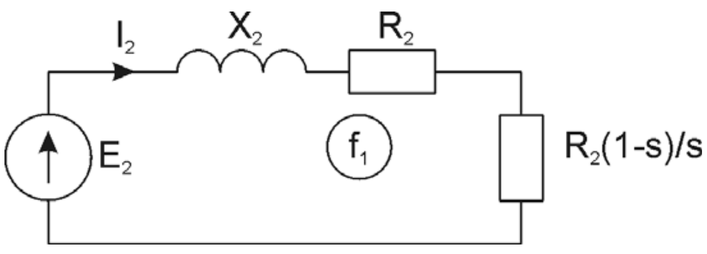
Such equation of rotor current can be illustrated by the following circuit (rotor equivalent circuit):



or when we take into consideration, that

$$\frac{R_2}{s} = R_2 + R_2\frac{1-s}{s}$$

rotor equivalent circuit has the form that will be applicable for further considerations of motor equivalent circuit:



EQUIVALENT CIRCUIT OF INDUCTION MACHINE

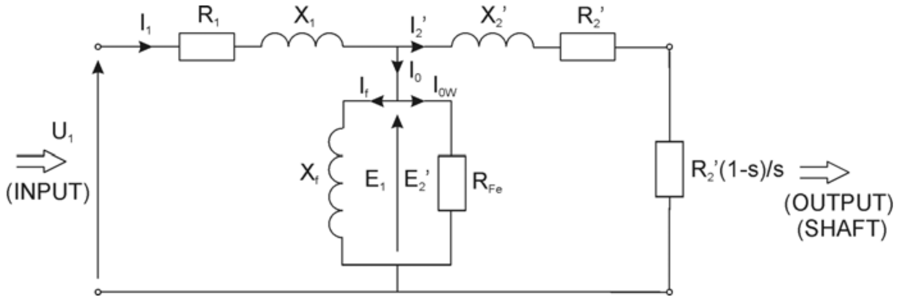
The circuit is for one phase only (symmetrical system assumed). Taking into consideration the stator (primary) circuit with its resistance and leakage reactance, the iron core with its losses and magnetization (including the magnetization of the air gap)

and

referring the rotor circuit to stator (primary) by assuming

$E_2' = \frac{N_1 k_{ws}}{N_2 k_{wr}} E_2$ where $\frac{N_1 k_{ws}}{N_2 k_{wr}}$ is "turn ratio"

we can draw the circuit representing entire induction machine (one phase):



Current I_{ow} corresponds to all no-load losses: in this case, besides the iron loss, we have also "mechanical loss" – due to friction & windage.

Notice that:

I_f - magnetizing current is much bigger than in transformer as bigger stator mmf is required to "magnetize" the air gap (to force the flux to flow through the air gap),

I_{ow} - active component of no-load current is bigger than in transformer,

I_o - no-load current is much bigger – it is about 30-50% of rated current,

$$R_2' \frac{1-s}{s} \text{ and the power } I_2'^{\,2} \cdot R_2' \cdot \frac{1-s}{s}$$

represent the output of the machine – "mechanical output" – the shaft and "shaft power".

Remember (!): rotor electrical circuit is always closed and there is no output there!. Electrical energy is converted into mechanical (torque, speed) and delivered to the receiver through the shaft of the machine. This "mechanical output" is represented by $R_2' \frac{1-s}{s}$ in electrical equivalent circuit.