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SOME STABILITY PROBLEMS OF CYLINDRICAL SHELLS

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1. A cylindrical shell of constant curvature, supported along all its four edges

Let the forces $q_x = \sigma_x h$, $q_y = \sigma_y h$, $q_{xy} = \tau_{xy} h$ (where h is the plate thickness) which are functions of the variables x, y independent of z, act on the shell. We assume that these forces are known and have been determined by solving the corresponding membrane problem. Our problem is that of solving in an accurate manner the problem of stability of a cylindrical shell of constant curvature.

The deflection of a cylindrical shell is described in the engineer's theory of shells developed by V. Z. Vlasov [1], by the following system of differential equations:

$$\frac{R}{Eh}\nabla^4\Phi - \frac{\partial^2 w}{\partial x^2} = 0$$

$$\frac{1}{R}\frac{\partial^2\Phi}{\partial x^2} + N\nabla^4 w = -\left(q_x\frac{\partial^2 w}{\partial x^2} + q_y\frac{\partial^2 w}{\partial y^2} + 2q_{xy}\frac{\partial^2 w}{\partial x\partial y}\right) \tag{1.1}$$

In these equations w denotes the deflection of the shell, Φ — the stress function, N — the flexural rigidity of the shell, E — Young's modulus. The assumed reference frame and the load are shown in Fig. 1.

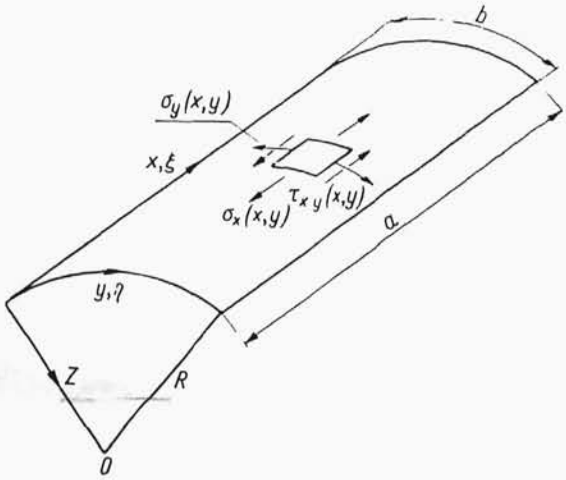

Fig. 1.

Using Vlasov's substitution,

$$w = \nabla^4 F,\quad \Phi = \frac{Eh}{R}\frac{\partial^2 F}{\partial x^2}$$

we reduce the system of equations (1.1) to one differential equation

$$N\nabla^8 F + \frac{Eh}{R^2}\frac{\partial^4 F}{\partial x^4} = -\left(q_x\frac{\partial^2 w}{\partial x^2} + q_y\frac{\partial^2 w}{\partial y^2} + 2q_{xy}\frac{\partial^2 w}{\partial x\partial y}\right) \tag{1.2}$$

The solution of this differential equation can be represented in the form of the integral relation

$$F(x,y) = -\iint_\Omega \bar{F}\left(q_\xi\frac{\partial^2 w}{\partial\xi^2} + q_\eta\frac{\partial^2 w}{\partial\eta^2} + 2q_{\xi\eta}\frac{\partial^2 w}{\partial\xi\partial\eta}\right)d\xi d\eta \tag{1.3}$$

The function $\bar{F}(x, y; \xi, \eta)$ is Green's function for the non-homogeneous differential equation (1.2) in the case of concentrated load $P = 1$ acting at the point (ξ, η) in the z-direction.

Performing the operation

$$\nabla^4 = \frac{\partial^4}{\partial\xi^4} + 2\frac{\partial^4}{\partial\xi^2\partial\eta^2} + \frac{\partial^4}{\partial\eta^4}$$

on both members of equation (1.3) and remembering that

$$\nabla^4 F\ (x, y) = w\ (x, y)$$
$$\nabla^4 \bar{F}\ (x, y;\ \xi, \eta) = \bar{w}\ (x, y;\ \xi, \eta)$$

we reduce equation (1.3) to the form

$$w(x, y) = -\iint_{\Omega} \bar{w}(x, y; \xi, \eta) \ [q_\xi(\xi, \eta) \frac{\partial^2 w(\xi, \eta)}{\partial \xi^2} + q_\eta \frac{\partial^2 w(\xi, \eta)}{\partial \eta^2} + 2q_{\xi\eta} \frac{\partial^2 w(\xi, \eta)}{\partial \xi \partial \eta}] \, d\xi d\eta \tag{1.4}$$

Through application of Green's transformation on a plane, equation (1.4) takes the form

$$w(x, y) = \int [(w \frac{\partial \bar{Q}_n}{\partial n} - \bar{Q}_n \frac{\partial w}{\partial n}) - (w \frac{\partial \bar{Q}_{ns}}{\partial s} - \bar{Q}_{ns} \frac{\partial w}{\partial s})] \, ds - \iint_{\Omega} w (\frac{\partial^2 \bar{Q}_\xi}{\partial \xi^2} + \frac{\partial^2 \bar{Q}_\eta}{\partial \eta^2} + 2 \frac{\partial^2 \bar{Q}_{\xi\eta}}{\partial \xi \partial \eta}) \, d\xi d\eta \tag{1.5}$$

where the following notations have been introduced:

$$\bar{Q}_n = \bar{w} q_n = \bar{w}(q_\xi \cos^2 \alpha + q_\eta \sin^2 \alpha + q_{\xi\eta} \sin 2\alpha) = \bar{Q}_\xi \cos^2 \alpha + \bar{Q}_\eta \sin^2 \alpha + \bar{Q}_{\xi\eta} \sin 2\alpha$$

$$\bar{Q}_{ns} = \bar{w} q_{ns} = -\bar{w} [(q_\eta - q_\xi) \frac{\sin 2\alpha}{2} + q_{\xi\eta} \cos 2\alpha] = -[(\bar{Q}_\eta - \bar{Q}_\xi) \frac{\sin 2\alpha}{2} + \bar{Q}_{\xi\eta} \cos 2\alpha]$$

We confine our considerations to the problem of stability of cylindrical shells simply supported or rigidly clamped along all edges.

In both cases the curvilinear integral in equation (1.5) is equal to zero, w and $\bar{w}$ being both equal to zero at the edges. Thus equation (1.5) reduces to

$$w(x, y) = -\iint_{\Omega} w(\xi, \eta) (\frac{\partial^2 \bar{Q}_\xi}{\partial \xi^2} + \frac{\partial^2 \bar{Q}_\eta}{\partial \eta^2} + 2 \frac{\partial^2 \bar{Q}_{\xi\eta}}{\partial \xi \partial \eta}) \, d\xi d\eta \tag{1.6}$$

or

$$w(x, y) = -\iint_{\Omega} w(\xi, \eta) [q_\xi \frac{\partial^2 \bar{w}}{\partial \xi^2} + q_\eta \frac{\partial^2 \bar{w}}{\partial \eta^2} + 2q_{\xi\eta} \frac{\partial^2 \bar{w}}{\partial \xi \partial \eta} + 2 (\frac{\partial q_\xi}{\partial \xi} \frac{\partial \bar{w}}{\partial \xi} + \frac{\partial q_\eta}{\partial \eta} \frac{\partial \bar{w}}{\partial \eta} + \frac{\partial q_{\xi\eta}}{\partial \xi} \frac{\partial \bar{w}}{\partial \eta} + \frac{\partial q_{\xi\eta}}{\partial \eta} \frac{\partial \bar{w}}{\partial \xi}) + \bar{w} (\frac{\partial^2 q_\xi}{\partial \xi^2} + \frac{\partial^2 q_\eta}{\partial \eta^2} + 2 \frac{\partial^2 q_{\xi\eta}}{\partial \xi \partial \eta})] \, d\xi \, d\eta \tag{1.7}$$

Since the equilibrium equations for plane stress have the form

$$\frac{\partial q_\xi}{\partial \xi} + \frac{\partial q_{\xi\eta}}{\partial \eta} + \varrho X h = 0, \qquad \frac{\partial q_{\xi\eta}}{\partial \xi} + \frac{\partial q_\eta}{\partial \eta} + \varrho X h = 0 \tag{1.8}$$

equation (1.7) may be considerably simplified if there are no mass forces ($X = 0, \ Y = 0$).
Then, we have

$$w(x, y) = -\iint w(\xi, \eta) [q_\xi \frac{\partial^2 \bar{w}}{\partial \xi^2} + q_\eta \frac{\partial^2 \bar{w}}{\partial \eta^2} + 2q_{\xi\eta} \frac{\partial^2 \bar{w}}{\partial \xi \partial \eta} + \bar{w} (\frac{\partial^2 q_\xi}{\partial \xi^2} + \frac{\partial^2 q_\eta}{\partial \eta^2} + \frac{2 \, \partial^2 q_{\xi\eta}}{\partial \xi \, \partial \eta})] \, d\xi d\eta \tag{1.9}$$

The problem of stability of a cylindrical shell with simply supported edges is then reduced to the solution of a Fredholm integral equation of the second type (equation (1.9)) with an asymmetric kernel (in the most general case of load).

2. A cylindrical shell simply supported at the edges

First, let us determine Green's function $\bar{w}$ $(x, y; \xi, \eta)$ for the case of free support along the edges.

The following differential equation should be solved:

$$N \nabla^8 \bar{F} + \lambda \frac{\partial^4 \bar{F}}{\partial x^4} = p, \quad \lambda = \frac{Eh}{R^2} \tag{2.1}$$

where p denotes the load of the shell, reduced here to the concentrated force $P = 1$ acting at the point (ξ, η). Assuming that

$$\bar{F} = \sum_{n, m}^{\infty} \bar{F}_{n, m} \sin \alpha_n x \sin \beta_m y$$

$$\alpha_n = \frac{n\pi}{a}, \quad \beta_m = \frac{m\pi}{b}$$

which enables the satisfaction of any boundary conditions for equation (2.1), and representing the concentrated load by the trigonometric series

$$p = \frac{4}{ab} \sum_{n, m}^{\infty} \sin \alpha_n \xi \sin \beta_m \eta \sin \alpha_n x \sin \beta_m y$$

we obtain the solution of equation (2.1) in the form:

$$\bar{F}(x, y; \xi, \eta) = \frac{4}{ab} \sum_{n, m}^{\infty} \frac{\sin \alpha_n \xi \sin \beta_m \eta}{N (\alpha^2_n + \beta^2_m)^4 + \lambda \alpha^4_n} \sin \alpha_n x \sin \beta_m y \tag{2.2}$$

In view of the relation $\nabla^4\bar{F} = \bar{w}$, we find

$$\bar{w}(x, y; \xi, \eta) = = \frac{4}{a\,b}\sum_{n,m}^{\infty}\frac{\sin\alpha_n\xi\sin\beta_m\eta}{\triangle_{n,m}}\sin\alpha_n x\sin\beta_m y \tag{2.3}$$

where

$$\triangle_{n,m} = N(\alpha^2_n + \beta^2_m)^2 + \lambda\frac{\alpha^4_n}{(\alpha^2_n + \beta^2_m)^2}$$

We assume further that the solution of the integral equation (1.9) can be represented in the form of the trigonometric series:

$$w(x, y) = \sum_{i,k}^{\infty} A_{ik}\sin\alpha_i x\sin\beta_k y \tag{2.4}$$

This assumption satisfies the boundary conditions of the problem, and the coefficients A_{ik} will be chosen to satisfy the integral equation (1.9).

Thus, we substitute the expressions (2.3) and (2.4) in equation (1.9).

After integration and rearrangement, we obtain the infinite system of equations

$$A_{i,k} = \frac{4}{ab\triangle_{ik}}\sum_{n,m}^{\infty} A_{n\,m}\,G_{nimk},\; i, k = 1,2,\ldots,\infty \tag{2.5}$$

By introducing the notations

$$q_\xi(\xi,\eta) = \overset{(0)}{q_\xi}\, r(\xi,\eta)$$

$$q_\eta(\xi,\eta) = \overset{(0)}{q_\eta}\, t(\xi,\eta)$$

$$q_{\xi,\eta}(\xi,\eta) = \overset{(0)}{q_{\xi,\eta}}\, s(\xi,\eta)$$

and substitution

$$a_{nimk} = \int_0^a\!\!\int_0^b r(\xi,\eta)\sin\alpha_i\xi\sin\beta_k\eta\sin\alpha_n\xi\sin\beta_m\eta\,d\xi\,d\eta$$

$$b_{nimk} = \int_0^a\!\!\int_0^b t(\xi,\eta)\sin\alpha_i\xi\sin\beta_k\eta\sin\alpha_n\xi\sin\beta_m\eta\,d\xi\,d\eta$$

$$c_{nimk} = \int_0^a\!\!\int_0^b \frac{\partial^2 r}{\partial\xi^2}\sin\alpha_i\xi\sin\beta_k\eta\sin\alpha_n\xi\sin\beta_m\eta\,d\xi\,d\eta \tag{2.6}$$

$$d_{nimk} = \int_0^a\!\!\int_0^b \frac{\partial^2 t}{\partial\eta^2}\sin\alpha_i\xi\sin\beta_k\eta\sin\alpha_n\xi\sin\beta_m\eta\,d\xi\,d\eta$$

$$e_{nimk} = \int_0^a\!\!\int_0^b \frac{\partial^2 s}{\partial\xi\partial\eta}\sin\alpha_i\xi\sin\beta_k\eta\sin\alpha_n\xi\sin\beta_m\eta\,d\xi\,d\eta$$

$$f_{nimk} = \int_0^a\!\!\int_0^b s(\xi,\eta)\cos\alpha_i\xi\cos\beta_k\eta\sin\alpha_n\xi\sin\beta_m\eta\,d\xi\,d\eta$$

the quantity G_{nimk} can be expressed by the relation

$$G_{nimk} = \overset{(0)}{q_\xi}\alpha^2_i\,a_{nimk} + \overset{(0)}{q_\eta}\beta_k^2\,b_{nimk} - \overset{(0)}{q_\xi}c_{nimk} + - \overset{(0)}{q_\eta}d_{nimk} - 2\overset{(0)}{q_{\xi\eta}}e_{nimk} - 2\alpha_i\beta_k\overset{(0)}{q_{\xi\eta}}f_{nimk} \tag{2.7}$$

Taking the determinant of the system (2.5) equal to zero, and assuming two of the three parameters, $\overset{(0)}{q_\xi}$, $\overset{(0)}{q_\eta}$, $\overset{(0)}{q_{\xi\eta}}$, to be constants, we have the buckling condition of the cylindrical shell.

It should be noted that the system of equations (2.5) is identical with that obtained using the Ritz-Timoshenko energy method, assuming that the deflection of the plate is expressed by the series (2.4).

The system of equations (2.6) holds also for the load (q) distributed in a non-uniform manner over the plate region. If q acts over a region Ω, constituting part of the shell, the coefficients a_{nimk}, ... f_{nimk} can be determined by integrating over the region Ω.

We use the system of equations (2.5) to determine the critical load in an approximate manner in the case when the shell is acted upon by a load $q_\xi(\eta)$ constituting a function of the variable η only. Consider first an auxiliary problem (Fig. 2). Let the shell be loaded by a concentrated force which will be defined as

$$P = \lim_{\epsilon\to 0} \overset{(0)}{q_\xi}\cdot 2\epsilon$$

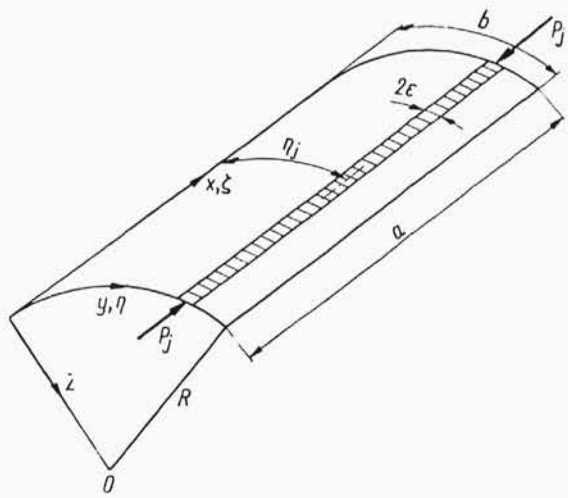

Fig. 2.

where a_{nimk} takes the form

$$a_{nimk} = \int_0^a\int_{\eta-\varepsilon}^{\eta+\varepsilon} \sin \alpha_n\xi \sin \alpha_i\xi \sin \beta_k\eta \sin \beta_m\eta \, d\xi \, d\eta = \qquad (2.8)$$

$$= a\epsilon \sin \beta_k\eta \sin \beta_m\eta \qquad i = n$$

The other integrals (2.6) are equal to zero.
We reduce the system of equations (2.5) to the form

$$A_k = \frac{2P_j a^2}{b\triangle_k} \sin \beta_k\eta_j \sum_{m=1}^{\infty} A_m \sin \beta_m \eta_j \quad (2.9)$$

Let us replace the load $\overset{(0)}{q_\xi}(\eta)$ by a system of forces $P_1, \dots P_r$. Then we can express the system of equations (2.5) in the form

$$\kappa A_k = \sum_{j=1}^{r} f_j \frac{\sin \beta_k \eta_j}{\Delta k} \cdot \sum_{m=1}^{8} A_m \sin \beta_m \eta_j \qquad (2.10)$$

$$k = 1,2, \dots, \infty$$

We introduce the notations

$$\kappa = \frac{b}{2a^2 P_0}, \, f_j = \frac{P_j}{P_0}$$

where P_0 is any of the forces P_j and $j = 1, \dots r$
We multiply the system of equations (2.10) by $\sin \beta_k\eta_p$ and perform summation with respect to k. Introducing the notation

$$S_\mu = \sum_{k=1}^{\infty} A_k \sin \beta_k \eta_\mu \qquad \mu = p, j$$

we reduce the infinite system of equations (2.10) to r equations

$$\sum_{j=1}^{j=r} (f_j \cdot a_{pj} - \kappa\delta_{pj}) S_j = 0 \qquad (2.11)$$

$$p = 1,2, \dots r$$

where δ_{pj} is Kronnecker's delta and

$$a_{pj} = \sum_{k=1}^{\infty} \frac{\sin \beta_k \eta_p \sin \beta_k \eta_j}{\triangle_k}$$

The buckling condition of the shell is that the determinant of the system of equations (2.11) should be equal to zero. With greater numbers of concentrated forces assumed, the critical load is determined accurately. However, as the number of forces increases, the difficulties of determining the elements of the system of equations (2.11) increase also. These difficulties may be avoided in the following manner. Note that the quantity a_{pj} is expressed by a rapidly converging series. It is sufficient, therefore, without impairing the accuracy of the calculation, to take only a few terms of the a_{pj} series.
We assume therefore that

$$a_{pj} \approx \sum_{k=1}^{k=s} \frac{\sin \beta_k \eta_p \sin \beta_k \eta_j}{\triangle_k}$$

Now, let us write the system of equations in the form

$$\kappa S_p = \sum_{k=1}^{k=s} \frac{\sin \beta_k \eta_p}{\triangle_k} \sum_{j=1}^{j=r} f_j \, S_j \sin \beta_k \eta_j \qquad (2.12)$$

$$p = 1,2, \dots, r$$

and multiply the above system by $f_p \sin\beta_\nu\eta_p$ ($\nu = 1,2,\dots, s$).
Let us perform summation of both members from $p = 1$ to r. After some simple transformations we obtain the following system of equations

$$\sum_{k=1}^{k=s} C_\nu (b_{\nu k} - \delta_{\nu k} \kappa) = 0 \qquad (2.13)$$

$$\nu = 1,2, \dots s$$

where

$$C_k = \sum_{j=1}^{j=r} f_j \, S_j \sin \beta_k \eta_j$$

$$b_{\nu k} = \frac{1}{\triangle_k} \sum_{p=1}^{p=r} f_p \sin \beta_\nu \eta_p \sin \beta_k \eta_p$$

The determinant set equal to zero constitutes the buckling condition of the shell

$$\left\| b_{\nu k} - \delta_{\nu k} \kappa \right\| = 0$$

3. A cylindrical shell with simply supported and rigidly clamped edges

Consider a shell simply supported along all edges and subjected to the action of normal and tangential forces q_x, q_y, q_{xy} and bending moments

$$M(y) = \sum_{k=1}^{\infty} M_k \sin \beta_k y$$

at the edge $x = 0$. The deflection of the plate takes the form

$$w(x, y) = -\iint_\Omega w \left(q_\xi \frac{\partial^2 \bar{w}}{\partial \xi^2} + \dots + 2\bar{w} \frac{\partial^2 q_{\xi\eta}}{\partial\xi\partial\eta}\right) d\xi \, d\eta + w_M(x, y) \qquad (3.1)$$

where $w_M(x, y)$ is the deflection of the shell due to the moment $M(y)$ at the edge $x = 0$, assuming that

$$q_\xi = q_\eta = q_{\xi\eta} = 0$$

This deflection will take the form [2][1]

$$w_M (x, y) = \frac{2}{a} \sum_{i,k}^{\infty} \frac{M_k \alpha_i}{\triangle_{ik}} \sin \alpha_i x \sin \beta_k y \tag{3.2}$$

By substituting (2.4) and (3.2) we obtain, after integration and rearrangement, the following system of equations:

$$A_{ik} = \frac{4}{ab \triangle_{ik}} \sum_{m,n}^{\infty} A_{n,m} G_{nimk} + \frac{2}{a} \frac{M_k \alpha_i}{\triangle_{ik}} \tag{3.3}$$

$$i, k = 1,2, \ldots, \infty$$

Now let us choose the function $M(y)$ to satisfy the condition of rigid clamping along the edge $x = 0$, i. e., the condition

$$\frac{\partial w\ (0, y)}{\partial y} = 0 \tag{3.4}$$

By use of the series (2.4), this can be written as

$$\sum_{i,k}^{\infty} A_{ik} \alpha_i \sin \beta_k y = 0$$

Since this sum should be equal to zero for any y, the condition of rigid clamping will be expressed as

$$\sum_{i=1}^{\infty} A_{ik} \alpha_i = 0 \tag{3.5}$$

Multiplying (3.3) by α_i and summing up with respect to i, we obtain

$$\frac{4}{ab} \sum_{i,k}^{\infty} \frac{\alpha_i}{\triangle_{ik}} \sum_{n,m}^{\infty} A_{n,m} G_{nimk} + \frac{2}{a} M_k \sum_{i=1}^{\infty} \frac{\alpha^2_i}{\triangle_{ik}} = 0 \tag{3.6}$$

From the last equation, let us determine M_k and sum up with respect to p instead of i. Substituting M_k in the system of equations (3.3) and rearranging, we obtain the system of equations:

$$A_{ik} = \frac{4}{ab \triangle_{ik}} \sum_{n,m}^{\infty} A_{n,m} \left(G_{nimk} - \alpha_i \frac{\sum_{p=1}^{\infty} \frac{\alpha_p}{\triangle_{pk}} G_{npmk}}{\sum_{p=1}^{\infty} \frac{\alpha^2_p}{\triangle_{pk}}} \right) \tag{3.7}$$

$$i, k = 1,2, \ldots, \infty$$

[1]) See equation (1.8) of the paper mentioned.

The determinant of this system taken equal to zero constitutes the buckling condition of a shell rigidly clamped at the edge $x = 0$, and simply supported along the remaining edges.

The system of equations (3.7) may be obtained in another way. As the starting point of our considerations let us assume the homogeneous integral equation

$$w\ (x, y) = - \iint_{\Omega} w \left(q_\xi \frac{\partial^2 \bar{w}}{\partial \xi^2} + q_\eta \frac{\partial^2 \bar{w}}{\partial \eta^2} + \ldots + 2 \bar{w} \frac{\partial^2 q_{\xi\eta}}{\partial \xi\, \partial \eta} \right) d\xi\, d\eta \tag{3.8}$$

In this equation $\bar{w}\,(x, y; \xi, \eta)$ denotes the deflection of the shell at the point (x, y) due to the action of the concentrated force $P = 1$ applied at the point (ξ, η) where the function $\bar{w}$ should be selected in such a way that the conditions of rigid clamping along the edge $x = 0$ and free support along the edges $x = a$, y, 0, $y = b$ are satisfied.

The function w should be written in the form (3.9)

$$\bar{w}\,(x, y; \xi, \eta) = \frac{4}{ab} \sum_{n,m}^{\infty} \frac{\sin \alpha_n \xi \sin \beta_m \eta}{\triangle_{n,m}} \sin \alpha_n x \times \sin \beta_m y + \frac{2}{a} \sum_{n,m}^{\infty} \frac{\bar{M}_m \alpha_n \sin \beta_m \eta}{\triangle_{nm}} \sin \beta_m y \sin \alpha_n x$$

Here, $\bar{M}_m$ is the Fourier coefficient of the clamping moment of the shell along the edge $x = 0$:

$$\bar{M}(y) = \sum_{m=1}^{\infty} \bar{M}_m \sin \beta_m y$$

From the condition of rigid clamping of the shell along the edge we find

$$\bar{M}_m = - \frac{\frac{2}{b} \sum_{\tau=1}^{\infty} \frac{\alpha_\tau \sin \alpha_\tau \xi}{\triangle_{\tau m}}}{\sum_{\tau=1}^{\infty} \frac{\alpha^2_\tau}{\triangle_{\tau m}}}$$

Substituting this value in the equation (3.9) we obtain finally

$$\bar{w}\,(x, y; \xi, \eta) = \frac{4}{ab} \sum_{n,m}^{\infty} \frac{\sin \beta_m \eta}{\triangle_{nm}} \sin \alpha_n x \times \sin \beta_m y \left(\sin \alpha_n \xi - \alpha_n \frac{\sum_{\tau=1}^{\infty} \frac{\alpha_\tau \sin \alpha_\tau \xi}{\triangle_{\tau m}}}{\sum_{\tau=1}^{\infty} \frac{\alpha^2_\tau}{\triangle_{\tau m}}} \right) \tag{3.10}$$

Substituting the series (2.4) and the function (3.10) in the integral equation (3.8) we obtain the system of equations (3.7) after performing the integrations prescribed.

Returning to the system of equations (3.3) and (3.6) we observe that if $G_{nimk} = 0$ for $n \neq i$, $m \neq k$ (a has a finite value for $i = n$, $m = k$), we can eliminate the coefficients A_{ik} from the above equations and reduce the systems of equations to the simple form

$$\sum_{i=1}^{\infty} \frac{\alpha^2{}_i}{\triangle_{ik} - \frac{4}{ab} G_{ii,kk}} = 0 \qquad (3.11)$$

$$(k = 1,2, \ldots\ldots, \infty)$$

It is easy to show, in view of the equation (2.5), that the form (3.11) is obtained only in the case where r = const, t = const. Then,

$$\frac{4}{ab} G_{ii,\ kk} = q_\xi^{(0)} \alpha^2{}_i + q_\eta^{(0)} \beta_k{}^2$$

For example, let us consider two special cases of the system of equations (3.7):

a) Let the shell be acted on by a unique load $q_\xi = q_\xi^{(0)}$ = const.
Then

$$G_{nimk} = q_\xi^{(0)}\ \alpha^2{}_i \frac{ab}{4},\ G_{n\tau mk} = q_\xi^{(0)} \alpha^2{}_\tau \frac{ab}{4} \qquad \begin{matrix} n = i \\ m = k \end{matrix}$$

The system of equations (3.7) will assume the form

$$A_{ik} = - \frac{\alpha_i\, q_\xi^{(0)}}{\triangle_{ik} - \alpha^2{}_i\, q_\xi^{(0)}} \frac{\sum\limits_{\tau=1}^{\infty} \frac{\alpha^3{}_\tau}{\triangle_{\tau k}} A_{\tau k}}{\sum\limits_{\tau=1}^{\infty} \frac{\alpha^2{}_\tau}{\triangle_{\tau k}}}$$

$$(i, k = 1,2, \ldots\ldots, \infty)$$

Using equation (3.11) we can replace this system by the sum, [2],

$$\sum_{i=1}^{\infty} \frac{\alpha^2{}_i}{\triangle_{ik} - \alpha^2{}_i q_\xi^{(0)}} = 0$$

b) Let the shell be acted on by tangential forces $q_{\xi\eta} = q_{\xi\eta}^{(0)}$ = const. In this case we find

$$G_{nimk} = - 2\alpha_i\, \beta_k\, q_{\xi\eta}^{(0)}\, f_{nimk} =$$

$$= 2\alpha_i \beta_k\, q_{\xi\eta}^{(0)} \frac{4\, \alpha_n\, \beta_m}{(\alpha^2{}_n - \alpha_i{}^2)\ (\beta^2{}_m - \beta^2{}_k)}$$

if $n \pm i$, $m \pm k$ are odd

$G_{nimk} = 0$ when $n \pm i$, $m \pm k$ are even.

Then, the system of equations (3.7) takes the form

$$A_{ik} = - \frac{32\alpha_i \beta_k q_{\xi\eta}^{(0)}}{ab\, \triangle_{ik}} \sum_{n,m}^{\infty} A_{nm} \frac{\alpha_n\, \beta_m}{(\beta^2{}_m - \beta^2{}_k)}$$

$$\left(\frac{1}{\alpha^2{}_n - \alpha^2{}_i} - \frac{\sum\limits_{r=1}^{\infty} \frac{\alpha^2{}_r}{\triangle_{rk}\, (\alpha^2{}_n - \alpha^2{}_r)}}{\sum\limits_{r=1}^{\infty} \frac{\alpha^2{}_r}{\triangle_{rk}}} \right) \qquad (3.12)$$

$$(i, k = 1,2, \ldots\ldots, \infty)$$

This procedure for a shell rigidly clamped along one edge can be generalised and applied to the case of rigid clamping along two, three and four edges.

Consider first the case of two opposite edges ($x = 0$ and $x = a$, for instance) clamped. Let

$$M\ (y) = \sum_{k=1}^{\infty} M_k \sin \beta_k y \text{ and } M'(y) = \sum_{k=1}^{\infty} M'{}_k \sin \beta_k y$$

denote the clamping moments along the edges $x = 0$ and $x = a$ due to forces exceeding their critical value and acting in the shell.

The integral equation of the problem will assume the form

$$w(x, y) = - \iint_\Omega w \left(q_\xi \frac{\partial^2 \bar{w}}{\partial \xi^2} + \ldots + 2\, \bar{w} \frac{\partial^2 q_{\xi\eta}}{\partial \xi \partial \eta}\right) d\xi d\eta +$$

$$+ w_M(x, y) + w_{M}{}'(x, y) \qquad (3.13)$$

and the boundary conditions are

$$\frac{\partial w\ (0,\ y)}{\partial x} = 0,\ \frac{\partial w\ (a,\ y)}{\partial x} = 0 \qquad (3.14)$$

The following notations are introduced in equation (3.13)

$$w_M = \frac{2}{a} \sum_{i=1}^{\infty} \frac{M_k\, \alpha_i}{\triangle_{ik}} \sin\, \alpha_i\, x \sin\ \beta_k\, y$$

$$w_{M'} = - \frac{2}{a} \sum_{i=1}^{\infty} \frac{M'{}_k \alpha_i \cos i\pi}{\triangle_{ik}} \sin\, \alpha_i\, x \sin \beta_k\, y$$

Substituting in equation (3.13) the expressions (2.4), (2.3) w_M and $w_{M'}$ we obtain the system of equations

$$A_{ik} = \frac{4}{ab\triangle_{ik}} \sum_{n,m}^{\infty} A_{n\,m}\, G_{nimk} +$$

$$+ \frac{2}{a\triangle_{ik}} (M_k\ \alpha_i - M'{}_k\ \alpha_i \ \cos\ i\pi) \qquad (3.15)$$

$$(i, k = 1,2, \ldots\ldots, \infty)$$

Taking the expressions (2.4) into consideration we rewrite the boundary conditions (3.14) in the form

$$\sum_{i=1}^{\infty} A_{ik}\,\alpha_i = 0, \quad \sum_{i=1}^{\infty} A_{ik}\,\alpha_i \cos i\pi = 0 \qquad (3.16)$$

Substituting in the equations (3.16) the value of A_{ik} we obtain from the system of equations (3.15) the system of equations

$$\frac{4}{ab} \sum_{i=1}^{\infty} \frac{\alpha_i}{\triangle_{ik}} \sum_{n,m}^{\infty} A_{nm}\;G_{nimk} +$$

$$+\ \frac{2}{a} \left(M_k \sum_{i=1}^{\infty} \frac{\alpha^2{}_i}{\triangle_{ik}} - M'_k \sum \frac{\alpha^2{}_i \cos i\pi}{\triangle_{ik}}\right) = 0$$

$$(3.17)$$

$$\frac{4}{ab} \sum_{i=1}^{\infty} \frac{\alpha_i \cos i\pi}{\triangle_{ik}} \sum_{n,m}^{\infty} A_{km}\;G_{nimk} +$$

$$+ \frac{2}{a} \left(M_k \sum_{i=1}^{\infty} \frac{\alpha^2{}_i \cos i\pi}{\triangle_{ik}} - M'_k \sum_{i=1}^{\infty} \frac{\alpha^2{}_i \cos^2 i\pi}{\triangle_{ik}}\right) = 0$$

Eliminating from the equations (3.17) the coefficients M_k, M'_k we obtain a unique system of equations. The determinant of this system set equal to zero constitutes the buckling condition of the shell.

If the forces acting on the shell are symmetrical in relation to the line $x = \alpha/2$ (the forces q_x and q_y for $q_{xy} = 0$ are the only ones remaining) assume $M_k = M'_k$ for symmetric and $M_k = -M'_k$ for asymmetric buckling.

In the special case where $q_\xi = q_\xi^{(0)}$ and $q_\eta = q_\eta^{(0)}$ we can, by eliminating A_{ik}, reduce the systems of equations (3.15) and (3.17) to the simple relation

$$\sum_{i=1}^{\infty} \frac{\alpha^2{}_i\,(M_k - M'_k \cos i\pi)}{\triangle_{ik} - (q_\xi^{(0)} \alpha^2{}_i + q_\eta^{(0)} \beta^2{}_k)} = 0 \quad (3.18)$$

$$(k = 1,2,\ldots\ldots, \infty)$$

For symmetric buckling of the shell with $M_k = M'_k$ the buckling condition will be given by the expression:

$$\sum_{i=1,3,5,\ldots}^{\infty} \frac{\alpha^2{}_i}{\triangle_{ik} - (q_\xi^{(0)} \alpha^2{}_i + q_\eta^{(0)} \beta^2{}_k)} = 0$$

$$(k = 1,2,\ldots\ldots, \infty)$$

For asymmetric buckling with $M_k = -M'_k$ the buckling condition is

$$\sum_{i=2,4,\ldots}^{\infty} \frac{\alpha^2{}_i}{\triangle_{ik} - (q_\xi^{(0)} \alpha^2{}_i + q_\eta^{(0)} \beta^2{}_k)} = 0$$

$$(k = 1,2,\ldots.. \infty)$$

Consider the case of a shell rigidly clamped along the edges $x = 0$, $y = 0$. The integral of the problem will take the form

$$w\,(x,\,y) = -\iint_{\Omega} w\,\Big[q_\xi \frac{\partial^2 \bar{w}}{\partial \eta^2} + \ldots +$$

$$+ 2\,\bar{w}\,\frac{\partial^2 q_{\xi\eta}}{\partial \xi\,\partial \eta}\Big]\,d\xi\,d\eta + w_M\,(x, y) + w_{M'}\,(x, y)$$

$$(3.19)$$

$$\frac{\partial w\,(0, y)}{\partial x} = 0, \quad \frac{\partial w\,(x, 0)}{\partial y} = 0$$

where

$$w_M = \frac{2}{a} \sum_{i,k}^{\infty} \frac{M_k\,\alpha_i}{\triangle_{ik}} \sin \alpha_i x \sin \beta_k y$$

$$(3.20)$$

$$w_M{}' = \frac{2}{b} \sum_{i,k}^{\infty} \frac{M'_i\,\beta_k}{\triangle_{ik}} \sin \alpha_i x \sin \beta_k y$$

Introducing (2.4), (2.3) and (3.20) in the integral equation (3.19), we obtain, after integration, the system of equations

$$A_{ik} = \frac{4}{ab\,\triangle_{ik}} \sum_{n,m}^{\infty} A_{nm}\;G_{nimk} +$$

$$(3.21)$$

$$+\ \frac{2}{a}\,\frac{M_k\,\alpha_i}{\triangle_{ik}} + \frac{2}{b}\,\frac{M'_i\,\beta_k}{\triangle_{ik}}$$

$$i, k = 1, \ldots, \infty$$

From the boundary conditions, which reduce to the relations

$$\sum_{i=1}^{\infty} A_{ik}\,\alpha_i = 0, \quad \sum_{k=1}^{\infty} A_{ik}\,\beta_k = 0 \qquad (3.22)$$

we obtain two systems of equations:

$$\frac{4}{ab} \sum_{i=1}^{\infty} \frac{\alpha_i}{\triangle_{ik}} \sum_{n,m}^{\infty} A_{nm}\;G_{nimk} + \frac{2}{a}\;M_k \sum_{i=1}^{\infty} \frac{\alpha^2{}_i}{\triangle_{ik}} +$$

$$+\ \frac{2\,\beta_k}{b} \sum_{i=k}^{\infty} \frac{M'_i\,\alpha_i}{\triangle_{ik}} = 0$$

$$(3.23)$$

$$\frac{4}{ab} \sum_{i=1}^{\infty} \frac{\beta_k}{\triangle_{ik}} \sum_{m,n}^{\infty} A_{nm}\;G_{nimk} + \frac{2}{a}\;\alpha_i \sum_{k=1}^{\infty} \frac{M_k\,\beta_k}{\triangle_{ik}} +$$

$$+\ \frac{2\,M'_i}{b} \sum_{k=1}^{\infty} \frac{\beta^2{}_k}{\triangle_{ik}} = 0$$

In the general case, the systems of equations (3.21) and (3.23) cannot be reduced to one system. This can be done in the special case

$$q_\xi = \overset{(0)}{q_\xi};\ q_\eta = \overset{(0)}{q_\eta},\ q_{\xi\eta} = 0,$$

where we have

$$G_{nimk} = \frac{ab}{4}\left(\overset{(0)}{q_\xi}\alpha^2{}_i + \overset{(0)}{q_\eta}\beta^2{}_k\right) \text{ for } n = i, m = k$$

The system of equations (3.21) takes the form

$$A_{ik} = \frac{2}{aD_{ik}}\left(M_k\,\alpha_i + \frac{a}{b}M'{}_i\,\beta_k\right)\ i,k = 1,2\ldots,\infty$$

where

$$D_{ik} = \triangle_{ik} - \left(\overset{(0)}{q_\xi}\alpha^2{}_i + \overset{(0)}{q_\eta}\beta^2{}_k\right)$$

Substituting A_{ik} in equations (3.23), we obtain

$$M_k\sum_{i=1}^{\infty}\frac{\alpha^2{}_i}{D_{ik}} + \frac{b}{a}\,\beta_k\sum_{i=1}^{\infty}\frac{M'{}_i\,\alpha_i}{D_{ik}} = 0$$

$$\alpha_i\sum_{k=1}^{\infty}\frac{M_k\,\beta_k}{D_{ik}} + \frac{b}{a}M'{}_i\sum_{k=1}^{\infty}\frac{\beta^2{}_k}{D_{ik}} = 0$$

Eliminating M'_i we obtain finally

$$M_k - \sum_{r=1}^{\infty}M_r\,K_{kr} = 0 \qquad k = 1,2,\ldots,\infty \quad (3.24)$$

where

$$K_{kr} = \frac{\beta_k\,\beta_r}{E_k}\sum_{t=1}^{\infty}\frac{\alpha_t^2}{F_t\,D_{tr}\,D_{tk}}$$

and

$$E_k = \sum_{\alpha=1}^{\infty}\frac{\alpha^2{}_\alpha}{D_{\alpha k}},\ F_t = \sum_{\beta=1}^{\infty}\frac{\beta^2{}_\beta}{D_{\beta t}}$$

The determinant of the system of equations (3.24) taken equal to zero is the buckling condition of the shell.

4. A cylindrical shell with ribs

Consider a cylindrical shell simply supported along the edges, subjected to the action of the force q_x, q_y, q_{xy} and having a rib parallel to the x-axis at the distance $\bar{y}$ from that axis. We assume that the rib is symmetric in relation to the shell and that its torsional rigidity is very small.

The deflection of the shell with a rib due to the action of a force exceeding its critical value can be described by the non-homogeneous integral equation

$$w(x, y) = -\iint_\Omega w\left(q_\xi\frac{\partial^2\bar{w}}{\partial\xi^2} + \ldots + 2\,\bar{w}\,\frac{\partial^2 q_{\xi\eta}}{\partial\xi\,\partial\eta}\right)d\xi\,d\eta + w_p(x, y) \quad (4.1)$$

In this equation $w_p(x, y)$ denotes the shell deflection due to the action of the rib on the shell. This force

$$p = \sum_{i=1}^{\infty}p_i\sin\alpha_i x$$

acts along the line $y = \bar{y}$ and the corresponding deflection of the shell will be equal to (see [2])

$$w_p(x, y) = \frac{2}{b}\sum_{i,k}^{\infty}\frac{p_i\sin\beta_k\bar{y}}{\triangle_{ik}}\sin\alpha_i x\sin\beta_k y \quad (4.2)$$

Introducing (2.4), (2.3) and (4.2) in equation (4.1), we transform this equation into the system of linear equations

$$A_{ik} = \frac{4}{ab\,\triangle_{ik}}\sum_{n=m}^{\infty}A_{n\,m}\,G_{nimk} + \frac{2\,p_i}{b}\cdot\frac{\sin\beta_n\bar{y}}{\triangle_{ik}} \quad (4.3)$$

$$i, k = 1,2,\ldots,\infty$$

The deflection of the rib is expressed by the differential equation

$$EI\frac{d^4 w_R}{dx^4} + Aq\frac{\partial^2 w_R}{\partial x^2} = -p \quad (4.4)$$

EI denoting the flexural rigidity of the rib, A — the cross-section of the rib. The solution of equation (4.4) can be expressed in the form of the trigonometric series

$$w_R = -\sum_{i=1}^{\infty}\frac{p_i\sin\alpha_i x}{EI\,\alpha^4{}_i - Aq\,\alpha^2{}_i} \quad (4.5)$$

The deflection of the shell and of the rib along the line $y = \bar{y}$ being the same, we have

$$w(x, \bar{y}) = w_R(x)$$

or

$$\sum_{i=k}^{\infty}A_{ik}\sin\alpha_i x\sin\beta_k\bar{y} = \sum_{i=1}^{\infty}\frac{p_i\sin\alpha_i x}{EI\,\alpha_i^4 - Aq\,\alpha^2{}_i}$$

This being valid for any x, we have

$$\sum_{k=1}^{\infty} A_{ik} \sin \beta_k \bar{y} + \frac{p_i}{EI\,\alpha_i^4 - Aq\,\alpha_i^2} = 0 \quad (4.6)$$

Substituting A_{ik} from equation (4.3) we obtain

$$\frac{4}{ab} \sum_{i=k}^{\infty} \frac{\sin \beta_k \bar{y}}{\Delta_{ik}} \sum_{n,m}^{\infty} A_{nm}\, G_{nimk} + \quad (4.7)$$

$$+ \frac{2}{b} p_i \sum_{i=1}^{\infty} \frac{\sin^2 \beta_k \bar{y}}{\Delta_{ik}} + \frac{p_i}{EI\,\alpha_i^4 - Aq\,\alpha_i^2} = 0$$

Eliminating the coefficient p_i from (4.3) and (4.7), we obtain the system of equations:

$$A_{ik} = \frac{4}{ab\,\Delta_{ik}} \sum_{n,m}^{\infty} A_{nm} \times$$

$$\times \left(G_{nimk} - \sin \beta_k \bar{y} \frac{\sum_{r=1}^{\infty} \frac{G_{nimr}}{\Delta_{ir}} \sin \beta_r \bar{y}}{\sum_{r=1}^{\infty} \frac{\sin^2 \beta_r \bar{y}}{\Delta_{ir}} + \frac{1}{EI\,\alpha_i^4 - Aq\,\alpha_i^2}} \right) \quad (4.8)$$

$$i, k = 1, 2, \ldots, \infty$$

The above system of equations can be obtained in another way. If the Green's function for a shell simply supported and stiffened by a rib is denoted by $\bar{w}(x, y; \xi, \eta)$ the stability problem reduces to the solution of the homogeneous integral equation

$$w(x, y) = -\iint_{\Omega} w \left(q_\xi \frac{\partial^2 \bar{w}}{\partial \xi^2} + q_\eta \frac{\partial^2 \bar{w}}{\partial \eta^2} + \ldots + \right.$$

$$\left. + \frac{2\, \bar{w}\, \partial^2 q_{\xi\eta}}{\partial \xi\, \partial \eta} \right) d\xi\, d\eta \quad (4.9)$$

where the Green's function takes the form

$$\bar{w}(x, y; \xi, \eta) = \frac{4}{ab} \sum_{n,m}^{\infty} \frac{\sin \alpha_n x \sin \beta_m y}{\Delta_{nm}} \times$$

$$\times \sin \alpha_n \xi \left(\sin \beta_m \eta - \right.$$

$$\left. - \sin \beta_m \bar{y} \frac{\sum_{r=1}^{\infty} \frac{\sin \beta_r \eta \, \sin \beta_r \bar{y}}{\Delta_{nr}}}{\sum_{r=1}^{\infty} \frac{\sin^2 \beta_r y}{\Delta_{nr}} + \frac{1}{EI\,\alpha_n^4 - Aq\,\alpha_n^2}} \right) \quad (4.10)$$

$$i, k = 1, 2, \ldots, \infty$$

Substituting the series (2.4) and (4.10) in equation (4.9) we obtain, after integration, the system of equations (4.8).

Let us consider two special problems.

a) Let the shell be acted on by a unique load $q_\xi = \overset{(0)}{q_\xi} = \text{const.}$

Then,

$$G_{nimk} = \overset{(0)}{q_\xi} \alpha_i^2 \frac{ab}{4}, \quad G_{nimr} = \overset{(0)}{q_\xi} \alpha_i^2 \frac{ab}{4} \qquad \begin{matrix} n = i \\ m = k \end{matrix}$$

The system of equations (4.8) takes the form

$$A_{ik} = \frac{\overset{(0)}{q_\xi} \sin \beta_k \bar{y}_i \alpha_i^2 \sum_{r=1}^{\infty} \frac{\sin \beta_r \bar{y}}{\Delta_{ir}} A_{ir}}{\left(\Delta_{ik} - \overset{(0)}{q_\xi} \alpha_i^2\right) \left(\sum_{r=1}^{\infty} \frac{\sin^2 \beta_r y}{\Delta_{ir}} + \frac{1}{EI\,\alpha_i^4 - Aq\,\alpha_i^2} \right)} \quad (4.11)$$

This system may be transformed.

From equation (4.3) we obtain

$$A_{ik} = \frac{2}{b}\, p_i \frac{\sin \beta_k \bar{y}}{\Delta_{ik} - \alpha_i^2 \overset{(0)}{q_\xi}},$$

and substituting A_{ik} in the relation (4.6) we have, ([2], [3])[1],

$$\frac{2}{b} \sum_{k=1}^{\infty} \frac{\sin^2 \beta_k \bar{y}}{\Delta_{ik} - \alpha_i^2 \overset{(0)}{q_\xi}} + \frac{1}{EI\,\alpha_i^4 - Aq\,\alpha_i^2} = 0 \quad (4.12)$$

$$(i = 1, 2, \ldots, \infty)$$

b) Consider a load $q_{\xi\eta} = \overset{(0)}{q_{\xi\eta}} = \text{const.}$, $q_\xi = q_\eta = 0$.

In this case we have

$$G_{nimk} = -\frac{8\, \alpha_n \beta_m \alpha_i \beta_k}{(\alpha_n^2 - \alpha_i^2)(\beta_m^2 - \beta_k^2)} \overset{(0)}{q_{\xi\eta}}$$

if $n \pm i$, $m \pm k$ are odd and $G_{nimk} = 0$ if $n \pm i$, $m \pm k$ are even.

We find in a similar manner

$$G_{nimr} = -\frac{8\, \alpha_n \beta_m \alpha_i \beta_r}{(\alpha_n^2 - \alpha_i^2)(\beta_m^2 - \beta_k^2)} \overset{(0)}{q_{\xi\eta}}$$

if $n \pm i$, $m \pm r$ are odd and $G_{nimr} = 0$; if $n \pm i$, $m \pm r$ are even. Substituting the above values in the system of equations (4.8) we obtain

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1) See equation (8.11) in Ref. [3] and equation (1.31) in Ref. [2].

$$A_{ik}=-\frac{32\alpha_i q_{\xi\eta}^{(0)}}{ab\,\Delta_{ik}}\sum_{n,m}^{\infty}A_{nm}\frac{\alpha_n\,\beta_m}{(\alpha_n^2-\alpha_i^2)}\left(\frac{\beta_k}{(\beta_m^2-\beta_k^2)}-\right.$$

$$\left.-\sin\beta_k\bar{y}\frac{\sum\limits_{r=1}^{\infty}\frac{\beta_r\sin\beta_r\bar{y}}{\Delta_{ir}(\beta^2_m-\beta^2_k)}}{\sum\limits_{r=1}^{\infty}\frac{\sin^2\beta_r\bar{y}}{\Delta_{ik}}+\frac{1}{EI\,\alpha_i^4-qA\,\alpha_i^2}}\right) \tag{4.13}$$

$$i,k=1,2,\ldots,\infty$$

Let us substitute $EI=\infty$, $A=\infty$ in the system of equations (4.8).

In this case we are concerned with a rigid rib. If we take the determinant of the system (4.8) (with $EI=\infty$ $A=\infty$) equal to zero, we obtain the buckling condition for a two-span cylindrical shell with the intermediate support at the distance $y=\bar{y}$ from the x-axis.

In the special case of load distribution symmetrical in relation to the axis $y=b/2$ and for a symmetrical form of buckling, we have, on the axis of symmetry, $w=0$ and $\partial w/\partial y=0$. We are concerned, therefore, with buckling of a shell with the sides equal to a and $b/2$, simply supported along the edges $x=0$, $x=a$, $y=0$, and rigidly fixed along the edge $b/2$.

Let us note, in addition, that in the case of a shell with a rib we can pass to the case of a shell rigidly clamped along the line $y=0$. Putting $EI=\infty$, $A=\infty$ in the system of equations (4.8), we move the rib to the support $y=0$.

For small values of y we can take $\sin\beta_k\bar{y}\rightarrow\beta_k\bar{y}$. The system of equations (4.8) will take the form

$$A_{ik}=\frac{4}{ab\,\Delta_{ik}}\sum_{n,m}^{\infty}A_{nm}\left(G_{nimk}-\beta_k\frac{\sum\limits_{r=1}^{\infty}\frac{\beta_r}{\Delta_{ir}}G_{nimr}}{\sum\limits_{r=1}^{\infty}\frac{\beta^2_r}{\Delta_{ir}}}\right)$$

$$i,k=1,2,\ldots,\infty$$

This formula is analogous to equation (3.7) derived in the case of a shell rigidly clamped at the edge $x=0$.

The above considerations for one rib can be generalised to the case of more longitudinal and transversal ribs.

Summary

The object of this paper was an accurate solution of the buckling problem of a cylindrical shell simply supported or rigidly clamped along the edges and having ribs. This aim has been attained by reducing the solution of an integral equation to an infinite system of secular equations. Our results are also applicable to rectangular plates. The passage from a shell to a plate can be effected by putting

$$\lambda=\frac{Eh}{R^2}\;(R\rightarrow\infty)$$

equal to zero in the expression Δ_{ik}.

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