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ON THE GALERKIN VECTORS FOR THE MICROPOLAR ELASTICITY

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In the paper [1] L. Sobrero has considered the propagation of elastic waves in Hooke's medium, using the resolving function which satisfies the repeated wave equation. This problem has been generalized by M. Iacovache [2], who applied the general algorithm divised by Gr.C. Moisil [3] and introduced the Galerkin vector into the elastodynamics.

Another method of determining the Iacovache function will be given here for the Hooke and Cosserat media; in our opinion it is rather simple and makes it possible to avoid tedious derivations involving the three and six order determinants.

Consider the equation of motion in terms of displacement for the Hookean medium

$$\Box_2 \underline{u} + (\lambda + \mu)\operatorname{grad}\operatorname{div}\underline{u} + \underline{X} = 0, \quad \Box_2 = \mu\nabla^2 - \rho\partial_t^2 \quad , \tag{1}$$

Here $\underline{X}$ denotes the body force for unit volume, $\underline{Y}$ -the body couple; $\underline{u}$ is the displacement vector and ρ the density. μ, λ are Lamé constants.

Applying the divergence operator to equation (1) we obtain

$$\Box_1 \operatorname{div}\underline{u} + \operatorname{div}\underline{X} = 0, \quad \Box_1 = (\lambda + 2\mu)\nabla^2 - \rho\partial_t^2 \ . \tag{2}$$

Next we apply the operator $\Box_1$ to equation (1). Making use of relation (2), we arrive at the following equation

$$\Box_1 \Box_2 \underline{u} = -[\Box_1 - (\lambda+\mu)\operatorname{grad}\operatorname{div}]\underline{X}. \tag{3}$$

Introducing into (3) the representation of the displacement $\underline{u}$ in the form

$$\underline{u} = [\Box_1 - (\lambda + \mu)\operatorname{grad}\operatorname{div}]\underline{F} \tag{4}$$

we obtain the repeated wave equation for the function $\underline{F}$

$$\Box_1 \Box_2 \underline{F} + \underline{X} = 0. \tag{5}$$

Here $\underline{F}$ is the Iacoveche vector function.

In the Cosserat-theory of elasticity the deformation of the body is described by a displacement vector $\underline{u}$ and an independent rotation vector $\underline{\varphi}$.

Consider the equations of motion in terms of displacement and rotations [4]:

$$(6) \qquad \Box_2 \underline{u} + (\lambda + \mu - \alpha)\,\mathrm{grad\ div}\ \underline{u} + 2\alpha\,\mathrm{curl}\ \underline{\varphi} + \underline{X} = 0,$$

$$(7) \qquad \Box_4 \underline{\varphi} + (\beta + \gamma - \epsilon)\,\mathrm{grad\ div}\ \underline{\varphi} + 2\alpha\,\mathrm{curl}\ \underline{u} + \underline{Y} = 0.$$

Here $\underline{X}$ is the vector of body forces, $\underline{Y}$ -vector of body-couples and $\alpha, \beta, \gamma, \epsilon, \mu, \lambda$ are material constants of the micropolar Cosserat medium.

The following notations have been introduced

$$\Box_2 = (\mu + \alpha)\nabla^2 - \rho\partial_t^2, \quad \Box_4 = (\gamma + \epsilon)\nabla^2 - 4\alpha - I\partial_t^2,$$

Here ρ -denotes the density and I-the rotational inertia.

Let us return to the system of coupled equations (6) (7). Applying the divergence operator, the system is split into two independent equations

$$(8) \qquad \Box_1 \mathrm{div}\ \underline{u} + \mathrm{div}\ \underline{X} = 0, \qquad \Box_1 = (\lambda + 2\mu)\nabla^2 - \rho\partial_t^2 ,$$

$$(9) \qquad \Box_3 \mathrm{div}\ \underline{\varphi} + \mathrm{div}\ \underline{Y} = 0, \qquad \Box_3 = (\beta + 2\gamma)\nabla^2 - 4\alpha - I\partial_t^2 .$$

After performing the curl operation on equation (6) (7) we obtain the relations

$$(10) \qquad \Box_2\,\mathrm{curl}\ \underline{u} + 2\alpha\,\mathrm{curl\ curl}\,\underline{\varphi} + \mathrm{curl}\ \underline{X} = 0,$$

$$(11) \qquad \Box_4\,\mathrm{curl}\ \underline{\varphi} + 2\alpha\,\mathrm{curl\ curl}\,\underline{u} + \mathrm{curl}\ \underline{Y} = 0.$$

Next we apply the operator $\Box_1\,\Box_4$ to equations (6) and we make use of the relations (8) and (11); subsequently we apply the operator $\Box_4\ \Box_3$ to equation (7) and we make use of the relation (9) and (10).

Finally we arrive at the equations

$$\square_1 (\square_2\square_4 + 4\alpha^2 \nabla^2)\underline{u} = - (\square_1\square_4 - \text{grad div}\,\Gamma)\underline{X} + 2\alpha\ \text{curl}\ \square_1\underline{Y}, \quad (12)$$

$$\square_3 (\square_2\square_4 + 4\alpha^2 \nabla^2)\underline{\varphi} = - (\square_2\square_3 - \text{grad div}\,\Theta)\underline{Y} + 2\alpha\ \text{curl}\ \square_3\underline{X}. \quad (13)$$

The following notations have been introduced here

$$\Gamma = (\lambda + \mu - \alpha)\square_4 - 4\alpha^2, \qquad \Theta = (\beta + \gamma - \epsilon)\square_2 - 4\alpha^2 .$$

We assume the following representation for the displacements and rotations, using the two vector functions $\underline{F}$ and $\underline{G}$

$$\underline{u} = (\square_1\square_4 - \text{grad div}\ \Gamma)\underline{F} - 2\alpha\ \text{curl}\ \square_3\underline{G}, \quad (14)$$

$$\underline{\varphi} = (\square_2\square_3 - \text{grad div}\ \Theta)\underline{G} - 2\alpha\ \text{curl}\ \square_1\underline{F}. \quad (15)$$

Introducing this representation into equations (12) and (13) we obtain two wave equations for the functions $\underline{F}$ and $\underline{G}$:

$$\square_1(\square_2\square_4 + 4\alpha^2\nabla^2)\underline{F} + \underline{X} = 0, \quad (16)$$

$$\square_3(\square_2\square_4 + 4\alpha^2\nabla^2)\underline{G} + \underline{Y} = 0. \quad (17)$$

The representation (14) (15) of displacement and rotation over the vector functions $\underline{F}$, $\underline{G}$ were established by N. Sandru [5], who applied the operational algorithm of Gr.C. Moisil [3].

Equations (16) (17) are very useful in determining the functions $\underline{u}$, $\underline{\varphi}$ due to the action of concentrated and instantaneous forces and couples.

REFERENCES

[1] SOBRERO, L.,:"Delle funzioni analoghe a potenziale intervenienti nella Fisica-Matematica". Atti Reale Acad. Lincei, Ser.VI, 21, 1935, 448.

[2] IACOVACHE, M.:"O extindere a metodei lui Galerkin pentru sistemul ecaütulov elasticitatii", Bul. Ştiint.Acad.Rep.Pop.Române, Ser.A, 1 (1949), 593.

[3] MOISIL, Gr.C.,:"Aspura sistemelor de equaţii cu derivata partiale lineare si cu coeficienti constanti," Bull. Ştiint.Acad. Rep.Pop. Române, Ser.A., (1949), 341.

[4] NOWACKI, W.,: "Theory of Micropolar Elasticity", CISM, Udine; Springer-Verlag, Wien, 1970.

[5] SANDRU, N.,:"On Some Problems of the Linear Theory of Asymmetric Elasticity", Int. Engng. Sci. 4, 1 (1966), 81.