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THE PLANE LAMB PROBLEM IN A SEMI-INFINITE MICROPOLAR ELASTIC BODY

W. NOWACKI and W. K. NOWACKI (WARSZAWA)

1. Introduction

The two-dimensional Lamb problem in the classical dynamic elasticity is formulated as follows [1]. It is required to determine the state of stress and strain in the elastic semi-space $x_1 \geqslant 0$ with the loaded boundary $x_1 = 0$, depending on the spatial variable x_2 and the time t.

In this paper we intend to solve the Lamb problem in the elastic micropolar medium. In the asymmetric elasticity the problem is rather complicated, for in such a medium the deformation is described by the displacement vector $\mathbf{u}$ and the rotation vector $\boldsymbol{\omega}$. We shall establish that the loadings in the generalized Lamb problem can be divided into two groups, the first generating the displacements $\mathbf{u} = (u_1, u_2, 0)$ and the rotations $\boldsymbol{\omega} = (0, 0, \omega_3)$ while the second, the displacements $\mathbf{u} = (0, 0, u_3)$ and the rotations $\boldsymbol{\omega} = (\omega_1, \omega_2, 0)$. To solve the Lamb problem we shall apply the double Fourier integral transform. In particular we shall examine the influence of loadings harmonic in time and we shall perform the transition from the two-dimensional to the one-dimensional problem.

2. The Fundamental Equations and Relations

The point of departure are the equations of the linear micropolar theory [2, 4]. We consider an elastic, homogeneous isotropic centro-symmetric body. Under the influence of external loadings there arises in the body the displacement field $\mathbf{u}(\mathbf{x}, t)$ and the rotation field $\boldsymbol{\omega}(\mathbf{x}, t)$, depending on the position of the point $\mathbf{x}$ and the time t.

The state of strain is determined by the non-symmetric strain tensor γ_{ji} and the curvature-twist tensor $\varkappa_{ji}$, where

$$\gamma_{ij} = u_{i,j} - \epsilon_{kji}\omega_k, \quad \varkappa_{ji} = \omega_{i,j}, \quad i, j, k = 1, 2, 3. \tag{2.1}$$

We apply here the tensorial index notation in the rectangular coordinate system; ϵ_{kji} denotes the unit antisymmetric tensor.

The state of stress is described by two non-symmetric tensors, the stress tensor σ_{ji} and the couple-stress tensor μ_{ji}.

The relations between the state of stress and the state of strain are linear; namely we have

$$\begin{aligned} \sigma_{ji} &= (\mu+\alpha)\gamma_{ji} + (\mu-\alpha)\gamma_{ij} + \lambda\gamma_{kk}\delta_{ij}, \\ \mu_{ji} &= (\gamma+\varepsilon)\varkappa_{ji} + (\gamma-\varepsilon)\varkappa_{ij} + \beta\varkappa_{kk}\delta_{ij}. \end{aligned} \tag{2.2}$$

The quantities μ, λ, α, β, γ, ε are material constants.

Inserting (2.2) into the equations of motion

$$\begin{aligned} &\sigma_{ji,j}+X_i = \varrho\ddot{u}_i, \\ &\epsilon_{ijk}\sigma_{jk}+\mu_{ji,j}+Y_i = J\ddot{\omega}_i \end{aligned} \tag{2.3}$$

and expressing the quantities γ_{ji}, $\varkappa_{ji}$ by the displacement u_i and the rotation ω_i, in accordance with the formulae (2.1) we arrive at a system of six differential equations, which we represent in the vectorial form

$$\begin{aligned} &(\mu+\alpha)\nabla^2\mathbf{u}+(\lambda+\mu-\alpha)\operatorname{grad}\operatorname{div}\mathbf{u}+2\alpha\operatorname{rot}\boldsymbol{\omega}+\mathbf{X} = \varrho\ddot{\mathbf{u}}, \\ &(\gamma+\varepsilon)\nabla^2\boldsymbol{\omega}+(\gamma+\beta-\varepsilon)\operatorname{grad}\operatorname{div}\boldsymbol{\omega}-4\alpha\boldsymbol{\omega}+2\alpha\operatorname{rot}\mathbf{u}+\mathbf{Y} = J\ddot{\boldsymbol{\omega}}. \end{aligned} \tag{2.4}$$

In Eqs. (2.3)–(2.4) $\mathbf{X}$ denotes the body force vector, $\mathbf{Y}$ the body couple vector, ϱ is the density and J the rotational inertia. The dots denote the time derivatives.

It is evident that Eqs. (2.4) are coupled. They become independent when $\alpha = 0$. In this case we have the following system of equations:

$$\begin{aligned} &\mu\nabla^2\mathbf{u}+(\lambda+\mu)\operatorname{grad}\operatorname{div}\mathbf{u}+\mathbf{X} = \varrho\ddot{\mathbf{u}}, \\ &(\gamma+\varepsilon)\nabla^2\boldsymbol{\omega}+(\gamma+\beta-\varepsilon)\operatorname{grad}\operatorname{div}\boldsymbol{\omega}+\mathbf{Y} = J\ddot{\boldsymbol{\omega}}. \end{aligned} \tag{2.5}$$

Equations $(2.5)_1$ are the displacement equations of the classical elasticity, while Eqs. $(2.5)_2$ describe a hypothetical elastic body in which only rotations occur.

Consider a particular case in which the external loadings, the body forces and moments and the vectors $\mathbf{u}$, $\boldsymbol{\omega}$ depend only on the variables x_1, x_2, t. In this particular case we are faced with two independent systems of equations

$$\begin{aligned} &(\mu+\alpha)\nabla^2 u_1+(\mu+\lambda-\alpha)\partial_1 e+2\alpha\partial_2\omega_3 = \varrho\ddot{u}_1, \\ &(\mu+\alpha)\nabla^2 u_2+(\mu+\lambda-\alpha)\partial_2 e-2\alpha\partial_1\omega_3 = \varrho\ddot{u}_2, \\ &(\gamma+\varepsilon)\nabla^2\omega_3-4\alpha\omega_3+2\alpha(\partial_1 u_2-\partial_2 u_1) = J\ddot{\omega}_3, \end{aligned} \tag{2.6}$$

and

$$\begin{aligned} &(\gamma+\varepsilon)\nabla^2\omega_1+(\gamma+\beta-\varepsilon)\partial_1\varkappa-4\alpha\omega_1+2\alpha\partial_2 u_3 = J\ddot{\omega}_1, \\ &(\gamma+\varepsilon)\nabla^2\omega_2+(\gamma+\beta-\varepsilon)\partial_2\varkappa-4\alpha\omega_2-2\alpha\partial_1 u_3 = J\ddot{\omega}_2, \\ &(\mu+\alpha)\nabla^2 u_3+2\alpha(\partial_1\omega_2-\partial_2\omega_1) = \varrho\ddot{u}_3. \end{aligned} \tag{2.7}$$

In the above equations the body forces and moments have been neglected ($X_i = Y_i = 0$); further, the following notations have been introduced:

$$\nabla^2 = \partial_1^2+\partial_2^2, \quad e = \partial_1 u_1+\partial_2 u_2, \quad \varkappa = \partial_1\omega_1+\partial_2\omega_2.$$

Consider first the system (2.6). The vectors $\mathbf{u} = (u_1, u_2, 0)$, $\boldsymbol{\omega} = (0, 0, \omega_3)$ are associated with the state of stress

$$\boldsymbol{\sigma} = \begin{vmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{21} & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{vmatrix}, \quad \boldsymbol{\mu} = \begin{vmatrix} 0 & 0 & \mu_{13} \\ 0 & 0 & \mu_{23} \\ \mu_{31} & \mu_{32} & 0 \end{vmatrix}. \tag{2.8}$$

Of course, the above stresses depend on the variables x_1, x_2 and the time t. According to the formulae (2.2) we obtain

$$\begin{aligned}
&\sigma_{11} = 2\mu\partial_1 u_1 + \lambda e, \quad \sigma_{22} = 2\mu\partial_2 u_2 + \lambda e, \quad \sigma_{33} = \lambda e,\\
&\sigma_{12} = \mu(\partial_1 u_2 + \partial_2 u_1) + \alpha(\partial_1 u_2 - \partial_2 u_1) - 2\alpha\omega_3, \qquad (2.9)\\
&\sigma_{21} = \mu(\partial_1 u_2 + \partial_2 u_1) - \alpha(\partial_1 u_2 - \partial_2 u_1) + 2\alpha\omega_3,
\end{aligned}$$

and

$$\begin{aligned}
&\mu_{13} = (\gamma+\varepsilon)\partial_1\omega_3, \quad \mu_{31} = (\gamma-\varepsilon)\partial_1\omega_3, \qquad (2.10)\\
&\mu_{23} = (\gamma+\varepsilon)\partial_2\omega_3, \quad \mu_{32} = (\gamma-\varepsilon)\partial_2\omega_3.
\end{aligned}$$

The system of Eqs. (2.7) is connected with the fields $\mathbf{u}(0, 0, u_3)$, $\boldsymbol{\omega} = (\omega_1, \omega_2, 0)$ and the state of stress

$$\boldsymbol{\sigma} = \begin{vmatrix} 0 & 0 & \sigma_{13} \\ 0 & 0 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & 0 \end{vmatrix}, \quad \boldsymbol{\mu} = \begin{vmatrix} \mu_{11} & \mu_{12} & 0 \\ \mu_{21} & \mu_{22} & 0 \\ 0 & 0 & \mu_{33} \end{vmatrix}. \qquad (2.11)$$

Furthermore,

$$\begin{aligned}
&\sigma_{13} = (\mu+\alpha)\partial_1 u_3 + 2\alpha\omega_2, \quad \sigma_{31} = (\mu-\alpha)\partial_1 u_3 - 2\alpha\omega_2,\\
&\sigma_{23} = (\mu+\alpha)\partial_2 u_3 - 2\alpha\omega_1, \quad \sigma_{32} = (\mu-\alpha)\partial_2 u_3 + 2\alpha\omega_1,
\end{aligned}$$

and

$$\begin{aligned}
&\mu_{11} = 2\gamma\partial_1\omega_1 + \beta\varkappa, \quad \mu_{22} = 2\gamma\partial_2\omega_2 + \beta\varkappa, \quad \mu_{33} = \beta\varkappa,\\
&\mu_{12} = \gamma(\partial_1\omega_2 + \partial_2\omega_1) + \varepsilon(\partial_1\omega_2 - \partial_2\omega_1).\\
&\mu_{21} = \gamma(\partial_1\omega_2 + \partial_2\omega_1) - \varepsilon(\partial_1\omega_2 - \partial_2\omega_1).
\end{aligned}$$

The components of the stress tensor (2.12) and the couple-stress tensor (2.13) are functions of the variables x_1, x_2 and the time t.

3. General Solution of the Eqs. (2.6)

The system of Eqs. (2.6) contains coupled functions u_1, u_2 and ω_3. The state of stress (2.8) indicates that the functions u_1, u_2, ω_3 may be generated by forces and moments on the boundary $x_1 = 0$, namely by normal and tangential forces on the boundary and a moment the vector of which lies along the x_3-axis. These loadings are connected with the state of stress through the boundary conditions

$$\begin{aligned}
&\sigma_{11}(0, x_2, t) = -f_1(x_2, t), \quad \sigma_{12}(0, x_2, t) = -f_2(x_2, t), \qquad (3.1)\\
&\mu_{13}(0, x_2, t) = -f_3(x_2, t).
\end{aligned}$$

Here $f_1 > 0$ is the normal loading directed along the positive x_1-axis, $f_2 > 0$ is the tangential loading lying in the plane $x_1 = 0$ and directed parallelly to the $+x_2$-axis. Finally $f_3 > 0$ is a moment the vector of which is parallel to the $+x_3$-axis.

Let us introduce the elastic potentials Φ, Ψ connected with the displacements u_1, u_2 by means of the relations

$$u_1 = \partial_1\Phi - \partial_2\Psi, \quad u_2 = \partial_2\Phi + \partial_1\Psi.$$

Inserting (3.2) into the system of equations (2.6) we arrive at the wave equations

$$
\begin{aligned}
&\left(\nabla^2-\frac{1}{c_1^2}\partial_t^2\right)\Phi = 0,\\
(3.3)\qquad &\left(\nabla^2-\frac{1}{c_2^2}\partial_t^2\right)\Psi - p\omega_3 = 0,\\
&\left(\nabla^2-\nu^2-\frac{1}{c_4^2}\partial_t^2\right)\omega_3 + s\nabla^2\Psi = 0,
\end{aligned}
$$

where we have introduced the notations

$$
c_1=\left(\frac{\lambda+2\mu}{\varrho}\right)^{1/2},\quad c_2=\left(\frac{\mu+\alpha}{\varrho}\right)^{1/2},\quad c_4=\left(\frac{\gamma+\varepsilon}{J}\right)^{1/2},\quad \nu^2=\frac{4\alpha}{\gamma+\varepsilon},
$$

$$
p=\frac{2\alpha}{\mu+\alpha},\quad s=\frac{2\alpha}{\gamma+\varepsilon},
$$

Eliminating Ψ or ω_3 from $(3.3)_2$ and $(3.3)_3$ we obtain the wave equations

$$
\begin{aligned}
(3.4)\qquad &\left[\left(\nabla^2-\frac{1}{c_2^2}\partial_t^2\right)\left(\nabla^2-\nu^2-\frac{1}{c_4^2}\partial_t^2\right)+\zeta^2\nabla^2\right](\Psi,\omega_3)=0,\\
&\zeta^2=\frac{4\alpha^2}{(\gamma+\varepsilon)(\mu+\alpha)}.
\end{aligned}
$$

Performing in $(3.3)_1$ and (3.4) the double Fourier integral transform defined by the formulae [5]

$$
\begin{aligned}
(3.5)\qquad &\tilde{\Phi}(x_1,\xi,\eta)=\frac{1}{2\pi}\int\int_{-\infty}^{\infty}\Phi(x_1,x_2,t)\exp[i(x_2\xi+\eta t)]\,dx_2\,dt,\\
&\Phi(x_1,x_2,t)=\frac{1}{2\pi}\int\int_{-\infty}^{\infty}\tilde{\Phi}(x_1,\xi,\eta)\exp[-i[(x_2\xi+\eta t)]\,d\xi\,d\eta,
\end{aligned}
$$

we obtain the following system of ordinary differential equations with the independent variable x_1:

$$
(3.6)\qquad (\partial_1^2-\xi^2+\sigma_1^2)\tilde{\Phi}=0,\quad (\partial_1^2-\lambda_1^2)(\partial_1^2-\lambda_2^2)(\tilde{\Psi},\tilde{\omega}_3)=0.
$$

Here

$$
\begin{aligned}
&(\lambda_1^2-\xi^2)+(\lambda_2^2-\xi^2)=\nu^2-\zeta^2-\sigma_2^2-\sigma_4^2,\\
&(\lambda_1^2-\xi^2)(\lambda_2^2-\xi^2)=\sigma_2^2(\sigma_4^2-\nu^2),\\
&\sigma_1=\frac{\eta}{c_1},\quad \sigma_2=\frac{\eta}{c_2},\quad \sigma_4=\frac{\eta}{c_4}.
\end{aligned}
$$

We assume that the loading acting on the boundary is bounded. Under this assumption the functions Φ, Ψ, ω_3 should tend to zero as $|x_1^2+x_2^2|\to\infty$. Consequently the solution of Eqs. (3.6) is taken in the form

$$\tilde{\Phi} = Ae^{-\delta x_1},$$

$$(3.7) \qquad \tilde{\Psi} = Be^{-\lambda_1 x_1} + Ce^{-\lambda_2 x_1},$$

$$\tilde{\omega}_3 = B'e^{-\lambda_1 x_1} + C'e^{-\lambda_2 x_1}, \quad \delta = (\xi^2 - \sigma_1^2)^{1/2}.$$

The quantities B' and B and C', C are related by means of Eq. $(3.3)_2$ or $(3.3)_3$.

Equating the coefficients of $e^{-\lambda_1 x_1}$ and $e^{-\lambda_2 x_1}$, we obtain from Eq. $(3.3)_2$ the relations

$$(3.8) \qquad B' = \varkappa_1 B, \quad C' = \varkappa_2 C,$$

where

$$\varkappa_1 = \frac{1}{p}(\lambda_1^2 - \xi^2 + \sigma_2^2), \quad \varkappa_2 = \frac{1}{p}(\lambda_2^2 - \xi^2 + \sigma_2^2).$$

It remains to calculate A, B and C. To this end we make use of the boundary conditions (3.1) which we represent in the form

$$|2\mu\partial_1 u_1 + \lambda e|_{x_1=0} = -f_1(x_2, t),$$

$$(3.9) \qquad |\mu(\partial_1 u_2 + \partial_2 u_1) + \alpha(\partial_1 u_2 - \partial_2 u_1) - 2\alpha\omega_3|_{x_1=0} = -f_2(x_2, t),$$

$$|(\gamma+\varepsilon)\partial_1 \omega_3|_{x_1=0} = -f_3(x_2, t).$$

These equations may be expressed in terms of the potentials Φ, Ψ. Performing the double Fourier integral transform we have

$$|2\mu(\partial_1^2\tilde{\Phi} + i\xi\partial_1\tilde{\Psi}) + \lambda(\partial_1^2 - \xi^2)\tilde{\Phi}|_{x_1=0} = -\tilde{f}_1(\xi, \eta),$$

$$(3.9') \qquad |\mu[(\partial_1^2 + \xi^2)\tilde{\Psi} - 2i\xi\partial_1\tilde{\Phi}] + \alpha(\partial_1^2 - \xi^2)\tilde{\Psi} - 2\alpha\tilde{\omega}_3|_{x_1=0} = -\tilde{f}_2(\xi, \eta),$$

$$|(\gamma+\varepsilon)\partial_1\tilde{\omega}_3|_{x_1=0} = -\tilde{f}_3(\xi, \eta).$$

Inserting the relations (3.7) into (3.9′), we are led to a system of three non-homogeneous linear equations for the quantities A, B and C. Thus, we have

$$A = \alpha_{11}\tilde{f}_1 + \alpha_{12}\tilde{f}_2 + \alpha_{13}\tilde{f}_3,$$

$$(3.10) \qquad B = \alpha_{21}\tilde{f}_1 + \alpha_{22}\tilde{f}_2 + \alpha_{23}\tilde{f}_3,$$

$$C = \alpha_{31}\tilde{f}_1 + \alpha_{32}\tilde{f}_2 + \alpha_{33}\tilde{f}_3,$$

where

$$\alpha_{11} = \frac{1}{\Delta}(a_1\varkappa_2\lambda_2 - a_2\varkappa_1\lambda_1), \qquad \alpha_{12} = \frac{(\varkappa_2 - \varkappa_1)}{\Delta} 2\mu i\xi\lambda_1\lambda_2,$$

$$\alpha_{13} = \frac{2\mu i\xi}{(\gamma+\varepsilon)\Delta}(\lambda_1 a_2 - \lambda_2 a_1), \qquad \alpha_{21} = -\frac{2\mu i\xi\varkappa_2\lambda_2\delta}{\Delta},$$

$$\alpha_{22} = [(2\mu+\lambda)\delta^2 - \lambda\xi^2]\frac{\varkappa_2\lambda_2}{\Delta}, \qquad \alpha_{23} = \frac{(a_2 + 4\mu^2\xi^2\lambda_2\delta)[(2\mu+\lambda)\delta^2 - \lambda\xi^2]}{(\gamma+\varepsilon)\Delta},$$

$$\alpha_{31} = \frac{2\mu i\xi\varkappa_1\lambda_1\delta}{\Delta}, \qquad \alpha_{32} = -\frac{[(2\mu+\lambda)\delta^2 - \lambda\xi^2]\varkappa_1\lambda_1}{\Delta},$$

$$\alpha_{33} = -\{4\mu^2\xi^2\lambda_1\delta - [(2\mu+\lambda)\delta^2 - \xi^2\lambda]a_1\}\frac{1}{(\gamma+\varepsilon)\Delta},$$

and

$$\Delta = 4\mu^2\xi^2\delta(\varkappa_2-\varkappa_1)\lambda_1\lambda_2+(\varkappa_1 a_2\lambda_1-\varkappa_2 a_1\lambda_2)[(2\mu+\lambda)\delta^2-\lambda\xi^2],$$

$$a_j = (\lambda_j^2+\xi^2)\mu+(\lambda_j^2-\xi^2)\alpha-2\alpha\varkappa_j, \quad j = 1, 2.$$

Inverting the Fourier transform in the expressions $(3.7)_1$ and $(3.7)_2$, we obtain the formulae

$$(3.11) \qquad \begin{aligned} \Phi &= \frac{1}{2\pi}\int\limits_{-\infty}^{\infty}\int A(\xi,\eta)e^{-\delta x_1}\exp[-i(\xi x_2+\eta t)]d\xi d\eta, \\ \Psi &= \frac{1}{2\pi}\int\limits_{-\infty}^{\infty}\int (Be^{-\lambda_1 x_1}+Ce^{-\lambda_2 x_1})\exp[-i(\xi x_2+\eta t)]d\xi d\eta; \end{aligned}$$

A, B and C are given by (3.11).

Making use of the relations (3.2) and $(3.7)_3$ we obtain the formulae for the displacements u_1, u_2 and the rotation ω_3:

$$(3.12) \qquad \begin{aligned} u_1 &= -\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}\int [\delta Ae^{-\delta x_1}-i\xi(Be^{-\lambda_1 x_1}+Ce^{-\lambda_2 x_1})]\exp[-i(x_2\xi+\eta t)]d\xi d\eta, \\ u_2 &= -\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}\int [i\xi Ae^{-\delta x_1}+(\lambda_1 Be^{-\lambda_1 x_1}+\lambda_2 Ce^{-\lambda_2 x_1}]\exp[-i(x_2\xi+\eta t)]d\xi d\eta, \\ \omega_3 &= \frac{1}{2\pi}\int\limits_{-\infty}^{\infty}\int [\varkappa_1 Be^{-\lambda_1 x_1}+\varkappa_2 Ce^{-\lambda_2 x_1}]\exp[-i(x_2\xi+\eta t)]d\xi d\eta. \end{aligned}$$

Knowing the displacements and the rotations we can calculate the deformations from the formulae (2.1) and the stresses from (2.9) and (2.10).

Consider now the particular case $\alpha = 0$. It is evident that here Eqs. (2.6) become independent. We obtain the system of equations

$$(3.13) \qquad \begin{aligned} \mu\nabla^2 u_1+(\mu+\lambda)\partial_1 e &= \varrho\ddot{u}_1, \\ \mu\nabla^2 u_2+(\mu+\lambda)\partial_2 e &= \varrho\ddot{u}_2, \\ (\gamma+\varepsilon)\nabla^2\omega_3 &= J\ddot{\omega}_3, \end{aligned}$$

and the wave Eqs. (3.7) take the form

$$(3.14) \qquad \begin{aligned} &\left(\nabla^2-\frac{1}{c_1^2}\partial_t^2\right)\Phi = 0, \quad \left(\nabla^2-\frac{1}{\hat{c}_2^2}\partial_t^2\right)\Psi = 0, \\ &\left(\nabla^2-\frac{1}{c_4^2}\partial_t^2\right)\omega_3 = 0, \quad \hat{c}_2 = \left(\frac{\mu}{\varrho}\right)^{1/2}. \end{aligned}$$

The solutions of these equations satisfying the condition $(\Phi, \Psi, \omega_3) \to 0$ as $|x_1^2+x_2^2| \to \infty$ (after having performed the Fourier integral transform), are the following:

(3.15)
$$\tilde{\Phi} = A^\circ e^{-\delta x_1}, \quad \delta = (\xi^2-\sigma_1^2)^{1/2},$$
$$\tilde{\Psi} = B^\circ e^{-\lambda_0 x_1}, \quad \lambda_0 = (\xi^2-\sigma_2^2)^{1/2},$$
$$\tilde{\omega}_3 = C^\circ e^{-\gamma_0 x_1}, \quad \gamma_0 = (\xi^2-\sigma_4^2)^{1/2}.$$

The constants A°, B°, C° are to be determined from the boundary conditions (3.10), in which we set $\alpha = 0$. Thus, we have

(3.16)
$$A^\circ = \alpha_{11}^0 \tilde{f}_1 + \alpha_{12}^0 \tilde{f}_2, \quad B^\circ = \alpha_{21}^0 \tilde{f}_1 + \alpha_{22}^0 \tilde{f}_2, \quad C^\circ = \alpha_{33}^0 \tilde{f}_3,$$

where

$$\alpha_{11}^0 = \frac{\mu(\xi^2+\lambda_0^2)}{\Delta_0}, \quad \alpha_{12}^0 = -\frac{2\mu i\xi\lambda_0}{\Delta_0},$$

$$\alpha_{21}^0 = \frac{2\mu i\xi\delta}{\Delta_0}, \quad \alpha_{22}^0 = -\frac{2\mu\delta^2-\lambda(\xi^2-\delta^2)}{\Delta_0},$$

$$\alpha_{33}^0 = \frac{1}{\gamma_0(\gamma+\varepsilon)}, \quad \Delta_0 = [2\mu\delta^2-\lambda(\delta^2-\xi^2)](\lambda_0^2+\xi^2)-4\mu^2\xi^2\delta\lambda_0.$$

Hence,

$$u_1 = -\frac{1}{2\pi}\int\int_{-\infty}^{\infty} [A^\circ \delta e^{-\delta x_1} - i\xi B^\circ e^{-\lambda_0 x_1}]\exp[-i(x_2\xi+\eta t)]d\xi d\eta,$$

$$u_2 = -\frac{1}{2\pi}\int\int_{-\infty}^{\infty} [i\xi A^\circ e^{-\delta x_1} + \lambda_0 B^\circ e^{-\lambda_0 x_1}]\exp[-i(x_2\xi+\eta t)]d\xi d\eta,$$

$$\omega_3 = \frac{1}{2\pi}\int\int_{-\infty}^{\infty} C^\circ e^{-\gamma_0 x_1}\exp[-i(x_2\xi+\eta t)]d\xi d\eta.$$

Observe that the displacements u_1, u_2 can be generated only by the loadings $\sigma_{11}(0, x_2, t) = -f_1(x_2, t)$ and $\sigma_{12}(0, x_2, t) = -f_2(x_2, t)$. The moment loading $\mu_{13}(0, x_2, t) = -f_3(x_2, t)$ produces the rotation ω_3. The formulae (3.17) for u_1 and u_2 describe the classical elastic body, while the formula $(3.17)_3$ refers to a hypothetical body in which only rotations and couple-stresses may exist.

4. General Solution of Eqs. (3.7)

In Eqs. (2.7) the displacement and rotation fields are described by the vectors $\mathbf{u} = (0, 0, u_3)$ and $\boldsymbol{\omega} = (\omega_1, \omega_2, 0)$. The state of stress (2.11) indicates that this field may arise only under the influence of loadings expressed by the boundary conditions

(4.1)
$$\mu_{11}(0, x_2, t) = -l_1(x_2, t), \quad \mu_{12}(0, x_2, t) = -l_2(x_2, t),$$
$$\sigma_{13}(0, x_2, t) = -l_3(x_2, t).$$

Here $f_1 > 0$ represents the moments with vectors parallel to the $+x_1$-axis, while $f_2 > 0$ describes moments the vectors of which are parallel to $+x_2$; $f_3 > 0$ is the tangential loading lying in the plane $x_1 = 0$, parallel to the $+x_3$-axis.

Introducing the potentials connected with the rotations by the formulae

$$\omega_1 = \partial_1\varphi - \partial_2\psi, \quad \omega_2 = \partial_2\varphi + \partial_1\psi, \tag{4.2}$$

we obtain from (2.7) the system of wave equations

$$\begin{aligned} &\left(\nabla^2 - \nu_0^2 - \frac{1}{c_3^2}\partial_t^2\right)\varphi = 0,\\ &\left(\nabla^2 - \nu^2 - \frac{1}{c_4^2}\partial_t^2\right)\psi - su_3 = 0,\\ &\left(\nabla^2 - \frac{1}{c_2^2}\partial_t^2\right)u_3 + p\nabla^2\psi = 0; \end{aligned} \tag{4.3}$$

here we have introduced the notations

$$\nu_0^2 = \frac{4\alpha}{2\gamma+\beta}, \quad c_3 = \left(\frac{2\gamma+\beta}{J}\right)^{1/2}, \quad s = \frac{2\alpha}{\gamma+\varepsilon}, \quad p = \frac{2\alpha}{\mu+\alpha}.$$

Eliminating ψ (or u_3) from Eqs. $(4.3)_2$ and $(4.3)_3$, we arrive at the wave equation

$$\left[\left(\nabla^2 - \frac{1}{c_2^2}\partial_t^2\right)\left(\nabla^2 - \nu^2 - \frac{1}{c_4^2}\partial_t^2\right) + \zeta^2\nabla^2\right](\psi, u_3) = 0, \tag{4.4}$$

identical with Eq. (3.4).

Let us perform in $(4.3)_1$ and (4.4) a double Fourier integral transform, defined by the formulae (3.5). We arrive then at a system of ordinary equations

$$(\partial_1^2 - \xi^2 - \nu_0^2 + \sigma_3^2)\tilde{\varphi} = 0, \quad (\partial_1^2 - \lambda_1^2)(\partial_1^2 - \lambda_2^2)(\tilde{\psi}, \tilde{u}_3) = 0, \tag{4.5}$$

where

$$\sigma_3 = \eta/c_3,$$

and λ_1^2, λ_2^2 have the same meaning as in Eq. (3.6). The solution of (4.5) has the form

$$\begin{aligned} &\tilde{\varphi} = Ae^{-\sigma x_1}, \quad \sigma = (\xi^2 + \nu^2 - \sigma_3^2)^{1/2},\\ &\tilde{\psi} = Be^{-\lambda_1 x_1} + Ce^{-\lambda_2 x_1},\\ &\tilde{u}_3 = B'e^{-\lambda_1 x_1} + C'e^{-\lambda_2 x_1}. \end{aligned} \tag{4.6}$$

The above functions satisfy the condition $(\varphi, \psi, u_3) \to 0$ as $|x_1^2 + x_2^2| \to \infty$. The quantities B', B are not independent, similarly to C' and C. Introducing $\tilde{\varphi}$ and $\tilde{u}_3$ into Eq. $(4.3)_3$, we obtain the relations

$$B' = \hat{\varkappa}_1 B, \quad C' = \hat{\varkappa}_2 C, \tag{4.7}$$

where

$$\hat{\varkappa}_1 = \frac{p(\xi^2 - \lambda_1^2)}{\lambda_1^2 + \sigma_2^2 - \xi^2}, \quad \hat{\varkappa}_2 = \frac{p(\xi^2 - \lambda_2^2)}{\lambda_2^2 + \sigma_2^2 - \xi^2}.$$

Let us now express the boundary conditions (4.1) by the rotations ω_1, ω_2 and the displacement u_3, and the latter by the potentials φ and ψ. Thus, we have

$$
\begin{aligned}
&|2\gamma(\partial_1^2\varphi-\partial_1\partial_2\psi)+\beta\nabla^2\varphi|_{x_1=0} = -l_1(x_2, t),\\
(4.8)\qquad &|\gamma[2\partial_1\partial_2\varphi+(\partial_1^2-\partial_2^2)\psi]+\varepsilon\nabla^2\psi|_{x_1=0} = -l_2(x_2, t),\\
&|(\mu+\alpha)\partial_1 u_3+2\alpha(\partial_2\varphi+\partial_1\psi)|_{x_1=0} = -l_3(x_2, t).
\end{aligned}
$$

Let us now apply to the boundary conditions the Fourier integral transform, making use of the relations (4.6). After some calculations we are led to a system of three equations for the determination of the quantities A, B and C. Thus, we have

$$
\begin{aligned}
&A = \beta_{11}\tilde{l}_1+\beta_{12}\tilde{l}_2+\beta_{13}\tilde{l}_3,\\
(4.9)\qquad &B = \beta_{21}\tilde{l}_1+\beta_{22}\tilde{l}_2+\beta_{23}\tilde{l}_3,\\
&C = \beta_{31}\tilde{l}_1+\beta_{32}\tilde{l}_2+\beta_{33}\tilde{l}_3,
\end{aligned}
$$

where

$$
\begin{aligned}
&\beta_{11} = \frac{d_2e_1-d_1e_2}{\Delta}, \qquad \beta_{12} = \frac{-2\gamma i\xi}{\Delta}(d_1\lambda_2-d_2\lambda_1),\\
&\beta_{13} = \frac{2\gamma i\xi}{\Delta}(\lambda_1e_2-\lambda_2e_1), \qquad \beta_{21} = \frac{2i\xi}{\Delta}(\alpha e_2-\gamma\sigma d_2),\\
&\beta_{22} = -\frac{4\alpha\gamma\xi^2\lambda_2-[\beta\xi^2-(2\gamma+\beta)\sigma^2]d_2}{\Delta},\\
(4.10)\qquad &\beta_{23} = -\frac{4\gamma^2\xi^2\sigma\lambda_2+[\beta\xi^2-(2\gamma+\beta)\sigma^2]e_2}{\Delta},\\
&\beta_{31} = -\frac{2\xi i}{\Delta}(\alpha e_1-\gamma\sigma d_1),\\
&\beta_{32} = \frac{4\alpha\xi^2\lambda_1+[\beta\xi^2-(2\gamma+\beta)\sigma^2]d_1}{\Delta},\\
&\beta_{33} = \frac{4\gamma^2\xi^2\lambda_1\sigma+[\beta\xi^2-(2\gamma+\beta)\sigma^2]e_1}{\Delta}
\end{aligned}
$$

and the following notations have been introduced:

$$
\begin{aligned}
&\Delta = 4\alpha\gamma\xi^2(\lambda_2e_1-\lambda_1e_2)+4\gamma^2\sigma\xi^2(\lambda_1d_2-\lambda_2d_1)-[\beta\xi^2+(2\gamma+\beta)\sigma^2](d_1e_2-d_2e_1),\\
(4.11)\qquad &d_1 = [2\alpha+(\mu+\alpha)\hat{\varkappa}_1]\lambda_1, \quad d_2 = [2\alpha+(\mu+\alpha)\hat{\varkappa}_2]\lambda_2,\\
&e_j = \gamma(\lambda_j^2+\xi^2)+\varepsilon(\lambda_j^2-\xi^2), \quad j = 1,2.
\end{aligned}
$$

Inverting the integral transform in $(4.6)_1$ and $(4.6)_2$, we have

$$
\begin{aligned}
&\varphi = \frac{1}{2\pi}\iint_{-\infty}^{\infty} Ae^{-\sigma x_1}\exp[-i(x_2\xi+\eta t)]\,d\xi\,d\eta,\\
(4.12)\qquad &\psi = \frac{1}{2\pi}\iint_{-\infty}^{\infty} (Be^{-\lambda_1x_1}+Ce^{-\lambda_2x_1})\exp[-i(x_2\xi+\eta t)]\,d\xi\,d\eta.
\end{aligned}
$$

Making use of the formulae (4.2) and the relation $(4.6)_3$ we finally obtain

$$\omega_1 = -\frac{1}{2\pi}\int\int_{-\infty}^{\infty}[\sigma Ae^{-\sigma x_1}-i\xi(Be^{-\lambda x_1}+Ce^{-\lambda_2 x_1})]\exp[-i(x_2\xi+\eta t)]d\xi d\eta,$$

$$\omega_2 = -\frac{1}{2\pi}\int\int_{-\infty}^{\infty}[i\xi Ae^{-\sigma x_1}+(\lambda_1 Be^{-\lambda_1 x_1}+\lambda_2 Ce^{-\lambda_2 x_1})]\exp[-i(x_2\xi+\eta t)]d\xi d\eta, \tag{4.13}$$

$$u_3 = \frac{1}{2\pi}\int\int_{-\infty}^{\infty}[\hat{\varkappa}_1 Be^{-\lambda_1 x_1}+\hat{\varkappa}_2 Ce^{-\lambda_2 x_1}]\exp[-i(x_2\xi+\eta t)]d\xi d\eta.$$

Knowing the functions ω_1, ω_2, u_3, we can determine the components of the state of stress σ_{ji}, μ_{ji} on the basis of the formulae (2.12) and (2.13).

Consider now the particular case $\alpha = 0$, which is characterized by independent equations (2.7):

$$(\gamma+\varepsilon)\nabla^2\omega_1+(\gamma+\beta-\varepsilon)\partial_1\varkappa = J\ddot{\omega}_1,$$

$$(\gamma+\varepsilon)\nabla^2\omega_2+(\gamma+\beta-\varepsilon)\partial_2\varkappa = J\ddot{\omega}_2, \tag{4.14}$$

$$\mu\nabla^2 u_3 = \varrho\ddot{u}_3.$$

The first two equations concern the hypothetical medium in which the deformation is described by the rotation angles ω_1 and ω_2. Equation $(4.14)_3$ describes the classical elastic medium; it constitutes the equation for a shear wave.

Introducing the potentials φ and ψ in accordance with the formulae (4.2) we obtain independent wave equations

$$\left(\nabla^2-\frac{1}{\partial_3^2}\partial_t^2\right)\varphi = 0, \quad \left(\nabla^2-\frac{1}{c_4^2}\partial_t^2\right)\psi = 0,$$

$$\left(\nabla^2-\frac{1}{\hat{c}_2^2}\partial_t^2\right)u_3 = 0, \quad \hat{c}_2 = \left(\frac{\mu}{\varrho}\right)^{1/2} \tag{4.15}$$

Performing the double Fourier integral transform and solving the resulting equations bearing in mind the condition $(\varphi, \psi\ \ u_3) \to 0$ as $|x_1^2+x_2^2| \to \infty$ we have

$$\tilde{\varphi} = A^0e^{-\sigma_0 x_1}, \quad \tilde{\psi} = B^0e^{-\gamma_0 x_1} \quad \tilde{u}_3 = C^0e^{-\lambda_0 x_1}. \tag{4.16}$$

where

$$\sigma_0 = (\xi^2-\sigma_3^2)^{1/2}, \quad \gamma_0 = (\xi^2-\sigma_4^2)^{1/2}, \quad \lambda_0 = (\xi^2-\hat{\sigma}_2^2)^{1/2}.$$

The constants A^0, B^0, C^0 are to be determined from the boundary conditions (4.8) in which we set $\alpha = 0$. Thus, we obtain

$$A^0 = \beta_{11}^0\tilde{l}_1+\beta_{12}^0\tilde{l}_2, \quad B^0 = \beta_{21}^0\tilde{l}_1+\beta_{22}^0\tilde{l}_2, \quad C^0 = \beta_{33}^0\tilde{l}_3, \tag{4.17}$$

where

$$\beta^0_{11} = -\frac{\gamma(\xi^2+\gamma_0^2)+\varepsilon(\gamma_0^2-\xi^2)}{\Delta_0}, \quad \beta^0_{12} = -\frac{2\gamma\gamma_0 i\xi}{\Delta_0},$$

$$\beta^0_{21} = \frac{2i\gamma\xi\sigma_0}{\Delta_0}, \quad \beta^0_{22} = -\frac{2\gamma\sigma_0^2+\beta(\sigma_0^2-\xi^2)}{\Delta_0}, \quad \beta^0_{33} = \frac{1}{\mu\lambda_0},$$

$$\Delta_0 = [2\gamma\sigma_0^2+\beta(\sigma_0^2-\xi^2)]\,[\gamma(\xi^2+\gamma_0^2)+\varepsilon(\gamma_0^2-\xi^2)]-4\gamma^2\gamma_0\xi^2\sigma_0.$$

Hence

$$\omega_1 = -\frac{1}{2\pi}\iint\limits_{-\infty}^{\infty}[\sigma_0 A^0 e^{-\sigma_0 x_1}-i\xi B^0 e^{-\gamma_0 x_1}]\exp[-i(\xi x_2+\eta t)]\,d\xi\,d\eta,$$

(4.18)
$$\omega_2 = -\frac{1}{2\pi}\iint\limits_{-\infty}^{\infty}[i\xi A^0 e^{-\sigma_0 x_1}+\gamma_0 B^0 e^{-\gamma_0 x_1}]\exp[-i(\xi x_2+\eta t)]\,d\xi\,d\eta,$$

$$u_3 = \frac{1}{2\pi}\iint\limits_{-\infty}^{\infty}C^0 e^{-\lambda_0 x_1}\exp[-i(x_2\xi+\eta t)]\,d\xi\,d\eta.$$

The formulae (4.17) and (4.18) indicate that the rotations ω_1, ω_2 are produced by an action of the moments $l_1(x_2, t)$, $l_2(x_2, t)$ on the boundary $x_1 = 0$. The displacement $u_3(x_1, x_2, t)$ is connected with an action of the loading $\sigma_{13}(0, x_2, t) = -l_3(x_2, t)$.

Observe that the relations (4.17)–(4.18) can also be derived from the formulae (4.9)-(4.13) by carrying out the transition $\alpha \to 0$.

5. Action of a Loading Harmonic in Time

We shall now examine a particular case of the loading, namely the loading harmonic n time. Assume first that the boundary conditions have the form

(5.1)
$$\sigma_{11}(0, x_2, t) = -f_1(x_2)e^{-i\omega t}, \quad \sigma_{12}(0, x_2, t) = 0, \quad \mu_{13}(0, x_2, t) = 0,$$

i.e. the loading is normal to the boundary $x_1 = 0$. This loading produces in the elastic semi-space the displacements u_1, u_2 and the rotation ω_3. In view of the formulae (3.11) and (3.12) we have

$$u_1 = -\frac{1}{2\pi}\iint\limits_{-\infty}^{\infty}[\alpha_{11}\delta e^{-\delta x_1}-i\xi(\alpha_{21}e^{-\lambda_1 x_1}+\alpha_{31}e^{-\lambda_2 x_1})]\tilde{f}_1\exp[-i(x_2\xi+\eta t)]\,d\xi\,d\eta,$$

(5.2)
$$u_2 = -\frac{1}{2\pi}\iint\limits_{-\infty}^{\infty}[i\xi\alpha_{11}e^{-\delta x_1}+(\lambda_1\alpha_{21}e^{-\lambda_1 x_1}+\lambda_2\alpha_{31}e^{-\lambda_2 x_1})]\tilde{f}_1\exp[-i(x_2\xi+\eta t)]\,d\xi\,d\eta,$$

$$\omega_3 = \frac{1}{2\pi}\iint\limits_{-\infty}^{\infty}[\varkappa_1\alpha_{21}e^{-\lambda_1 x_1}+\varkappa_2\alpha_{31}e^{-\lambda_2 x_1}]\tilde{f}_1\exp[-i(x_2\xi+\eta t)]\,d\xi\,d\eta,$$

where

(5.3)
$$\tilde{f}_1(\xi,\eta)=\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}\int f_1(x_2)e^{-i\omega t}\exp[i(x_2\xi+\eta t)]\,dx_2\,dt,$$
$$=\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}f_1(x_2)e^{ix_2\xi}dx_2\int\limits_{-\infty}^{\infty}e^{-it(\omega-\eta)}dt.$$

Introducing the notations

$$\tilde{f}_1^*(\xi)=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty}f_1(x_2)e^{ix_2\xi}dx_2,$$

and making use of the known relation [5]

(5.4)
$$\int\limits_{-\infty}^{\infty}e^{ith}dt=2\pi\delta(h),$$

where $\delta(\ )$ denotes the Dirac function [7], we obtain

(5.5)
$$\tilde{f}_1(\xi,\eta)=\sqrt{2\pi}\,\delta(\eta-\omega)\tilde{f}_1^*(\xi).$$

Substituting now (5.5) into (5.2) and performing the required integration we arrive at the formulae

(5.6)
$$u_1=-\frac{e^{-i\omega t}}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty}[\alpha_{11}\delta e^{-\delta x_1}-i\xi(\alpha_{21}e^{\lambda_1x_1}+\alpha_{31}e^{-\lambda_2x_1})]_{\eta=\omega}\tilde{f}_1^*(\xi)e^{-ix_2\xi}d\xi,$$
$$u_2=-\frac{e^{-i\omega t}}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty}[i\xi\alpha_{11}e^{-\delta x_1}+(\lambda_1\alpha_{21}e^{-\lambda_1x_1}+\lambda_2\alpha_{31}e^{-\lambda_2x_1})]_{\eta=\omega}\tilde{f}_1^*(\xi)e^{-ix_2\xi}d\xi,$$
$$\omega_3=\frac{e^{-i\omega t}}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty}[\varkappa_1\alpha_{21}e^{-\lambda_1x_1}+\varkappa_2\alpha_{31}e^{-\lambda_2x_1}]_{\eta=\omega}\tilde{f}^*(\xi)e^{-ix_2\xi}d\xi.$$

In the expression in the parenthesis η should be replaced by ω. For instance, we have

$$|\alpha_{21}|_{\eta=\omega}=-\left|\frac{2\mu i\xi\delta\varkappa_2\lambda_2}{\Delta}\right|_{\eta=\omega}.$$

The expressions δ, λ_2, $\varkappa_2$, Δ contain the quantities

$$\sigma_1=\frac{\eta}{c_1},\qquad\sigma_2=\frac{\eta}{c_2},\qquad\sigma_4=\frac{\eta}{c_3}.$$

In all these quantities η has to be replaced by ω. In particular, the expression

(5.7)
$$\Delta_{\eta=\omega}=|4\mu^2\xi^2\delta(\varkappa_2-\varkappa_1)\lambda_1\lambda_2+(\varkappa_1a_2\lambda_1-\varkappa_2a_1\lambda_2)\,[(2\mu+\lambda)\,\delta^2-\lambda\xi^2]|_{\eta=\omega}=0,$$

may be regarded as the condition of existence of surface waves in the elastic semi-space. In fact, if we consider the homogeneous system of Eqs. (3.10) for monochromatic vibrations, the condition of consistency of this system is the vanishing of its determinant, which leads to Eq. (5.7).

Equation (5.7) was derived in the paper [6].

Consider now the particular case $\alpha = 0$. In view of the formulae (3.15) and (3.16) we have

$$\Phi = \frac{1}{2\pi} \int\int_{-\infty}^{\infty} \alpha_{11}^0 e^{-\delta x_1} \tilde{f}_1(\xi, \eta) \exp[-i(x_2\xi + \eta t)] d\xi d\eta, \tag{5.8}$$

$$\Psi = \frac{1}{2\pi} \int\int_{-\infty}^{\infty} \alpha_{21}^0 e^{-\lambda_0 x_1} \tilde{f}_1(\xi, \eta) \exp[-i(x_2\xi + \eta t)] d\xi d\eta.$$

Inserting (5.5) into (5.8) and denoting by star the amplitudes of the potentials, we have

$$\Phi^* = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} |\alpha_{11}^0 e^{-\delta x_1}|_{\eta=\omega} \tilde{f}_1^*(\xi) e^{-ix_2\xi} d\xi, \tag{5.9}$$

$$\Psi^* = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} |\alpha_{21}^0 e^{-\lambda_0 x_1}|_{\eta=\omega} \tilde{f}_1^*(\xi) e^{-ix_2\xi} d\xi,$$

where

$$\alpha_{11}^0|_{\eta=\omega} = \frac{\mu\left(2\xi^2 - \frac{\omega^2}{c_2^2}\right)}{D}, \quad \alpha_{21}^0|_{\eta=\omega} = \frac{2\mu i\xi\left(\xi^2 - \frac{\omega^2}{c_1^2}\right)^{1/2}}{D},$$

$$D = \left[2\mu\left(\xi^2 - \frac{\omega^2}{c_1^2}\right) - \lambda\frac{\omega^2}{c_1^2}\right]\left(2\xi^2 - \frac{\omega^2}{\hat{c}_2^2}\right) - 4\mu^2\xi^2\left(\xi^2 - \frac{\omega^2}{c_1^2}\right)^{1/2}\left(\xi^2 - \frac{\omega^2}{\hat{c}_2^2}\right)^{1/2}.$$

For the particular case of a concentrated force $f_1(x_2, t) = P\delta(x_2)e^{-i\omega t}$ acting at the origin of the coordinate system we obtain [1]

$$\Phi^* = -\frac{P}{\mu\pi} \int_0^{\infty} \frac{2\xi\nu_2}{R(\xi)} e^{-\nu_1 x_1} \sin \xi x_2 d\xi, \tag{5.10}$$

$$\Psi^* = -\frac{P}{\mu\pi} \int_0^{\infty} \frac{\left(2\xi^2 - \frac{\omega^2}{c_2^2}\right)}{R(\xi)} e^{-\nu_2 x_1} \cos \xi x_2 d\xi,$$

where

$$R(\xi) = \left(2\xi^2 - \frac{\omega^2}{\hat{c}_2^2}\right) - 4\xi^2\sqrt{\xi^2 - \omega^2/c_1^2}\,\sqrt{\xi^2 - \omega^2/\hat{c}_2^2}.$$

Let us now examine a loading varying harmonically in time and defined by the boundary conditions

$$\mu_{11}(0, x_2, t) = -l_1(x_2)e^{-i\omega t}, \quad \mu_{12}(0, x_2, t) = 0, \quad \mu_{13}(0, x_2, t) = 0. \tag{5.11}$$

We are faced here with the loading of the plane $x_1 = 0$ by moments the vectors of which are parallel to the $+x_1$-axis. This loading generates in the body the displacement u_3 and the rotations ω_1, ω_2. Bearing in mind the formulae (4.13) we have

$$\omega_1 = -\frac{1}{2\pi}\int\int_{-\infty}^{\infty}[\sigma\beta_{11}e^{-\sigma x_1} - i\xi(\beta_{21}e^{-\lambda_1 x_1}+\beta_{31}e^{-\lambda_2 x_1})]\tilde{l}_1(\xi,\eta)\exp[-i(x_2\xi+\eta t)]d\xi\, d\eta,$$

(5.12)
$$\omega_2 = -\frac{1}{2\pi}\int\int_{-\infty}^{\infty}[i\xi\beta_{11}e^{-\sigma x_1}+(\lambda_1\beta_{21}e^{-\lambda_1 x_1}+\lambda_2\beta_{31}e^{-\lambda_2 x_1})]\times \tilde{l}_1(\xi,\eta)\exp[-i(\xi x_2+\eta t)]d\xi\, d\eta,$$

$$u_3 = \frac{1}{2\pi}\int\int_{-\infty}^{\infty}[\hat{\varkappa}_1\beta_{21}e^{-\lambda_1 x_1}+\hat{\varkappa}_2\beta_{31}e^{-\lambda_2 x_1}]\tilde{l}_1(\xi,\eta)\exp[-i(\xi x_2+\eta t)]d\xi\, d\eta.$$

Similarly to (5.5) we have here

(5.13)
$$\tilde{l}_1(\xi,\eta) = \sqrt{2\pi}\delta(\eta-\omega)\tilde{l}_1^*(\xi),$$

where

$$\tilde{l}_1^*(\xi) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} l_1(x_2)e^{ix_2\xi}dx_2.$$

Substituting (5.13) into (5.12) and performing the required integration, we obtain

(5.14)
$$\omega_1 = -\frac{e^{-i\omega t}}{\sqrt{2\pi}}\int_{-\infty}^{\infty}[\sigma\beta_{11}e^{-\sigma x_1}-i\xi(\beta_{21}e^{-\lambda_1 x_1}+\beta_{31}e^{-\lambda_2 x_1})]_{\eta=\omega}\tilde{l}_1^*(\xi)e^{-ix_2\xi}d\xi,$$

$$\omega_2 = -\frac{e^{-i\omega t}}{\sqrt{2\pi}}\int_{-\infty}^{\infty}[i\xi\beta_{11}e^{-\sigma x_1}+(\lambda_1\beta_{21}e^{-\lambda_1 x_1}+\lambda_2\beta_{31}e^{-\lambda_2 x_1})]_{\eta=\omega}\tilde{l}_1^*(\xi)e^{-ix_2\xi}d\xi,$$

$$u_3 = \frac{e^{-i\omega t}}{\sqrt{2\pi}}\int_{-\infty}^{\infty}[\hat{\varkappa}_1\beta_{21}e^{-\lambda_1 x_1}+\hat{\varkappa}_2\beta_{31}e^{-\lambda_2 x_1}]_{\eta=\omega}\tilde{l}^*(\xi)e^{-ix_2\xi}d\xi.$$

In particular, the condition

$$|\beta_{21}|_{\eta=\omega} = \left|\frac{2i\xi(\alpha e_2-\gamma\sigma d_2)}{\Delta}\right|_{\eta=\omega}$$

means that in the expressions for e_2, d_2, σ and Δ the parameter η is to be replaced by ω.

Obviously, in the expressions (5.14), $\Delta_{\eta=\omega} \neq 0$.

If we seek the solution of the system (2.7) assuming that the boundary conditions (4.1) are homogeneous, then Eqs. (4.8) are homogeneous and lead to homogeneous linear equations for the quantities A, B and C. Equating to zero the determinant, i.e. requiring that the system be consistent, we arrive at the equation

(5.15)
$$\Delta(\xi,\omega) = 0,$$

where $\Delta(\xi,\omega)$ is given by the formula (4.11).

Equation (5.15) is identical with the characteristic equation for Love waves occurring in the elastic micropolar semi-space [6].

Consider the particular case $\alpha = 0$. Taking into account the boundary conditions (5.11) and bearing in mind the relations (4.16) we have

(5.16)
$$\varphi = \frac{e^{-i\omega t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} |\beta_{11}^0 e^{-\sigma_0 x_1}|_{\eta=\omega} \tilde{l}_1^*(\xi) e^{-i\xi x_2} d\xi,$$
$$\psi = \frac{e^{-i\omega t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} |\beta_{21}^0 e^{-\gamma_0 x_1}|_{\eta=\omega} \tilde{l}_1^* e^{-i\xi x_2} d\xi.$$

Here

$$\beta_{11}^0\big|_{\eta=\omega} = -\frac{2\gamma\xi^2 - (\gamma+\varepsilon)\omega^2/c_4^2}{D_0}, \quad \beta_{21}^0\big|_{\eta=\omega} = \frac{2i\gamma\xi(\xi^2-\omega^2/c_3^2)^{1/2}}{D_0},$$

$$D_0 = \gamma^2\left[\left(2\xi^2 - \frac{\omega^2}{\hat{c}_4^2}\right)^2 - 4\xi^2\left(\xi^2 - \frac{\omega^2}{c_3^2}\right)^{1/2}\left(\xi^2 - \frac{\omega^2}{c_4^2}\right)^{1/2}\right],$$
$$\hat{c}_4 = (\gamma/J)^{1/2}.$$

The formulae

(5.17)
$$\omega_1 = \partial_1\varphi - \partial_2\psi, \quad \omega_2 = \partial_2\varphi + \partial_1\psi$$

make it possible to calculate the rotation field $\boldsymbol{\omega} = (\omega_1, \omega_2, 0)$ in the hypothetical elastic medium in which no displacement can occur.

6. One-dimensional Problem of Wave Propagation in the Elastic Semi-space

Assume that all factors generating deformation depend on the variables x_1 and t only. In this case all components of the displacement vector and the rotation vector also depend only on x_1 and t.

Assume first that the following boundary conditions are prescribed on the plane $x_1 = 0$:

(6.1)
$$\sigma_{11}(0, x_2, t) = -f_1(t), \quad \sigma_{12}(0, x_2, t) = -f_2(t), \quad \mu_{13}(0, x_2, t) = -f_3(t).$$

The displacements $u_1(x_1, t)$, $u_2(x_1, t)$, $\omega_3(x_1, t)$ corresponding to this loading can be calculated from the formulae (3.12) for the two-dimensional problem. Let us first calculate the quantity $\tilde{f}_1(\xi, \eta)$ appearing in the quantities A, B and C [see (3.11)]. We have here

$$\tilde{f}_1(\xi, \eta) = \frac{1}{2\pi} \int\int_{-\infty}^{\infty} f_1(t) e^{i(x_2\xi+\eta t)} dx_2\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} f_1(t) e^{i\eta t} dt \int_{-\infty}^{\infty} e^{ix_2\xi} dx_2.$$

Since

$$\int_{-\infty}^{\infty} e^{ix_2\xi} dx_2 = 2\pi\delta(\xi),$$

we obtain

$$\tilde{f}_1(\xi,\eta)=\delta(\xi)\sqrt{2\pi}\bar{f}_1(\eta),\quad \bar{f}_1(\eta)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(t)e^{i\eta t}dt. \tag{6.2}$$

The formula (3.11) can be represented in the form

$$\begin{aligned}A&=[\alpha_{11}\bar{f}_1(\eta)+\alpha_{12}\bar{f}_2(\eta)+\alpha_{13}\bar{f}_3(\eta)]\sqrt{2\pi}\delta(\xi),\\ B&=[\alpha_{21}\bar{f}_1(\eta)+\alpha_{22}\bar{f}_2(\eta)+\alpha_{23}\bar{f}_3(\eta)]\sqrt{2\pi}\delta(\xi),\\ C&=[\alpha_{31}\bar{f}_1(\eta)+\alpha_{32}\bar{f}_2(\eta)+\alpha_{33}\bar{f}_3(\eta)]\sqrt{2\pi}\delta(\xi).\end{aligned} \tag{6.3}$$

Substituting the above quantities into (3.12) and performing the integration we arrive at the expressions

$$\begin{aligned}u_1&=-\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}[\delta\alpha_{11}e^{-\delta x_1}]_{\xi=0}\bar{f}_1(\eta)e^{-i\eta t}d\eta,\\ u_2&=-\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}[\lambda_1e^{-\lambda_1x_1}(\alpha_{22}\bar{f}_2+\alpha_{23}\bar{f}_3)+\lambda_2e^{-\lambda_2x_1}(\alpha_{32}\bar{f}_2+\alpha_{33}\bar{f}_3)]_{\xi=0}e^{-i\eta t}d\eta,\\ \omega_3&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}[\varkappa_1(\alpha_{22}\bar{f}_2+\alpha_{23}\bar{f}_3)e^{-\lambda_1x_1}+\varkappa_2(\alpha_{32}\bar{f}_2+\alpha_{33}\bar{f}_3)e^{-\lambda_2x_2}]_{\xi=0}e^{-i\eta t}d\eta.\end{aligned} \tag{6.4}$$

Since

$$[\delta\alpha_{11}]_{\xi=0}=\left|(\xi^2-\sigma_1^2)^{1/2}\frac{a_1\varkappa_2\lambda_2-a_2\varkappa_1\lambda_1}{\Delta}\right|_{\xi=0}=\frac{i}{c_1\varrho\eta},$$

we have

$$u_1=-\frac{i}{\varrho\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{\bar{f}_1(\eta)}{\eta}e^{i(\sigma_1x_1-\eta t)}d\eta. \tag{6.5}$$

The quantities $a_1, a_2, \varkappa_1, \varkappa_2, \lambda_1, \lambda_2, \Delta$ in the above formulae are to be taken from the formulae (3.6). In (6.4) for u_2 and ω_3. we have

$$\lambda_1\big|_{\xi=0}=r_1.\quad \lambda_2\big|_{\xi=0}=r_2,\quad \alpha_{22}\big|_{\xi=0}=\frac{r_2\varrho_2}{D},\quad \alpha_{23}\big|_{\xi=0}=\frac{b_2}{D(\gamma+\varepsilon)},$$

$$\alpha_{33}\big|_{\xi=0}=-\frac{b_1}{D(\gamma+\varepsilon)},\quad \alpha_{32}=-\frac{r_1\varrho_1}{D},\quad D=\varrho_1r_1b_2-\varrho_2r_2b_1,$$

where

$$\begin{aligned}r_{1,2}^2&=-\frac{1}{2}[\sigma_2^2+\sigma_4^2-\nu^2\pm\sqrt{(\sigma_2^2+\sigma_4^2-\nu^2)^2+4\sigma_2^2(\nu^2-\sigma_4^2)}],\\ b_j&=r_j^2(\mu+\alpha)-2\alpha\varrho_j,\quad j=1,2,\\ \varrho_j&=\frac{1}{p}(r_j^2+\sigma_2^2),\qquad j=1,2.\end{aligned}$$

Assume now that in the considered one-dimensional problem we deal with loadings varying harmonically in time. Thus, the conditions (6.1) take the form

$$\sigma_{11}(0, x_2, t) = -f_1^0 e^{-i\omega t}, \quad \sigma_{12}(0, x_2, t) = -f_2^0 e^{-i\omega t}, \quad \mu_{13}(0, x_2, t) = -f_3^0 e^{-i\omega t} \tag{6.6}$$

where f_1^0, f_2^0, f_3^0 are constants. Following the procedure of Sec. 5 we insert into the relations (6.4) the quantities

$$\tilde{f}_\alpha(\eta) = \sqrt{2\pi}\,\delta(\eta-\omega) f_\alpha^0, \quad \alpha = 1, 2.$$

For instance, from (6.4) we obtain

$$u_1(x_1, t) = \frac{i f_1^0}{c_1 \varrho \omega} e^{-i\omega\left(t - \frac{x_1}{c_1}\right)}. \tag{6.7}$$

Let us now return to the general one-dimensional problem and consider the boundary conditions

$$\mu_{11}(0, x_2, t) = -l_1(t), \quad \mu_{12}(0, x_2, t) = -l_2(t), \quad \mu_{13}(0, x_2, t) = -l_3(t). \tag{6.8}$$

The rotations $\omega_1(x_1, t)$, $\omega_2(x_1, t)$ and the displacements $u_3(x_1, t)$ generated in the semi-space can be calculated from the formulae (4.14), taking into account that

$$\tilde{l}_j(\xi, \eta) = \sqrt{2\pi}\,\delta(\xi)\bar{l}_j(\eta), \quad \bar{l}_j(\eta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} l_j(t) e^{i\eta t} dt, \quad j = 1, 2, 3. \tag{6.9}$$

Representing the constants A, B and C entering (4.9) in the form

$$\begin{aligned} A &= [\beta_{11}\bar{l}_1(\eta) + \beta_{12}\bar{l}_2(\eta) + \beta_{13}\bar{l}_3(\eta)]\sqrt{2\pi}\,\delta(\xi), \\ B &= [\beta_{21}\bar{l}_1(\eta) + \beta_{22}\bar{l}_2(\eta) + \beta_{23}\bar{l}_3(\eta)]\sqrt{2\pi}\,\delta(\xi), \\ C &= [\beta_{31}\bar{l}_1(\eta) + \beta_{32}\bar{l}_2(\eta) + \beta_{33}\bar{l}_3(\eta)]\sqrt{2\pi}\,\delta(\xi), \end{aligned} \tag{6.10}$$

and inserting (6.10) into (4.13) we obtain

$$\begin{aligned} \omega_1 &= -\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [\sigma\beta_{11} e^{-\sigma x_1}]_{\xi=0} \bar{l}_1(\eta) e^{-i\eta t} d\eta, \\ \omega_2 &= -\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [\lambda_1(\beta_{22}\bar{l}_2 + \beta_{23}\bar{l}_3) e^{-\lambda_1 x_1} + \lambda_2(\beta_{32}\bar{l}_2 + \beta_{33}\bar{l}_3) e^{-\lambda_2 x_1}]_{\xi=0} e^{-i\eta t} d\eta, \\ \omega_3 &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [\hat{\varkappa}_1(\beta_{22}\bar{l}_2 + \beta_{23}\bar{l}_3) e^{-\lambda_1 x_1} + \hat{\varkappa}_2(\beta_{32}\bar{l}_2 + \beta_{33}\bar{l}_3) e^{-\lambda_2 x_1}]_{\xi=0} e^{-i\eta t} d\eta. \end{aligned} \tag{6.11}$$

The quantities $\beta_{11}, \ldots, \beta_{33}$ are given by (4.10). In these formulae we set $\xi = 0$. We have

$$\begin{aligned} &\beta_{12}\big|_{\xi=0} = \beta_{13}\big|_{\xi=0} = \beta_{31}\big|_{\xi=0} = \beta_{21}\big|_{\xi=0} = 0. \\ &\beta_{11}\big|_{\xi=0} = \frac{1}{(\beta+2\gamma)(\gamma^2-\sigma_3^2)}, \quad \beta_{22}\big|_{\xi=0} = \frac{\eta_2 r_2}{N}, \\ &\beta_{23}\big|_{\xi=0} = -\frac{p_2}{N}, \quad \beta_{32}\big|_{\xi=0} = \frac{\eta_1 r_1}{N}, \quad \beta_{33}\big|_{\xi=0} = -\frac{p_1}{N}, \end{aligned} \tag{6.12}$$

where

$$r_1 = |\lambda_1|_{\xi=0}, \quad r_2 = |\lambda_2|_{\xi=0}, \quad r_{1,2}^2 = -\frac{1}{2}[\sigma_2^2+\sigma_4^2-\nu^2 \pm \sqrt{(\sigma_2^2+\sigma_4^2-\nu^2)+4\sigma_2^2(\nu^2-\sigma_4^2)}],$$

$$n_j = 2\alpha+(\mu+\alpha)\eta_j, \quad p_j = (\gamma+\varepsilon)r_j^2,$$

$$n_j = -\frac{pr_j^2}{r_j^2+\sigma_2^2}, \qquad j = 1, 2,$$

$$N = n_1 r_1 p_2 - n_2 r_2 p_1.$$

In the particular case of forced vibrations varying harmonically in time on the boundary $x_1 = 0$, we obtain for instance from the boundary condition $\mu_{11}(0, x_2, t) = -l_1^0 e^{-i\omega t}$ the expression

$$\tilde{l}_1(\eta) = \sqrt{2\pi}\,\delta(\eta-\omega) l_1^0.$$

Substituting the above expression into (6.11) and bearing in mind the value of $\beta_{11}|_{\xi=0}$ in (6.12) we finally obtain

$$\omega_1(x, t) = \frac{il_1^0 \exp\left[-i\omega\left(t-\frac{x_1}{c_3}\sqrt{1-\frac{r_0^2 c_3^2}{\omega^2}}\right)\right]}{Jc_3\sqrt{\omega^2-\omega_0^2}}. \tag{6.13}$$

This is a wave propagated in the direction of the x_1-axis, undergoing a dispersion. The wave exists when $\omega^2 > r_0^2 c_3^2 = \omega_0^2$ or when $\omega^2 > 4\alpha/J$. Consider now the case $\alpha = 0$ in which a total seperation of the plane waves in the elastic semi-space occurs. Here we obtain the system of wave equations

$$\left(\partial_1^2 - \frac{1}{c_1^2}\partial_t^2\right) u_1(x_1, t) = 0,$$

$$\left(\partial_1^2 - \frac{1}{c_3^2}\partial_t^2 - \nu_0^2\right) \omega_1(x_1, t) = 0,$$

(6.14)

$$\left(\partial_1^2 - \frac{1}{\hat{c}_2^2}\partial_t^2\right) (u_2, u_3) = 0,$$

$$\left(\partial_1^2 - \frac{1}{c_4^2}\partial_t^2\right) (\omega_2, \omega_3) = 0, \quad \hat{c}_2 = \left(\frac{\mu}{\varrho}\right)^{1/2}.$$

Bearing in mind the conditions (6.1) and (6.8), we represent the solution of the above equations in the form

$$u_1 = \frac{i}{\varrho\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\tilde{f}_1(\eta)}{c_1\eta} \exp[i\sigma_1 x_1 - i\eta t]\, d\eta,$$

$$u_2 = \frac{i}{c_2\varrho\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\tilde{f}_2(\eta)}{\eta} \exp(i\sigma_1 x_1 - i\eta t)\, d\eta,$$

(6.15)
$$\omega_3 = \frac{i}{Jc_4\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\tilde{f}_3(\eta)}{\eta} \exp[i\sigma_4 x_1 - i\eta t] d\eta,$$

$$\omega_1 = \frac{i}{c_3 J\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\tilde{l}_1(\eta)}{\eta} \exp[i\sigma_3 x_1 - i\eta t] d\eta,$$

$$\omega_2 = \frac{i}{Jc_4\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\tilde{l}_2(\eta)}{\eta} \exp[i(\sigma_4 x_1 - \eta t)] d\eta,$$

$$u_3 = \frac{i}{\varrho c_2\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\tilde{l}_3(\eta)}{\eta} \exp[i(\sigma_2 x_1 - \eta t)] d\eta.$$

where the following notations have been introduced:

$$\sigma_1 = \frac{\eta}{c_1}, \quad \sigma_2 = \frac{\eta}{c_2}, \quad \sigma_3 = \frac{\eta}{c_3}, \quad \sigma_4 = \frac{\eta}{c_4}.$$

Consider now forced vibrations varying harmonically in time; the conditions (6.6) and the boundary conditions

(6.16) $\mu_{11}(0, x_2, t) = -l_1^0 e^{-i\omega t}, \quad \mu_{12}(0, x_2, t) = -l_2^0 e^{-i\omega t}, \quad u_3(0, x_2, t) = -l_3^0 e^{-i\omega t},$

lead to the following expressions deduced from (5.15):

$$u_1 = \frac{if_1^0}{\varrho c_1 \omega} e^{-i\omega\left(t - \frac{x_1}{c_1}\right)}, \quad u_2 = \frac{if_2^0}{\varrho c_2 \omega} e^{-i\omega\left(t - \frac{x_1}{c_2}\right)},$$

$$u_3 = \frac{il_3^0}{\varrho c_2 \omega} e^{-i\omega\left(t - \frac{x_1}{c_2}\right)}, \quad \omega_1 = \frac{il_1^0}{Jc_3 \omega} e^{-i\omega\left(t - \frac{x_1}{c_3}\right)};$$

$$\omega_2 = \frac{il_2^0}{Jc_4 \omega} e^{-i\omega\left(t - \frac{x_1}{c_4}\right)}, \quad \omega_3 = \frac{if_3^0}{Jc_4 \omega} e^{-i\omega\left(t - \frac{x_1}{c_4}\right)}.$$

Observe that the waves u_1, u_2, u_3 concern the classical elastic medium, whereas the waves ω_1, ω_1, ω_3 occur in the hypothetical medium, in which the particles of the body cannot undergo any displacements.

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Streszczenie

PŁASKIE ZAGADNIENIE LAMBA W MIKROPOLARNEJ PÓŁPRZESTRZENI SPRĘŻYSTEJ

W pracy przedstawiono rozwiązanie tzw. zagadnienia dwuwymiarowego Lamba w sprężystej mikropolarnej półprzestrzeni sprężystej. Obciążenia (siłami i momentami) na brzegu $x_1 = 0$ rozdzielono na dwie grupy, z których jedna wywołuje związane z sobą przemieszczenie u_1 i u_2 obrót ω_3, a druga obroty ω_1, ω_2 i przemieszczenie u_3. Do rozwiązania zagadnienia użyto podwójnej transformacji całkowej Fouriera. Rozważono jako przypadek szczególny działanie obciążenia zmiennego w sposób harmoniczny w czasie. Wreszcie rozpatrzono zagadnienie jednowymiarowe.

Резюме

ПЛОСКАЯ ЗАДАЧА ЛАМБА В МИКРОПОЛЯРНОМ УПРУГОМ ПОЛУПРОСТРАНСТВЕ

В работе дается решение так наз. двумерной задачи Ламба в упругом микрополярном полупространстве. Нагрузки (силами и моментами) на краю $x_1 = 0$, разделены на две группы, одна из которых вызывает связанные с собой перемещения u_1, u_2 и оборот ω_3, а вторая обороты ω_1, ω_2 и перемещение u_3. При решении задачи использовалось двойное интегральное преобразование Фурье. В качестве особого случая рассматривается действие, изменяющейся гармонически во времени нагрузки. Наконец рассматривается одномерная задача.

INSTITUTE OF FUNDAMENTAL TECHNICAL RESEARCH
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