BULLETIN
DE
L'ACADÉMIE POLONAISE DES SCIENCES

SÉRIE DES SCIENCES TECHNIQUES

Volume XII, Numéro 9

VARSOVIE 1964

BULLETIN DE L'ACADÉMIE
POLONAISE DES SCIENCES
4. Série des sciences techniques
Volume XII, No. 9 — 1964

APPLIED MECHANICS

Green Functions for the Thermoelastic Medium. II

by

W. NOWACKI

Presented on June 17, 1964

1. In paper [1] have been determined the Green functions for the displacements and temperature produced in the infinite thermoelastic space by the concentrated forces. These functions have been obtained from wave equations as derived from the fundamental differential equations of the thermoelasticity by decomposition of the displacement vector into the potential and ıotational parts.

In this paper a more direct method of obtaining the Green functions will be given starting from the disjoint system of differential equations of the theımoelasticity.

The disjoint equations have been derived for the quasi-static problem by V. Ionescu-Cazimir, [2], and later generalized by S. Kaliski, [3], and in another way by J. S. Podstrigach, [4], and D. Rüdiger, [5]. These equations are for the thermoelasticity of analogous importance as the Galerkin functions for elastostatics and elastodynamics.

We shall consider successively the Green functions for the dynamic problem in the cases of concentrated and distributed forces and heat sources.

We proceed in our considerations from the linearized equations of thermoelasticity, [5],

$$\mu\nabla^2 \boldsymbol{u} + (\lambda+\mu)\,\mathrm{grad}\,\mathrm{div}\,\boldsymbol{u} + \boldsymbol{X} = \gamma\,\mathrm{grad}\,\theta + \varrho\ddot{\boldsymbol{u}}, \tag{1.1}$$

$$\nabla^2\theta - \frac{1}{\varkappa}\dot{\theta} - \eta\,\mathrm{div}\,\dot{\boldsymbol{u}} = -\frac{Q}{\varkappa}. \tag{1.2}$$

The first of the above relations is the equation of motion, in terms of displacements, while the second — the generalized heat conduction equation. In these equations $\boldsymbol{u}$ denotes the displacement vector, $\boldsymbol{X}$ — the vector of body forces, $\theta = T - T_0$ is the difference between the absolute temperature T and the temperature T_0 which describes the natural state of the body, Q is a function characterizing the intensity of the heat sources, μ and λ are the Lamé constants related to the isothermic state, $\varkappa = \lambda_0/\varrho c_\varepsilon$ is a coefficient where λ_0 denotes the heat conductivity coefficient, while ϱ is the density, and c_ε — the specific heat for constant deformation. Further, we have $\eta = \gamma T_0/\lambda_0$ where $\gamma = (3\lambda+2\mu)\, a_t = 3Ka_t$, a_t is the coefficient of linear thermal expansion, and K — the bulk modulus. Moreover, we have denoted $Q = W/\varrho c_\varepsilon$, where W is the quantity of heat created per unit volume and unit time. The functions $\mathbf{u}, \theta, X$ and Q are functions of coordinates and time. The derivatives of these functions with respect to the time variable will be marked with points.

We introduce four solving functions, vectorial functions $\boldsymbol{\varphi} = (\varphi_1, \varphi_2, \varphi_3)$ and the scalar function ψ connecting them with the displacement $\mathbf{u}$ and the temperature θ by the following relations:

$$\mathbf{u} = \Omega\boldsymbol{\varphi} - \operatorname{grad}\operatorname{div}(\Gamma\boldsymbol{\varphi}) + \gamma_0 \operatorname{grad}\psi, \tag{1.3}$$

$$\theta = \eta\partial_t \operatorname{div}(\Box_2^2\, \boldsymbol{\varphi}) + (1+a)\, \Box_1^2\, \psi, \tag{1.4}$$

where

$$\Omega = (1+a)\, \Box_1^2\, \Box_3^2 - \gamma_0\, \eta\partial_t \nabla^2, \quad \Gamma = a\, \Box_3^2 - \gamma_0\, \eta\partial_t \tag{1.5}$$

and wherein we have denoted

$$\Box_\alpha^2 = \nabla^2 - \frac{1}{c_\alpha^2}\partial_t^2, \quad \alpha = 1, 2, \quad c_1^2 = \frac{\lambda+2\mu}{\varrho}, \quad c_2^2 = \frac{\mu}{\varrho},$$

$$\Box_3^2 = \nabla^2 - \frac{1}{\varkappa}\partial_t, \quad a = \frac{\lambda+\mu}{\mu}, \quad \gamma_0 = \frac{\mu}{\mu}.$$

Introducing (1.3) and (1.4) into (1.1) and (1.2), we arrive at the system of four disjoint equations

$$\Box_2^2 (\Box_1^2\, \Box_3^2 - \eta m\partial_t \nabla^2)\, \varphi_i + \frac{X_i}{\varrho c_1^2} = 0, \quad i = 1, 2, 3, \tag{1.6}$$

$$(\Box_1^2\, \Box_3^2 - \eta m\partial_t \nabla^2)\, \psi + \frac{Q\mu}{\varrho c_1^2} = 0. \tag{1.7}$$

The above equations will be used for determining the Green functions in the case of concentrated forces and heat sources acting in an infinite thermoelastic medium.

2. Effect of concentrated forces

Let us first examine the effect of the body force $X = (X_1, 0, 0)$ acting in the infinite thermoelastic body, along the axis x_1. In this case, assuming that no heat sources occur, we have $\varphi_2 = \varphi_3 = 0$ and $\psi = 0$.

The system of Eqs. (1.6) and (1.7) reduces to the equation

$$\Box_2^2 (\Box_1^2\, \Box_3^2 - \eta m\partial_t \nabla^2)\, \varphi_1 + \frac{X_1}{\varrho c_1^2} = 0. \tag{2.1}$$

Let us assume that the body force $X_1(x, t)$ is a harmonic function of the time variable. Then, substituting it into Eq. (2.1)

$$X_1 = (x, t) = X_1^*(x)\, e^{i\omega t}, \quad \varphi_1(x, t) = \varphi_1^*(x, \omega)\, e^{i\omega t},$$

we reduce Eq. (2.1) to the form

$$(\nabla^2 - k_1^2)(\nabla^2 - k_2^2)(\nabla^2 + \tau^2)\,\varphi_1^* = -\frac{X_1^*}{\varrho c_1^2}. \tag{2.2}$$

Here we used the following notations

$$k_1^2 + k_2^2 = q\,(1+\varepsilon) - \sigma^2, \quad k_1^2 k_2^2 = -\,q\sigma^2,$$

$$q = \frac{i\omega}{\varkappa}, \quad \sigma^2 = \frac{\omega^2}{c_1^2}, \quad \tau^2 = \frac{\omega^2}{c_2^2}, \quad \varepsilon = \eta m \varkappa, \quad m = \frac{\gamma}{c_1^2 \varrho}.$$

The quantities k_1^2 and k_2^2 are the roots of the biquadratic equation

$$k^4 + k^2\,[\sigma^2 - q\,(1+\varepsilon)] - q\sigma^2 = 0.$$

The roots k_α ($\alpha = 1, 2, 3, 4$) are complex quantities, functions of the parameter ω. A detailed analysis of these roots has been given in [6]. In what follows we consider only the roots k_1, k_2 with positive real parts.

Eq. (2.2) will be presented in the operator form

$$D_1 D_2 D_3\,\varphi_1^* = -\frac{X_1^*}{\varrho c_1^2}, \tag{2.3}$$

where

$$D_\alpha = \nabla^2 - k_\alpha^2, \quad \alpha = 1, 2, \quad D_3 = \nabla^2 + \tau^2.$$

The particular integrals Eq. (2.3) can be presented in the form

$$\varphi_1^* = \frac{1}{k_1^2 - k_2^2}\left(\frac{F_1^*}{k_1^2+\tau^2} - \frac{F_2^*}{k_2^2+\tau^2} + \frac{k_1^2 - k_2^2}{(k_1^2+\tau^2)(k_2^2+\tau^2)}F_3^*\right), \tag{2.4}$$

where the functions F_α^*, ($\alpha = 1, 2$) satisfy the familiar wave equation

$$D_\alpha F_\alpha^* = -\frac{X_1^*}{\varrho c_1^2}, \quad \alpha = 1, 2, 3. \tag{2.5}$$

Let now the concentrated force, acting along the axis x_1, be applied at the origin of the coordinate system. Then we have $X_1^* = \delta(x_1)\,\delta(x_2)\,\delta(x_3)$, and the particular integral of the Eqs. (2.5) are given in the form of functions:

$$F_\alpha^* = \frac{1}{4\pi\varrho c_1^2 R}\,e^{-k_\alpha R}, \quad \alpha = 1, 2, \quad F_3^* = \frac{1}{4\pi\varrho c_1^2 R}\,e^{-i\tau R}, \tag{2.6}$$

$$R = (x_1^2 + x_2^2 + x_3^2)^{1/2}.$$

Inserting (2.6) into (2.4), we obtain the following expression for the function φ_1^*:

$$\varphi_1^* = \frac{1}{4\pi\varrho c_1^2 (k_1^2 - k_2^2) R}\left[\frac{c^{-k_1 R}}{k_1^2+\tau^2} - \frac{c^{-k_2 R}}{k_2^2+\tau^2} + \frac{k_1^2 - k_2^2}{(k_1^2+\tau^2)(k_2^2+\tau^2)}\,e^{-i\tau R}\right]. \tag{2.7}$$

The functions u_i^* and θ^* will be determined from (1.3) and (1.4), thus

$$(2.8)\quad u_1^* = \Omega\varphi_1^* - \partial_1^2 \Gamma^* \varphi_1^*, \quad u_2^* = -\partial_1 \partial_2 \Gamma^* \varphi_1^*, \quad u_3^* = -\partial_1 \partial_3 \Gamma^* \varphi_1^*,$$
$$\theta^* = \eta i\omega (\nabla^2 + \tau^2) \partial_1 \varphi_1^*,$$

where

$$\Omega^* = (1+a)(\nabla^2 - k_1^2)(\nabla^2 - k_2^2), \quad \Gamma^* = a(\nabla^2 - q) - (1+a)\varepsilon q.$$

Introducing (2.7) into (2.8) and taking into account the relations

$$(k_a^2 - q)(k_a^2 + \sigma^2) = k_a^2 q\varepsilon, \quad \alpha = 1, 2,$$
$$(k_1^2 + \tau^2)(k_2^2 + \tau^2) = \sigma^2 [a(\tau^2 + q) + (1+a) q\varepsilon],$$

we arrive, after simple transformations, at the following formulae for the amplitudes of the displacements and the temperature

$$(2.9)\quad \begin{cases} u_j^* = -\dfrac{1}{4\pi c_1^2} \partial_1 \partial_j E(R,\omega) + \dfrac{1}{4\pi\varrho c_2^2 R} e^{-i\tau R} \delta_{1j}, \quad j = 1, 2, 3 \\ \theta^* = -\dfrac{q\varepsilon}{4\pi\varrho c_1^2 m} \partial_1 F(R,\omega), \end{cases}$$

where

$$(2.10)\quad \begin{cases} E(R,\omega) = \dfrac{1}{k_1^2 - k_2^2}\left[\dfrac{k_2^2 - q}{k_2^2} \dfrac{e^{-k_2 R}}{R} - \dfrac{k_1^2 - q}{k_1^2} \dfrac{e^{-k_1 R}}{R} - \dfrac{k_1^2 - k_2^2}{\sigma^2} \dfrac{e^{-i\tau R}}{R}\right], \\ F(R,\omega) = \dfrac{1}{k_1^2 - k_2^2}\left[\dfrac{e^{-k_2 R}}{R} - \dfrac{e^{-k_1 R}}{R}\right]. \end{cases}$$

Let us now translate the concentrated force from the origin of the coordinate system to the point (ξ). Then the quantities u_j^* and θ^* will be the Green functions for the case where the concentrated force is acting along the axis x_1. The formulae (2.9) remain valid, provided R now denotes the distance between the points (x) and (ξ).

In general, if the concentrated force acts along to the axis x_j $(j = 1, 2, 3)$, then the Green functions for the displacements and the temperature are given by the formulae

$$(2.11)\quad u_k^{*(j)}(x, \xi, \omega) = -\frac{1}{4\pi\varrho c_1^2}\left((\partial_j \partial_k E(R,u) - \frac{c_1^2}{c_2^2} \delta_{jk} \frac{e^{-i\tau R}}{R}\right),$$

$$(2.12)\quad \theta^{*(j)}(x, \xi, \omega) = -\frac{q}{4\pi\varrho c_1^2 m} \partial_j F(R,\omega), \quad j, k = 1, 2, 3.$$

$$(2.13)\quad u_k^{(j)}(x, \xi, t) = Re\,[e^{i\omega t} u_k^{*(j)}(x, \xi, \omega)],$$
$$\theta^{(j)}(x, \xi, t) = Re\,[e^{i\omega t} \theta^*(x, \xi, \omega)].$$

The formulae (2.11) and (2.12) coincide with those derived by the author in [7] by another method.

Neglecting the coupling of the deformation and temperature fields ($\varepsilon = 0$, $\eta = 0$) i.e., taking into account the relations $k_1^2 = q$, $k_2^2 = -\sigma^2$, we have

$$(2.14)\qquad u_k^{*(j)}(x, \xi, \omega) = -\frac{1}{4\pi\omega^2\varrho}\left[\partial_j\,\partial_k\left(\frac{e^{-i\sigma R} - e^{-i\tau R}}{R}\right) - \tau^2\frac{e^{-i\tau R}}{R}\right],$$

$$(2.15)\qquad \theta^{*(j)}(x, \xi, \omega) = 0, \quad j, k = 1, 2, 3.$$

Let us consider the effect of the uniformly distributed force $X_1(x, t) = X_1(x_1, x_2)\, e^{i\omega t} = \delta(x_1)\,\delta(x_2)\, e^{i\omega t}$ applied at the origin of the coordinate system and acting along the axis x_1. The problem examined is two-dimensional, independent of the variable x_3. Thus, in Eqs. (2.2) we should assume $\nabla^2 = \partial_1^2 + \partial_2^2$. The formulae (2.4) and (2.5) remain valid, however the functions F_α^* occurring there will assume the form

$$(2.16)\qquad F_1^* = \frac{1}{2\pi\varrho c_1^2}K_0(k_1 r), \qquad F_2^* = \frac{1}{2\pi\varrho c_1^2}K_0(k_2 r), \qquad F_3^* = \frac{1}{2\pi\varrho c_1^2}K_0(i\tau r),$$

where $K_0(z)$ is the modified Bessel function of the third kind, and $r = (x_1^2 + x_2^2)^{1/2}$.

Substituting (2.16) into (2.8), and bearing in mind that the derivatives with respect to the variable x_3 vanish, we obtain

$$(2.17)\qquad u_j^* = \frac{1}{2\pi\varrho c_2^2}K_0(i\tau r) - \frac{1}{2\pi\varrho c_1^2}\partial_1\,\partial_j E_0(r_1\,\omega),$$

$$(2.18)\qquad \theta^* = -\frac{q\varepsilon}{2\pi\varrho c_1^2 m}\partial_1 F_0(r_1\,\omega), \quad j = 1, 2,$$

where

$$E_0(r, \omega) = \frac{1}{k_1^2 - k_2^2}\left[\frac{k_2^2 - q}{k_2^2}K_0(k_2 r) - \frac{k_1^2 - q}{k_1^2}K_0(k_1 r) - \frac{k_1^2 - k_2^2}{\sigma^2}K_0(i\tau r)\right],$$

$$F_0(r, \omega) = \frac{1}{k_1^2 - k_2^2}\left[K_0(k_2 r) - K_0(k_1 r)\right].$$

In general, if the uniformly distributed force is applied at the point (ξ), and acts along the axis x_j, then the Green functions for the amplitudes of the displacements and temperature assume the form

$$(2.19)\qquad u_k^{*(j)} = -\frac{1}{2\pi\varrho c_1^2}\left[\partial_j\,\partial_k E_0(r, \omega) - \frac{c_1^2}{c_2^2}K_0(i\tau r)\,\delta_{jk}\right],$$

$$(2.20)\qquad \theta^{*(j)} = -\frac{q}{2\pi\varrho c_1^2 m}\partial_j F_0(r, \omega), \quad j, k = 1, 2.$$

If the coupling of the deformation and temperature fields can be neglected ($\varepsilon = 0$, $k_1^2 = q$, $k_2^2 = -\sigma^2$), then the formulae (2.19) and (2.20) reduce to the relations

$$(2.21)\qquad u_k^{*(j)} = -\frac{1}{2\pi\omega^2\varrho}\left[\partial_j\,\partial_k\left(K_0(i\sigma r) - K_0(i\tau r)\right)\right] - \tau^2 K_0(i\tau r)\,\delta_{jk}], \; -j, k = 1, 2,$$

$$(2.22)\qquad \theta^{*(j)} = 0.$$

3. The effect of heat sources

Let in the infinite thermoelastic medium act the heat source $Q(x, t) = Q(x)\, e^{i\omega t}$. If no body forces occur, then the function $\boldsymbol{\varphi}$ vanishes, and we have at our disposal the equation

$$(\square_1^2 \square_3^2 - \eta m \partial_t)\, \psi + \frac{Q\mu}{\varkappa \varrho c_1^2} = 0, \tag{3.1}$$

whence we have

$$D_1 D_2 \psi = -\frac{Q^* \mu}{\varkappa \varrho c_1^2}. \tag{3.2}$$

The particular integral of the above equation is

$$\psi^* = \frac{1}{k_1^2 - k_2^2} (F_1^* - F_2^*). \tag{3.3}$$

The functions F_a^*, $(a = 1, 2)$, will be determined as the particular integrals of the equations

$$D_a F_a^* = -\frac{Q^* \mu}{\varkappa \varrho c_1^2}. \tag{3.4}$$

For the concentrated heat source $Q^*(x) = \delta(x_1)\, \delta(x_2)\, \delta(x_3)$ acting at the origin of the coordinate system, we obtain

$$F_a^* = \frac{\mu}{4\pi \varrho c_1^2 \varkappa} \frac{e^{-k_a R}}{R}, \quad a = 1, 2. \tag{3.5}$$

Thus, we have

$$\psi^* = -\frac{\mu}{4\pi \varrho c_1^2 \varkappa} F(R_1 \omega), \tag{3.6}$$

where $F(R, \omega)$ is determined as in the previous section.

The amplitudes of the displacements and temperature will be found from the formulae (1.3) and (1.4)

$$u_j^* = \gamma_0 \partial_i \psi^*, \quad \theta^* = (1 + a)(\nabla^2 + \sigma^2)\, \psi^*, \quad j = 1, 2, 3. \tag{3.7}$$

Inserting (3.6) into (3.7), we arrive at the relations

$$u_j^* = -\frac{m}{4\pi\varkappa} \partial_j F(R, \omega), \quad j = 1, 2, 3, \tag{3.8}$$

$$\theta^* = -\frac{1}{4\pi\varkappa} H(R, \omega), \tag{3.9}$$

where

$$H(R, \omega) = \frac{1}{k_1^2 - k_2^2} \left[(k_2^2 + \sigma^2) \frac{e^{-k_2 R}}{R} - (k_1^2 + \sigma^2) \frac{e^{-k_1 R}}{R} \right].$$

Translating the concentrated unit heat source from the origin of the coordinate system to the point $(\boldsymbol{\xi})$, denoting by R the distance between the points $(\boldsymbol{x})$ and $(\boldsymbol{\xi})$, and taking the real parts of the functions, we obtain the following Green functions for the displacements and the temperature

$$u_k(\boldsymbol{x}, \boldsymbol{\xi}, t) = -\frac{m}{4\pi\varkappa} \operatorname{Re}[e^{i\omega t} \partial_k F R, \omega)], \quad k = 1, 2, 3, \tag{3.10}$$

$$\theta(\boldsymbol{x}, \boldsymbol{\xi}, t) = -\frac{1}{4\pi\varkappa} \operatorname{Re}[e^{i\omega t} H(R, \omega)]. \tag{3.11}$$

For the elastic medium where the coupling of the deformation and temperature fields can be neglected, the formulae (3.10) and (3,11), with $\varepsilon = 0$, $k_1^2 = q$, $k_2^2 = -\sigma^2$, simplify considerably. In this case we have

$$u_k(\boldsymbol{x}, \boldsymbol{\xi}, t) = -\frac{m}{4\pi\varkappa R} \operatorname{Re}\left[\frac{e^{i\omega t}}{q+\sigma^2}(e^{-i\sigma R} - e^{-\sqrt{q}R})\right], \tag{3.12}$$

$$\theta(\boldsymbol{x}, \boldsymbol{\xi}, t) = \frac{1}{4\pi\varkappa R} \operatorname{Re}[e^{i\omega t - R\sqrt{q}}]. \tag{3.13}$$

If a uniformly distributed heat source of unit intensity is acting along the straight line passing through the point $(\boldsymbol{\xi})$, and parallel to the axis x_3, we obtain

$$\psi^* = -\frac{\mu}{2\pi\varrho c_1^2 \varkappa}[K_0(k_2 r) - K_0(k_1 r)], \tag{3.14}$$

where $r = [(x_1 - \xi_1)^2 + (x_2 - \xi_2)^2]^{1/2}$. The displacements and the temperature will be given by the formulae

$$u_j(\boldsymbol{x}, \boldsymbol{\xi}, t) = -\frac{m}{2\pi\varkappa} \operatorname{Re}\left\{\frac{e^{i\omega t}}{k_1^2 - k_2^2}[K_0(k_2 r) - K_0(k_1 r)]\right\}, \quad j = 1, 2, \tag{3.15}$$

$$\theta(\boldsymbol{x}, \boldsymbol{\xi}, t) = -\frac{1}{2\pi\varkappa} \operatorname{Re}\left\{\frac{e^{i\omega t}}{k_1^2 - k_2^2}[(k_2^2+\sigma^2) K_0(k_2 r) - (k_1^2+\sigma^2) K_0(k_1 r)]\right\}. \tag{3.16}$$

If no coupling between the deformation and temperature fields occurs ($\varepsilon = 0$, $k_1^2 = q$, $k_2^2 = -\sigma^2$), we have

$$u_j(\boldsymbol{x}, \boldsymbol{\xi}, t) = -\frac{m}{2\pi\varkappa} \operatorname{Re}\left\{\frac{e^{i\omega t}}{q+\sigma^2}[K_0(i\sigma r) - K_0(r\sqrt{q})]\right\}, \quad j = 1, 2, \tag{3.16}$$

$$\theta(\boldsymbol{x}, \boldsymbol{\xi}, t) = \frac{1}{2\pi\varkappa} \operatorname{Re}\{e^{i\omega t} K_0(r\sqrt{q})\}. \tag{3.17}$$

DEPARTMENT OF MECHANICS OF CONTINUOUS MEDIA, INSTITUTE OF FUNDAMENTAL TECHNICAL PROBLEMS, POLISH ACADEMY OF SCIENCES

(ZAKŁAD MECHANIKI OŚRODKÓW CIĄGŁYCH, INSTYTUT PODSTAWOWYCH PROBLEMÓW TECHNIKI, PAN)

REFERENCES

[1] W. Nowacki, *Green functions for the thermoelastic medium. I*, Bull. Acad. Polon. Sci., Sér. sci. techn., **12** (1964), 315 [391].

[2] V. Ionescu-Cazimir, *Aspura equaţiilor echilibrului thermoelastic. IV, Cazul plan*, Bull. Ştiintific, sect. de ştiinte matem. şi fizice, Acad. Rep. Pop. Române, **4** (1952), No. 3.

[3] S. Kaliski, *Pewne problemy dynamicznej teorii sprężystości i cial niesprężystych* [in Polish], [*Some dynamic problems of the theory of elasticity and inelastic bodies*], Warsaw, 1957.

[4] I. S. Podstrigach, *Zagalnyi rozviazok niestatsionarnoi zadachi termopruzhnosti* [in Ukrainian], [*General solution of the non-steady thermoelastic problem*], Prikl. Mech., Kiev, **6** (1960), No. 2.

[5] D. Rüdiger, *Bemerkung zur Integration der thermoelastischen Grundgleichungen*, Österr. Ing. Arch., **18** (1964), Nos. 1—2.

[6] P. Chadwick, I. N. Sneddon, *Plane waves in elastic solid conducting heat*, J. Mech. Phys. Solids, **6** (1958).

[7] W. Nowacki, *Some dynamic problems of thermoelasticity*, Arch. Mech. Stos., **11** (1959), No. 1959.

[8] M. A. Biot, *Thermoelasticity and irreversible thermodynamics*, J. of Appl. Phys., **27** (1956), No. 3.

В. НОВАЦКИЙ, **ФУНКЦИИ ГРИНА ДЛЯ НЕОГРАНИЧЕННОЙ ТЕРМОУПРУГОЙ СРЕДЫ. II.**

Во второй части работы приводятся в замкнутом виде функции перемещений и температуры Грина для случая воздействия точечных и линейных сосредоточенных сил, а также воздействия точечных и линейных источников тепла. Исходной точкой при решении этой проблемы были функции Галеркина, обобщенные на термоупругую среду.