# THE INFLUENCE OF RIBS WELDED TO THE ROLLED I-BEAMS UPON THEIR RESISTANCE 



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# THE INFLUENCE OF RIBS WELDED TO THE ROLLED I-BEAMS UPON THEIR RESISTANCE 

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Rolled steel beams are calculated for bending moments according to the formula:

$$
\begin{equation*}
\sigma=\frac{M}{W}, \tag{1}
\end{equation*}
$$

where $\sigma$ is the normal stress on the edge of the flange of the cross section, while $W$ is the section modulus. We assume as max $\sigma$ the admissible stress $k$, being the $\frac{1}{n}$ part of the ultimate stress ( $n=$ about 3 ), eventually of the yield point ( $n=$ about 2 ). The application of the above formula (1) is justified, if we are certain, that by increasing the moment $M$ we will arrive at the limit of resistance of the beam, i. e. the breakage in the plane of the load's action will take place. This is the case when treating a long and low beam, suitably protected against deformation in horizontal plane (sidewise buckling). The section modulus $W$ is here really an indicator of the beam's resistance. By increasing $W$ we obtain the proportional increasing of the moment $M$, which the beam can safely bear. Yet this rule is of a real value to a certain limit only. When the beam is relatively short and high, normal stresses in horizontal section through the web $\sigma_{z}$ at the point of the concentrated force's action grow more important and can easily become more dangerous than the normal stresses The increasing of the bending moment $M$ in such a case results finally in crushing of the flange, directly below the acting load, and of the web, this causing too early breakage of the beam, occasioned besides other purposes by a sudden diminution of the section modulus. Attention to stresses $\sigma_{z}$ was paid by Prof. Huber. ${ }^{1}$ )

The danger of crushing can be delayed, if not avoided, by means of stiffeners (ribs) joined to the I-beams by aid of welding, similar to the stiffeners in the plate-girders. Those ribs allow to apply again formula (1) even to relatively high and short beams, which we cau see often in practice (stringers and floor beams in bridges, girders). Followiug tests were executed to find the influence of such ribs upon the resistance of the I-beams.

Tests were conducted in two series. The first one comprised 16 I-beams Nos 16, 20, 24 and 30 , the second one 6 beams Nos 32 and 34 . The „number" of a beam is equal to its depth in centimeters. All these beams of a span of $L=2,00$ meters were submitted to ben-
${ }^{1}$ ) M. T. Huber. Studja nad belkami dwuteowemi. Wydawnictwo Warszawskiego Towarzystwa Politeehnicznego.
ding tests, a concentrated force haring been applied in the centre of the span on a 200 tons Amsler machine.

Research works were conducted with 3 kinds of beams:

1. Beams without ribs (one of each depth of I-beams).
2. Beams with 3 ribs, placed over the supports and under the concentrated force, at a distance of 1 m from each other (two such beams were taken in each depth for series I and one beam in series II).
3. Beams with 5 ribs, every 50 ctm ., 3 of which were placed as above at the points of the concentrated forces action. (reaction and force $P$ ), one such beam being taken in each depth.

TABLE I.

| I No <br> (depth of the <br> beam in cm) | 0 | 3 | Quantity of ribs |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 3 |  |  |  |
| 16 | 8,6 | 7,425 | 7,6 |  |  |
| 20 | 15,4 | 13,75 | 15,8 |  |  |
| 24 | 22,9 | 23,85 | 26,3 |  |  |
| 30 | 39,9 | 48,45 | 48,3 |  |  |
| 32 | 46 | 58,5 | 59,5 |  |  |
| 34 | 51 | 69,5 | 72,5 |  |  |



Fig. 1.

Table I represents the maximal values of the load, which have been born by tested beams. The load was concentrated one in the middle of the span $L=2,00 \mathrm{~m}$.

In table II, this value of $R$ from table I is called $R_{o}$, which is corresponding to beams without ribs, white $R_{3}$ and $R_{5}$ are corresponding to beams with 3 and 5 ribs respectively. The columns of the table give us the differences in tons and percents of the substractive.

TABLE II.

| I No | $\mathrm{R}_{3}-\mathrm{R}_{0}$ |  | $\mathrm{R}_{5}-\mathrm{R}_{3}$ |  | $\mathrm{R}_{5}-\mathrm{R}_{0}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Tons | $\%$ | Tons | $\%$ | Tons | $\%$ |
| 16 | $-1,175$ | $-13,7$ | 0,175 | 2,36 | $-1,0$ | $-11,6$ |
|  | $-1,75$ | $-11,3$ | 2,05 | 14,9 | 0,4 | 2,6 |
| 24 | 0,95 | 4,15 | 2,45 | 10,27 | 3,4 | 14,8 |
| 30 | 8,55 | 21,4 | $-0,15$ | $-0,31$ | 8,4 | 21,0 |
| 32 | 12,5 | 27,2 | 1,0 | 1,71 | 13,5 | 29,4 |
| 34 | 18,5 | 36,3 | 3,0 | 4,6 | 12,5 | 42,2 |

The first column of both Tables represents the depth of the I-beams in centimeters (number of I-beam). The values $R_{3}-R_{0}$ in the column 2 show us that the higher the I-beam the greater the increase of its resistance obtained by addition of 3 stiffeners under the concentrated loads in its middle and over the supports. No increase has been obtained in I-beams Nr. 16 and 20.

The relatively small increase of resistance and even a diminution of resistance of I-beam No 16 beams may be accounted for by thermical stresses resulting from the welding of ribs, which stresses can even occasion a local weakening of the beam's material in smaller and thinner beams.

The adding of two more ribs between the acting forces generally increases the resistance (No 30 excepting), but in a less distinct manner (differences $R_{5}-R_{3}$ ). The last
column states an increase of I-beam's resistance which is obtained by aid of adding of 5 ribs (No 16 excepting). The increase augments in percents accordingly to the depth of the beam.

In formula (1) let us assume $\sigma=1200 \mathrm{~kg} / \mathrm{cm}^{2}$ and

$$
\begin{equation*}
M=\frac{P L}{4} \tag{2}
\end{equation*}
$$

$L=200 \mathrm{~cm}$, a greatest safe load to be born will be then obtained $P_{b}$ :

$$
P_{b}=\frac{4 \sigma W}{L}=\frac{4 \cdot 1200}{200} W=24 W
$$

The factor of safety:

$$
n=\frac{R}{P}
$$

or the relation of the greatest load $R$ towards the safe load $P_{b}$ is given by Table III.
T A B L E III.

| Series | I <br> $\mathbf{N r}$. | W <br> $\mathbf{c m}^{3}$ | $\mathbf{P}_{b}$ <br> Tons | $\mathbf{n}_{0}$ | $\mathbf{n}_{3}$ | $\mathrm{n}_{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | 16 | 117 | 2,81 | 3,06 | 2,98 | 3,05 |
|  | 20 | 214 | 5,14 | 3 | 2,68 | 3,08 |
|  | 24 | 354 | 8,50 | 2,7 | 2,80 | 3,10 |
|  | 30 | 653 | 15,67 | 2,55 | 3,09 | 3,08 |
| II | 32 | 782 | 18,75 | 2,45 | 3,12 | 3,16 |
|  | 34 | 923 | 22,32 | 2,28 | 3,12 | 3,25 |

From this table we can conclude: the adding of stiffeners to the I-beams increases their security, chiefly the adding of 3 stiffeners in the plans of acting of the concentrated forces.

By substituting the corresponding $R$ from table I to $P$ and taking $W$ from table III, we worked out table IV, which eliminates to some degree the influence of the l-beams variety in depth, i. e. the influence of section modulus, and allows us to find out the influence of other factors upon the beams during the bending of the beam.


Fig. 3.

TABLE IV.

| $\begin{gathered} \mathrm{I} \\ \mathrm{Nr} . \end{gathered}$ | Quantity of stiffeners | $\sigma \mathrm{kg} / \mathrm{mm}^{2}$ |
| :---: | :---: | :---: |
| 16 | 0 | 36,8 |
|  | 3 | 31,7 |
|  | 5 | 32,4 |
| 20 | 0 | 36 |
|  | 3 | 32,2 |
|  | 5 | 36,9 |
| 24 | 0 | 32,4 |
|  | 3 | 33,8 |
|  | 5 | 37,2 |
| 30 | 0 | 30,6 |
|  | 3 | 37 |
|  | 5 | 37 |
| 32 | 0 | 29,4 |
|  | 3 | 37,4 |
|  | 5 | 38,0 |
| 34 | 0 | 27,7 |
|  | 3 | 37,7 |
|  | 5 | 39,3 |

Table IV is illustrated by Fig. 2-4. The depth of the beams on the horizontal axes is measured in centimeters and the stresses on the ordinate axes are measured in $\mathrm{kg} / \mathrm{cm}^{2}$. Fig. 2 relates to beams without ribs, Fig. 3 to beams with 3 ribs, while Fig. 3 refers to beams with 5 ribs. If the conditions of the tests had been ideal, the material of the beams absolutely uniform, excluding all possibilities of sidewise buckling and furthermore if the value of formula :(1) had exclusively decided of resistance, then the lines $\sigma$ would have been horizontal.

If the diagram of stresses had corresponded furthermore to Hooke's law, in the dangerous cross section of the beam, (at the moment of reaching the value $R$ by the loading force, as seen on Fig. 5a), then the ordinates


Fig. 3. of diagram $R$ would have been equal to the yield point, respectively to the ultimate stress (limit of resistance) of the steel. Due to plasticity, however, the diagram of stresses assumes the shape of a broken line (fig. 5b), after


Fig. 4. the outside fibers reach the limit of plasticity, and the beam's resistance is exhausted only after the straight inclined comes entirely close to the neutral axis (fig. 5 c ). This would mean an increase of $50 \%$ of the ultimate load for a rectangular beam, while about $17 \%$ would be obtained in an I-beam. The section modulus increases then to the value $1,17 \mathrm{~W}$, if formula (1) is to be used when $P$ reaches the value $R$, accordingly to Table $1^{1}$ ).

Thus the values of Table IV should be divided by 1,17 , and the ordinates of the diagrams should be diminished proportionately in Fig. $2-4$. We can notice that the diagram in fig. 2 is somewhat falling down, while the diagrams $R_{3}$ and $R_{5}$ (fig. 3 and 4) are rising to the right. The falling down of $R_{0}$ when the depth of the beams increases, would hawe been still more noticeable, had the tested beams been protected against sidewise buckling. They have specially affected the beam I No 16, weakest of all ones. This sidewise buckling only can explain the reason of rising of the $R_{3}$ and $R_{5}$ diagrams. The

${ }^{1}$ ) Bleich: Stahlhochbauten, I Bd. Berlin 1932, page 400.

$\mathrm{kg} / \mathrm{mm}^{2} \quad h=20 \mathrm{~cm}$.

higher the tested reinforced I-beam, the greater its resistance against sidewise buckling, which will then appear later and damage less the beam. Timoshenko studied the phenomena of sidewise buckling by using his approximative method and setting formulas for critical stresses, beyond which the usual rule of equilibre is not to be applied. He assumed that the ends of the beam are restrained from rotating in the manner that the broken beam remains on the supports in the vertical plane. Such a restraining did not exist in our tests and thence the sidewise buckling could result much more easily. We cannot therefore study it according to Timoshenko's formulas.

The relation between normal stresses $\sigma$ and the possibility of the waves' formation in the web in places of the biggest moment, as well as the influence of shearing stresses $\tau$ upon the eventual waves formation in the web, in places of biggest shearing forces, have been studied by Timoshenko too. He studied also the influence of ribs in both cases and found that they were of no avail in the first one. The thickness of the rolled I-beams web is sufficient, however, to counteract the influence of $\sigma$ upon the waves' formation. In the second case the influence of ribs is advantageous and the critical shearing stress $\tau$ depends from the relation of $h: a$, where $a$ is the distance between the ribs, while $h$ represents the beam's depth. For the given span of the beam $L$, for the depth $h$, and the given kind of beam's loading, max $\tau$ is proportionnal to max $\sigma$. Taking this under consideration, the establishing of a relation between $h: a$ and the value $\sigma$ from Table IV for each depth of the beam, would be quite appropriate here. The increase of resistance by adding three ribs and its further increase by adding two more ribs, is shown clearly in these diagrams (Fig. 6-11). Yet the value of $\tau$ cannot explain here the advantageous influence of ribs, considering that no waves were found on the web of damaged beams, which would result from surpassing the critical $\tau$. The thickness of rolled I-beams web accounts very sufficiently for it (in the case as in our load at least), taking into consideration both the critical $\tau$ and critical $\sigma$. Where is the reason then its decreasing the resistence with the growing depth $h$ in beams without ribs, and how can we explain the increase of resistance when adding the ribs, the more the higher the beams? A study of tests' shape will enable us to find an answer to this.

The sidewise buckling of the beams provided with ribs shows two half-waves with the point of deflexion in the center (fig. 15), while one flange remains unaltered. In beams without ribs, Fig. 13, the sidewise buckling can be noticed in one half-wave. The I-beams Nos 30 and 24 got bent to the side on both flanges; only one flange of I-beam No 20 was bent, while the other one remained nearly straight (Fig. 17). Finally one flange of the I-beam Ne 16 (Fig. 20) remained straight and the second one (the compressed one) was bent $S$-like (in two half waves). It is evident from these examples that the ribs facilitate the formation of two half waves instead of one only, increasing thus the critical force which starts the sidewise buckling.

More details are furnished by photos showing the web. The beams with ribs got bent vertically as well at the upper as at the lower flange. Fig. 14, 16 and 18. Beams without ribs remained straight (No 30 and 24), Fig. 12, or got bent only imperceptibly. (No 20 and 16), Fig. 19. I-beams No 16 show less bending, whether provided with ribs or without them. The crushing of the upper chord under the load can be seen in beams without ribs and its size depends from the depth of the beams. (Fig. 12). I-beam No 16 presents no sign of such a crushing. Fig. 19.

The influence of ribs upon the deformation of beams grows with their depth. The resistance to bending of the beams with ribs was almost exhausted and it required but little to break them, while beams without ribs got damaged by crushing before reaching this limit. In the I-beam No 16 the failure was caused by breaking, independently from ribs. The above crushing observed in deeper I-beams without ribs, with relatively small stresses $\sigma$, makes us think that the decisive role was played by normal stresses in horizontal section through the web directly below the flange, at the place of the concentrated load acting, as mentioned in the beginning. Huber calls them transversal stresses and wrote


Fig. 12.


Fig. 13.


Fig. 14.


Fig. 16.


Fig. 17.
several chapters about them in his above mentioned work „Studja nad belkami dwuteowemi - Studies of beams with double - T section".

Huber found in the case of a concentrated force $P$ the highest transversal stress under this force

$$
\sigma_{z}=\% \frac{P}{F}
$$

where $F$ is the cross section of the l-beam. The coefficient $\%$ is given in Table V.

TABLE V.

| I NP | $\chi$ |
| :---: | :---: |
| 10 | 7,33 |
| 20 | 7,75 |
| 30 | 7,63 |
| 40 | 7,52 |
| 50 | 7,41 |

on in average

$$
\%=7,53
$$

Let us try to find here the influence


Fig. 18. of ribs, according to Huber, who studied, however, their influence in plate girders only, uniformly loaded. The ribs form, according to him, a rigid support for the flange, which rests like a beam upon an elastic substratum. In our case the force $P$ acts directly over the rib. Supposing the rib distributes equally the


Fig. 19.


Fig. 20.
action of the force in both flanges, the highest transversal compression stresses will be found directly below the upper flange, while the aperisthic tensil stresses will appear immediately above the lower one. Assuming the rectilineal law of transversal stress variations, and calling
$h_{1}$ the height and $\delta$ the thickness of the web,
$y$ the deflection of the flange, $p$ it's pressure upon the web for each unit of its lenghth, we shall obtain a unit deformation (elongation or shortening) of the web's height above (or below) the neutral axis:

$$
\frac{2 y}{h_{1}}=\frac{1}{2} \frac{P}{2 \sigma E}
$$

The caracteristic of the substratum will be then:

$$
k=\frac{p}{y}=4 \frac{\sigma E}{h_{1}}
$$

Let us call

$$
\alpha^{4}=\frac{k}{4 E I_{s}}=\frac{\delta}{h_{1} I_{s}}
$$

( $I_{s}$ represents the moment of incrtia of the flange's cross section with respect to the horizontal axis passing through it's center of gravity), then the deflection of the flange at the distance of $x$ from $P$ amounts to:

$$
y=\frac{P}{2} \frac{1}{8 E I_{s} \alpha^{3}} e^{-\alpha x}(\cos \alpha x+\sin \alpha x)
$$

The max. deflection:

$$
f=y_{\max .}=\frac{1}{16} \frac{P}{E I_{s} \alpha^{3}}=\frac{P \alpha}{16 E I_{s} \alpha^{4}}=\frac{P \alpha h_{1}}{16 E \delta}
$$

The transversal stress below the force:

$$
\sigma_{z}=\frac{k f}{\delta}=\frac{\alpha}{4} \cdot \frac{P}{\partial}
$$

Assuming for beams without ribs:

$$
{ }_{v}^{4}=\frac{0.4 \delta}{h_{1} I_{s}}=0,4 a^{4}
$$

Prof. Huber found

$$
\sigma_{0}=\frac{P}{2 \delta} \alpha_{0}=\frac{P}{2 \delta} \sqrt[4]{0,4} \cdot \alpha=2 \sqrt[4]{0,4} \sigma_{z}
$$

Consequently:

$$
\sigma_{z}=\frac{\sigma_{0}}{2 \sqrt[4]{0,4}}=\frac{\sigma_{0}}{1,59}
$$

In other words: the transversal stress will diminish 1,59 times if the web is stiffened with a rib in place of the concentrated force's action.

Formula 4 will be then:

$$
\begin{equation*}
\sigma_{z}=\frac{\%}{1,59} \frac{P}{F} \tag{5}
\end{equation*}
$$

TABLE VI,

| I <br> Nr. | $F$ <br> $\mathrm{~cm}^{2}$ | $\chi$ | $\boldsymbol{F}$ <br> $\%$ <br> $\mathbf{c m}^{2}$ | $1,59 F$ <br> $\chi$ <br> $\mathbf{c m}^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| 16 | 22,8 | 7,58 | 3,01 | 4,79 |
| 20 | 33,5 | 7,75 | 4,32 | 6,86 |
| 24 | 46,1 | 7,70 | 6,00 | 9,54 |
| 30 | 69,1 | 7,63 | 9,05 | 14,36 |
| 32 | 77,8 | 7,61 | 10,2 | 16,2 |
| 34 | 86,8 | 7,59 | 11,4 | 18,15 |

Table VI gives us the values $\chi$ interpolated from Table $V$ as well as auxiliary values for formulas 4) and 5) and Table VII. Transversal stresses, accordingly to formula 4) and 5) are shown in Table VII, substituting suitably $R$ from Table I to $P$. Table VII is analogical to Table IV, which gives us longitudinal stresses $\sigma$ in the same cases. The greatest value of $\sigma$ and $\sigma_{z}$ being the probable cause of the failure, is market on the corresponding Table.

From comparing both Tables we can conclude that the longitudinal stresses were more dangerous for the lowest beams (I No 16 and 20) while transversal stresses presented most danger for the deepest ones (I No 34). For other beams transversal stresses were dangerous for not reinforced beams and longitudinal stresses for reinforced ones.

These conclusions resulting from our tests confirm the theory.
If we admit that the greater of both stresses $\sigma$ or $\sigma_{z}$ is in this case the deciding factor, then $\sigma_{z}$ will be decisif if $\sigma_{\tilde{z}}>\sigma$, consequently for beams without ribs according to 4 and 1

$$
\begin{equation*}
\chi \frac{P}{F}>\frac{M}{W} \tag{6}
\end{equation*}
$$

or in an average

$$
\frac{p}{M}>2,36
$$

In our case according to (2) we have

$$
\begin{equation*}
\frac{M}{P}=\frac{L}{4} \tag{7}
\end{equation*}
$$

The above conditions will be therefore represented by an unequality
or

$$
\begin{align*}
& h: L>1:(2,36.4) \\
& h: L>1: 9,44 \tag{8}
\end{align*}
$$

and as $L=200 \mathrm{~cm}$. therefore $h>200: 9.44=21,2 \mathrm{~cm}$. Actually for $h=20 \mathrm{~cm}$. we have found:
table vil.

| $\begin{gathered} 1 \\ \mathrm{Nr} \end{gathered}$ | Quantity of stiffeners | $\sigma_{z} \mathrm{~kg} \mathrm{~cm}^{2}$ |
| :---: | :---: | :---: |
| 16 | 0 | 28,7 |
|  | 3 | 15,5 |
|  | 5 | 15,8 |
| 20 | 0 | 35,6 |
|  | 3 | 20 |
|  | 5 | 23 |
| 24 | 0 | 38,2 |
|  | 3 | 25 |
|  | 5 | 27,6 |
| 30 | 0 | 44 |
|  | 3 | 33,7 |
|  | 5 | 33,6 |
| 32 | 0 | 45 |
|  | 3 | 36,2 |
|  | 5 | 36,7 |
| 34 | 0 | 44,8 |
|  | 3 | 38,8 |
|  | 5 | 39,8 |

$$
\begin{aligned}
& \sigma=36 \quad \mathrm{~kg} / \mathrm{cm}^{2} \\
& \sigma_{z}=35,6 \quad \text { " that is } \sigma>\sigma_{z} \\
& \sigma=32,4 \mathrm{~kg} / \mathrm{cm}^{2} \\
& \sigma_{z}=38,2 \quad \text {, that is } \sigma>\sigma_{z}
\end{aligned}
$$

while for $h=24 \mathrm{~cm}$

For beams with ribs, taking into consideration (5), the unquality (6) will be:
or

$$
\begin{aligned}
& \frac{\%}{1,59} \frac{P}{F}>\frac{M}{W} \\
& \frac{M}{P}<\frac{2,36}{1,59}
\end{aligned}
$$

or in our case

$$
\begin{equation*}
h: L>1,59: 9,44=1: 5,93 \tag{9}
\end{equation*}
$$

For $L=200 \mathrm{~cm}$ we obtain $h>33,8 \mathrm{~cm}$. We actually obtained for $h=34 \mathrm{~cm} \sigma_{z}>\sigma$ and for $h=32 \mathrm{~cm} \sigma>\sigma_{z}$.

Prof. Huber's theory refferring to beams without ribs and the above given contribution referring to the beams with ribs were thus entirely confirmed by these tests.

It is worth while noticing that the unequalities (8) and (9) are of a more general importance, approximately at least, than could be thought, since the relation $M: P$ is varying between narrow limits and does not in any practical case deviate much from the values $\frac{L}{4}$, It should be noticed namely that in cases of many concentrated forcess, respectively a distributed load, the reaction in the greatest concentrated force $P$.

Therefore, in an extreme case of constant and uniform load, we have as well:

$$
M: P=\frac{1}{8} p L: \frac{1}{2} p L=L: 4, \text { as in equ. } 7
$$

Attention, however, must be given in deep beams without ribs, like in I No 34, to $\sigma_{z}$ values, which are rather high. The values $\sigma_{z}$ for deeper beams and $\sigma$ for lower ones ought to be equal, being the limit of resistance for both kinds of beams. The same state of destruction should be determined by an equal value $\sigma_{r e d}$. We can conclude from the above that either the formula (4) gives too high values or $\sigma_{r e d}<\sigma_{z}$. Both eventualities take place. Due to plasticity and to the crushing of the flange by the loading roller, the mathematical line of the roller and flange's contact changed into a narrow band of limited breadth of a few centimeters. The loading force ceased to be a concentrated one and changed into a load distributed over the surface of this band. If we determine the breadth of the band by $c$ and the length of the deflection's buckling half-wave by $L$ (and considering that approximatively $2 L=0,8 h+2,4) \mathrm{cm}$, then the influence of breadth $c$ can be, according to Huber, determined very closely, by dividing the value from the equation (4) by $N=1+$ $+\frac{4}{7}\left(\frac{c}{L}\right)^{2}$. For I-beam No 30 (Fig. 12) $c=10 \mathrm{~cm}$, therefore $\frac{4}{7}\left(\frac{10}{30}\right)^{2}=\frac{4}{7.9}$, while $N=1,0636$, so that 44 will be replaced by $\sigma_{z}=44: N=41,5 \mathrm{~kg} / \mathrm{mm}^{2}$.

A further reduction of $\sigma_{z}$ will be obtained considering the shearing stresses between the web and flange, neglected in formulas 4 and 5 . If the flange had been resting on the web like on an elastic substratum and had there been no shearing stresses between the flange and the web, then - in case of an uniform distribution of load on the upper flange $q$ for length unit - a transversal stress would been noticiable directly under the flange $\sigma_{z}=\frac{q}{\delta}$. Huber found in an exact manner this value smaller from 8 to $10 \%$ instead. We can therefore safely multiply by 0,91 the values of formulas (4) and (5), as the influence of shearing stresses is surely not smaller in a concentrated force's case. Considering that the values $\sigma$ from Table V should be also divided by 1,17 (compare fig. $5 a, b$ and $c$ ), the unequalities $6-9$ will not change considerably by it. In the case of INo 30 it will be then $\sigma_{z}=0,91 \cdot 41,5=37,6 \mathrm{~kg} / \mathrm{mm}^{2}$ instead of $44 \mathrm{~kg} / \mathrm{mm}^{2}$. Similarly for I No $34 \frac{0,91}{1,0035} \cdot 44,8=$ $=38,4 \mathrm{~kg} / \mathrm{mm}^{2}$ instead of $45,7 \mathrm{~kg} / \mathrm{mm}^{2}$. The value thus obtained is still, hewever, considurably bigger than an average value, about $30-36 \mathrm{~kg} / \mathrm{mm}^{2}$, proving that for high beams $\sigma_{\text {red }}<\sigma_{\tilde{z}}$. This can be explained by the fact that, according to formula 4, compressions diminishes very quickly when the distance of the force grows. It is well known that the local compression stresses, for instance, in a concrete or steel bearing plate can attain very high values, when the plate is loaded only on a small part of the surface; they are much higher than if the plate had been entirely loaded.

The deflections $f$ figured im mm in Table VIII and IX correspond to forces $P$ in tons for all I-beams. They increase of course when the force $P$ increases. In the last 4 columns of the Table we see the quotionts $f: P$ resp. $10 f: P$. Theoretically $f=\frac{P L^{3}}{48 E 1}$, where $L=200 \mathrm{~cm}, E=2150 \mathrm{t} / \mathrm{cm}^{2}$

| For I No. | 16 | 20 | 24 | 30 |
| :---: | :---: | :---: | :---: | :---: |
| $1=$ | 935 | 2142 | 4246 | 980 |
| $f: P=$ | 0,83 | 0,382 |  |  |
| $10 f: P=$ |  |  | 1,83 | 0,775 |

TABLE VIII.

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline \multirow[t]{2}{*}{$$
\begin{gathered}
\mathrm{I} \\
\mathrm{Nr} .
\end{gathered}
$$} \& \multirow[t]{2}{*}{$$
\begin{gathered}
P \\
\text { Tons }
\end{gathered}
$$} \& \multicolumn{4}{|c|}{$f \mathrm{~mm}$} \& \multicolumn{4}{|c|}{$f: P$} <br>
\hline \& \& 1 \& 2 \& 3 \& 3 \& 1 \& 2 \& 3 \& 4 <br>
\hline 16 \& $$
\begin{aligned}
& 1 \\
& 2 \\
& 3 \\
& 4 \\
& 5 \\
& 6 \\
& 7
\end{aligned}
$$ \& $$
\begin{gathered}
0,5 \\
1 \\
1,5 \\
3,5 \\
4,4 \\
5,5 \\
23,5
\end{gathered}
$$ \& $$
\begin{aligned}
& 0,6 \\
& 1,6 \\
& 2,5 \\
& 3,5 \\
& 4,3 \\
& 5,3 \\
& 9,3
\end{aligned}
$$ \& 1,1
2,6
4,9 \& 1,7
3
5,8 \& $$
\begin{aligned}
& 0,5 \\
& 0,5 \\
& 0,5 \\
& 0,87 \\
& 0,88 \\
& 0,92 \\
& 3,25
\end{aligned}
$$ \& $$
\begin{aligned}
& 0,6 \\
& 0,8 \\
& 0,83 \\
& 0,87 \\
& 0,86 \\
& 0,88 \\
& 1,32
\end{aligned}
$$ \& $$
\begin{aligned}
& 0,55 \\
& 0,65 \\
& 0,82
\end{aligned}
$$ \& 0,85
0,75
0,95 <br>
\hline 20 \& $$
\begin{array}{r}
2 \\
3 \\
5 \\
6 \\
8 \\
10 \\
12 \\
14 \\
15
\end{array}
$$ \& 0,2
1

24,2 \& 2,1

4,2 \& $$
\begin{gathered}
1 \\
1,8 \\
\\
2,5 \\
3,3 \\
4,3 \\
11,7
\end{gathered}
$$ \& \[

$$
\begin{array}{r}
0,8 \\
1,7 \\
2,6 \\
3,4 \\
4,1 \\
5,4 \\
47,6
\end{array}
$$

\] \& \[

$$
\begin{aligned}
& 0,04 \\
& 0,10 \\
& 1,61
\end{aligned}
$$
\] \& 0,42

0,42 \& $$
\begin{aligned}
& 0,5 \\
& 0,45 \\
& \\
& 0,42 \\
& 0,41 \\
& 0,43 \\
& 0,98
\end{aligned}
$$ \& \[

$$
\begin{aligned}
& 0,4 \\
& 0,425 \\
& \\
& 0,43 \\
& 0,425 \\
& 0,41 \\
& 0,45 \\
& 1,25
\end{aligned}
$$
\] <br>

\hline
\end{tabular}

TABLE IX.

| $\begin{gathered} \text { I } \\ \text { Nr. } \end{gathered}$ | $\begin{gathered} P \\ \text { Tons } \end{gathered}$ | $f \mathrm{~mm}$ |  |  |  | $f: P$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| 24 | 5 | 0,5 | 0,9 | 0,5 | 0,7 | 1,0 | 1,8 | 1,0 | 1,4 |
|  | 10 | 1,7 | 1,9 | 1,0 | 1,9 | 1,7 | 1,9 | 1,0 | 1,9 |
|  | 15 | 2,7 | 3 | 2 | 3,2 | 1,8 | 2 | 1,33 | 2,13 |
|  | 20 | 6,5 | 7 | 6 | 5,3 | 3,25 | 3,5 | 3 | 2,65 |
|  | 25 |  |  |  | 28,9 |  |  |  | 11,5 |
| 30 | 5 | 0,5 | 0,4 | 0,9 | 0,1 | 1 | 0,8 | 1,8 | 0,2 |
|  | 10 | 1,0 | 1,2 | $1,2$ | 0,9 | 1 | 1,2 | 1,8 | 0,9 |
|  | 15 | 1,5 | 1,6 | 2,1 | 1,2 | 1 | 1,07 | 1,4 | 0,8 |
|  | 20 | 2 | 2,3 | 2,5 | 1,9 | 1 | 1,15 | 1,25 | 0,95. |
|  | 25 | $2,7$ | 2,9 |  | 2,5 |  | 1,16 | 1,2 |  |
|  | 30 | 3,8 | 3,3 | 3,9 | 3,1 | 1,26 | 1,1 | 1,3 | 1,04 |
|  | 35 | 18,7 | 4,4 | 5 | 4 | 5,33 | 1,26 | 1,42 | 1,14 |
|  | 40 |  | 14,2 | 14 | 13 |  | 3,55 | 3,5 | 3,15 |
|  | 45 |  | 34,2 | 34 | 35 |  | 7,6 | 7,5 | 7,8 |

We can notice that within the limits of proportionality between deflection and the acting force, the theoretical value for I No 16 corresponds approximately to the results of the tests, while in deeper beams the tests show values $f: P$ larger than the theoretical ones. It is due to the fact that thevalue $f$ is not only influenced by the deflection of the tested beam, but also by the deflection of the supporting beam of the testing machine. The mutual relation of these deflections is proportional to the moment of inertia of both, the tested and the supporting beams. This relation is very small in I No 16, so that the supporting beam's deflection has hardly any influence at all. The case differs, however, with deeper beams.

Table X gives us deflections, the entire deflections $f$ as well as the constant deflections $f^{\prime}$. Elastic deflection $f^{\prime \prime}=f-f^{\prime}$. The figures in the last column of the Table confirm to some extent the conclusion arrived at by Wöhler and Bauschinger as

TABLE $X$.

| I Nr. | Ribs | $\begin{gathered} P \\ \text { Tons } \end{gathered}$ | $f$ | $f^{\prime}$ mm | $f^{\prime \prime}$ | $\frac{10 f}{p}$ | $\frac{10 f^{\prime \prime}}{p}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 32 | 0 | $\begin{aligned} & 10,5 \\ & 20,5 \\ & 29 \\ & 35 \end{aligned}$ | $\begin{aligned} & 1,4 \\ & 2,68 \\ & 3,9 \\ & 5,68 \end{aligned}$ | $\begin{array}{r} 0,28 \\ 38 \\ 68 \\ 1,93 \end{array}$ | $\begin{aligned} & 1,12 \\ & 2,3 \\ & 3,22 \\ & 3,75 \end{aligned}$ | $\begin{aligned} & 1,33 \\ & 1,31 \\ & 1,34 \\ & 1,62 \end{aligned}$ | $\begin{aligned} & 1,068 \\ & 1,122 \\ & 1,11 \\ & 1,07 \end{aligned}$ |
|  | 3 | $\begin{aligned} & 10,5 \\ & 20 \\ & 30 \\ & 35 \\ & 39,5 \\ & \hline \end{aligned}$ | $\begin{aligned} & 1,3 \\ & 2,57 \\ & 4,16 \\ & 4,90 \\ & 6,05 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0,02 \\ & 0,22 \\ & 0,8 \\ & 1 \\ & 1,65 \\ & \hline \end{aligned}$ | $\begin{aligned} & 1,28 \\ & 2,35 \\ & 3,35 \\ & 3,90 \\ & 4,40 \\ & \hline \end{aligned}$ | $\begin{aligned} & 1,14 \\ & 1,285 \\ & 1,384 \\ & 1,4 \\ & 1,53 \\ & \hline \end{aligned}$ | $\begin{aligned} & 1,22 \\ & 1,18 \\ & 1,12 \\ & 1,11 \\ & 1,11 \end{aligned}$ |
|  | 5 | 30 | 4,4 | 1,65 | 3,35 | 1,47 | 1,12 |
| 34 | 0 | $\begin{array}{r} 6 \\ 10 \\ 15 \\ 40 \end{array}$ | $\begin{aligned} & 1 \\ & 1,35 \\ & 2 \\ & 4,85 \end{aligned}$ | $\begin{aligned} & \hline 0 \\ & 0 \\ & 0,1 \\ & 1,8 \end{aligned}$ | $\begin{aligned} & 1 \\ & 1,35 \\ & 1,9 \\ & 3,05 \\ & \hline \end{aligned}$ | $\begin{aligned} & 1,666 \\ & 1,35 \\ & 1,33 \\ & 1,21 \end{aligned}$ | $\begin{aligned} & 1,666 \\ & 1,35 \\ & 1,27 \\ & 0,76 \end{aligned}$ |
|  | 3 | $\begin{aligned} & 10,5 \\ & 20,5 \\ & 30 \\ & 40 \\ & 50,2 \\ & 56 \end{aligned}$ | $\begin{gathered} 1,15 \\ 2,14 \\ 3,15 \\ 4,48 \\ 6,33 \\ 10,1 \end{gathered}$ |  | $\begin{aligned} & 1,07 \\ & 2,0{ }^{\prime} \\ & 2,9 \\ & 3,87 \\ & 4,73 \\ & 5,08 \end{aligned}$ | $\begin{aligned} & 1,1 \\ & 1,05 \\ & 1,05 \\ & 1,12 \\ & 1,26 \\ & 1,8 \end{aligned}$ | $\begin{aligned} & 1,02 \\ & 1 \\ & 0,97 \\ & 0,9^{\prime} \\ & 0,9_{4}^{\prime} \\ & 0,9 \end{aligned}$ |
|  | 5 | $\begin{aligned} & 10,5 \\ & 20 \\ & 30 \\ & 40 \\ & 45 \end{aligned}$ | $\begin{aligned} & 1,475 \\ & 2,37 \\ & 3,5 \\ & 4,8 \\ & 5,91 \end{aligned}$ |  | $\begin{aligned} & 1,375 \\ & 2,095 \\ & 2,825 \\ & 3,85 \\ & 4,52 \end{aligned}$ | $\begin{aligned} & 1,405 \\ & 1,18 \\ & 1,17 \\ & 1,20 \\ & 1,31 \end{aligned}$ | $\begin{aligned} & 1,31 \\ & 1,05 \\ & 0,942 \\ & 0,962 \\ & 1,00 \end{aligned}$ |

the result of a series of tests: the proportionality between elastic strain and stress even after exceeding the yield point.

The most important conclusions, resulting from these tests, may be, therefore, formulated as follows:

1. The reinforcement of the I-beams by aid of ribs, welded to their web at the places of the concentrated force's action, increase their resistance to bending. Such increase is proportionate to the depth of the beam. In the case of the tested beams, the increase of resistance attained $40 \%$ for a NP 30 I-beam, but was absolutely unoticeable for a NP 16 beam. When the ribs are fixed to the web between the points of the concentrated force's action the resistance of the beam increases, but to a much lesser extent.
2. When the depth of the beams, increases their resistance rises more slowly than the $W$. The greatest tensions, therefore, obtained from formula $\sigma=\frac{M}{W}$ decrease with the growing depth of the beams. This formula can be no longer used for determining the resistance of high beams subjected to the action of concentrated forces, as the beams are not destroyed by breakage but by crushing. If ribs are welded at the place of the concentrated force's action, the danger of crushing is delayed and the above formula can be used.

For lower beams, which are not subject to crushing, but only to bending the role played by ribs is of much lesser importance, for very low ones even without any result.

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